



Pre-IB AI Mathematics

Structured Summer Preparation

for Future AI SL Mathematics Students

A five-week bridge from IGCSE Mathematics into IB Mathematics: Applications and Interpretation SL.

Learning goal	Review the most important IGCSE foundations so that students enter IB Mathematics AI SL with stronger number fluency, modelling confidence, graph sense, geometric reasoning, statistical literacy and probability skills.
Syllabus bridge	IGCSE Mathematics foundations leading into IB Mathematics AI SL: number and algebra, functions as models, geometry and trigonometry, transformations, statistics, probability and technology-supported interpretation.
Big idea	AI students need to move confidently between calculations, diagrams, tables, graphs, models and written interpretations. The emphasis is not only on getting answers, but on explaining what those answers mean in context.
How to use this pack	Complete one week at a time. Each practice section contains 48 varied questions , moving from skills to contexts and interpretation.

Week 1
Number and percentages

Week 2
Algebra and sequences

Week 3
Graphs and models

Week 4
Geometry and trigonometry

Week 5
Statistics and probability

Pre-IB boundary. This pack is designed to rebuild fluency before the IB course begins. The examples and questions are deliberately contextual, graphical and interpretive because those habits matter strongly in Mathematics: Applications and Interpretation SL.



Contents



Teacher overview: scope and weekly structure

Week	Focus	Main purpose
1	Number fluency and proportional reasoning	Secure percentages, ratio, rates, standard form, indices, bounds and financial calculations.
2	Algebra and sequences	Review algebraic manipulation, equations, formulae, inequalities, simultaneous equations, sequences and proportional models.
3	Coordinate geometry and functions as models	Build graph sense, straight lines, transformations, function notation, regression language and model interpretation.
4	Geometry, mensuration, trigonometry and transformations	Review diagrams, measurement, right and non-right triangle methods, bearings, transformations and simple vectors.
5	Statistics, histograms and probability	Strengthen data handling, grouped data, box plots, scatter diagrams, regression interpretation, probability, Venn diagrams and tree diagrams.

Suggested pace. Students should spend about 2–3 focused sessions on each week: one for notes and examples, one for practice, and one for checking corrections.



Week 1: Number fluency, percentages, standard form and indices

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Percentages, proportional reasoning, financial mathematics, rates, bounds, standard form and index laws.

Key results

- Percentage multiplier: increase by $r\%$ means multiply by $1 + \frac{r}{100}$.
- Percentage multiplier: decrease by $r\%$ means multiply by $1 - \frac{r}{100}$.
- Compound growth: $A = P(1 + r)^n$ when r is written as a decimal.
- Standard form: $a \times 10^n$ where $1 \leq a < 10$.
- Average speed = $\frac{\text{distance}}{\text{time}}$.
- Density = $\frac{\text{mass}}{\text{volume}}$.

reverse percentages

After a 20% discount, a jacket costs €96. A 20% discount means the sale price is 80% of the original price.

$$0.8 \times \text{original} = 96$$

$$\text{original} = \frac{96}{0.8} = 120.$$

Answer: €120.

Practice questions: Week 1

There are 48 varied questions in this section. Show working where appropriate and write interpretations in context.

- Convert 0.375 to a fraction in simplest form and to a percentage.
- Convert $\frac{7}{20}$ to a decimal and to a percentage.
- Find 18% of 350.
- Increase 240 by 12%.
- Decrease 680 by 15%.
- After a 20% discount, a jacket costs €96. Find the original price.
- A price of €80 is increased by 10% and then decreased by 10%. Find the final price.
- €1500 is invested at 4% compound interest per year for 3 years. Find the final value.
- €900 is invested at 3% simple interest per year for 5 years. Find the final amount.
- Simplify the ratio 42 : 56 : 70.
- Divide 720 in the ratio 2 : 3 : 4.
- Which is better value: 400 g for €2.20 or 650 g for €3.38? Justify your answer.
- A car travels 126 km in 1 hour 45 minutes. Find its average speed in km/h.
- A material has mass 468 g and volume 180 cm³. Find its density in g/cm³.
- An exchange rate is £0.86 for €1. Convert €300 to pounds.
- A map has scale 1 : 50000. A route measures 6.4 cm on the map. Find the actual distance in km.
- Write 0.0000482 in standard form.
- Write 6.31×10^4 as an ordinary number.
- Evaluate $(3.2 \times 10^5)(4 \times 10^{-3})$, giving the answer in standard form.



20. Evaluate $\frac{7.5 \times 10^6}{2.5 \times 10^2}$, giving the answer in standard form.
21. Estimate $\frac{49.6 \times 19.7}{0.51}$ by rounding each number to 1 significant figure.
22. Round 0.007846 to 2 significant figures.
23. Round 89450 to 2 significant figures.
24. A length is recorded as 12.6 cm to the nearest 0.1 cm. Write the lower and upper bounds.
25. A distance is recorded as 480 m to the nearest 10 m. Write the lower and upper bounds.
26. Simplify $2^5 \times 2^{-3}$.
27. Simplify $\frac{(a^3)^4}{a^5}$.
28. Expand and simplify $(3x^2y)^2$.
29. Evaluate $81^{1/2} + 27^{1/3}$.
30. Simplify $\sqrt{50}$.
31. 5% of a number is 18. Find the number.
32. Which is larger: 14% of 80 or 18% of 60?
33. A 20% sales tax is added to €48. Find the price including tax.
34. A price increases from €75 to €90. Find the percentage increase.
35. A population decreases from 500 to 420. Find the percentage decrease.
36. A population of 24000 grows by 3% each year for 2 years. Find the new population.
37. A car worth €8500 depreciates by 12% each year for 3 years. Find its value after 3 years.
38. A car uses 42 litres of fuel to travel 630 km. Find the fuel economy in km per litre.
39. Petrol costs €1.84 per litre. Find the cost of 36.5 litres.
40. A rectangle has length 12.4 cm and width 7.8 cm, each correct to the nearest 0.1 cm. Find the upper bound for its area.
41. Evaluate $3.6 \times 10^{-4} + 2.1 \times 10^{-4}$ in standard form.
42. Evaluate $0.00072 \div (3 \times 10^{-5})$.
43. Simplify $\frac{6x^3y^{-2}}{3x^{-1}y}$.
44. y is directly proportional to x . When $x = 6$, $y = 18$. Find y when $x = 14$.
45. y is inversely proportional to x . When $x = 3$, $y = 20$. Find y when $x = 12$.
46. For the model $C = 8 + 0.45n$, where C is cost in euros and n is the number of pages printed, interpret the numbers 8 and 0.45.
47. A taxi costs €3.50 plus €1.80 per kilometre. Write a formula for the cost C for d km and find the cost for 12 km.
48. A bank balance changes by +120, -35, -18 and +60 euros. Find the overall change.

Answer key: Week 1

1. $\frac{3}{8} = 37.5\%$.
2. $0.35 = 35\%$.
3. 63.
4. 268.80.
5. 578.
6. €120.
7. €79.20.
8. €1687.30 to the nearest cent.
9. €1035.



10. $3 : 4 : 5$.
11. 160, 240, 320.
12. 650 g for $e3.38$; it is about $e5.20$ per kg, compared with $e5.50$ per kg.
13. 72 km/h.
14. 2.6 g/cm^3 .
15. £258.
16. 3.2 km.
17. 4.82×10^{-5} .
18. 63100.
19. 1.28×10^3 .
20. 3×10^4 .
21. About 2000.
22. 0.0078.
23. 89000.
24. $12.55 \leq x < 12.65$.
25. $475 \leq d < 485$.
26. 4.
27. a^7 .
28. $9x^4y^2$.
29. 12.
30. $5\sqrt{2}$.
31. 360.
32. 14% of 80 is larger: $11.2 > 10.8$.
33. $e57.60$.
34. 20%.
35. 16%.
36. 25461.6, so about 25462 people.
37. $e5794.62$ to the nearest cent.
38. 15 km/litre.
39. $e67.16$.
40. 97.7325 cm^2 .
41. 5.7×10^{-4} .
42. 24.
43. $\frac{2x^4}{y^3}$.
44. 42.
45. 5.
46. 8 is the fixed cost; 0.45 is the cost per page.
47. $C = 3.50 + 1.80d$; $C = e25.10$ when $d = 12$.
48. $+e127$.



Week 2: Algebra, equations, inequalities and sequences

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Manipulating expressions, solving equations, rearranging formulae, arithmetic/geometric sequences and proportional models.

Key results

- Expand brackets carefully before collecting like terms.
- Factorising reverses expanding.
- When solving inequalities, reversing the sign is needed when multiplying or dividing by a negative.
- Arithmetic sequences add a constant difference.
- Geometric sequences multiply by a constant ratio.
- A formula is useful because it describes a relationship for many possible inputs.

a sequence as a simple model

A subscription costs €25 to join and then €12 per month. A model for the cost after h months is

$$C = 25 + 12h.$$

If the total cost is €97, then

$$25 + 12h = 97 \Rightarrow 12h = 72 \Rightarrow h = 6.$$

Answer: The subscription has lasted 6 months.

Practice questions: Week 2

There are 48 varied questions in this section. Show working where appropriate and write interpretations in context.

1. Evaluate $3x^2 - 2x + 5$ when $x = -2$.
2. Simplify $4a + 3b - 2a + 5b$.
3. Expand and simplify $5(x - 3) + 2(2x + 1)$.
4. Expand $(x + 7)(x - 2)$.
5. Factorise $6x + 15$.
6. Factorise $x^2 + 9x + 20$.
7. Factorise $2x^2 + 7x + 3$.
8. Solve $4x - 7 = 21$.
9. Solve $3(2x - 5) = 27$.
10. Solve $\frac{x + 4}{3} = 6$.
11. Solve $5x + 2 = 2x - 10$.
12. Solve $x^2 - 7x + 12 = 0$.
13. Solve $2x^2 - 5x - 3 = 0$.
14. Rearrange $P = 2l + 2w$ to make l the subject.
15. Rearrange $V = \pi r^2 h$ to make h the subject.
16. Rearrange $y = 3x - 4$ to make x the subject.
17. Solve $2x + 5 < 17$.



18. Solve $7 - 3x \leq 1$.
19. Solve $-4 \leq 2x + 6 < 14$.
20. Find the n th term of the arithmetic sequence $5, 9, 13, 17, \dots$
21. Find the 15th term of the sequence $5, 9, 13, 17, \dots$
22. Find the sum of the first 20 terms of the arithmetic sequence with first term 3 and common difference 4.
23. Find the n th term of the geometric sequence $81, 27, 9, 3, \dots$
24. Find the 6th term of $81, 27, 9, 3, \dots$
25. The model $P = 500(1.04)^t$ gives the value of an investment after t years. Find P when $t = 5$.
26. The model $C = 25 + 12h$ gives the cost in euros of hiring a bike for h hours. Find h when $C = 97$.
27. Solve simultaneously: $x + y = 12$ and $2x - y = 6$.
28. Solve simultaneously: $3x + 2y = 19$ and $x - y = 2$.
29. Explain the difference between an equation and an identity.
30. Write an expression for three more than twice a number n .
31. The table $n = 1, 2, 3, 4$ gives $y = 7, 10, 13, 16$. Find a formula for y in terms of n .
32. Using your formula from the previous question, find n when $y = 52$.
33. BMI is given by $B = \frac{m}{h^2}$. Find B when $m = 72$ and $h = 1.8$.
34. A value begins at 250 and is multiplied by 1.08 each year. Find its value after 2 years.
35. Show the solution $x \geq -1$ on a number line.
36. Solve $2(x - 1) = 3(x + 4)$.
37. Solve $\frac{x}{4} + \frac{x}{6} = 5$.
38. Simplify $\frac{x^2 - 16}{x + 4}$, stating any restriction.
39. Expand and simplify $(2x - 3)^2$.
40. Write $x^2 + 6x + 14$ in completed-square form.
41. State the minimum value of $x^2 + 6x + 14$.
42. Solve $x^2 + 6x + 5 = 0$.
43. Let $A = \{2, 4, 6, 8\}$ and $B = \{1, 2, 3, 4\}$. Find $A \cap B$.
44. Using the same sets, find $A \cup B$.
45. In a class of 30, 18 study French, 16 study Spanish and 7 study both. How many study neither?
46. y is directly proportional to x^2 . When $x = 3$, $y = 45$. Find y when $x = 5$.
47. I is inversely proportional to d^2 . When $d = 2$, $I = 80$. Find I when $d = 5$.
48. Give one limitation of using a straight-line model to predict a quantity far beyond the data collected.

Answer key: Week 2

- | | |
|-----------------------|------------------------|
| 1. 21. | 7. $(2x + 1)(x + 3)$. |
| 2. $2a + 8b$. | 8. $x = 7$. |
| 3. $9x - 13$. | 9. $x = 7$. |
| 4. $x^2 + 5x - 14$. | 10. $x = 14$. |
| 5. $3(2x + 5)$. | 11. $x = -4$. |
| 6. $(x + 4)(x + 5)$. | 12. $x = 3, 4$. |



13. $x = 3, -\frac{1}{2}$.
14. $l = \frac{P}{2} - w$.
15. $h = \frac{V}{\pi r^2}$.
16. $x = \frac{y+4}{3}$.
17. $x < 6$.
18. $x \geq 2$.
19. $-5 \leq x < 4$.
20. $4n + 1$.
21. 61.
22. 820.
23. $81(\frac{1}{3})^{n-1}$.
24. $\frac{1}{3}$.
25. e608.33 to the nearest cent.
26. $h = 6$.
27. $x = 6, y = 6$.
28. $x = \frac{23}{5}, y = \frac{13}{5}$.
29. An equation is true only for particular values; an identity is true for all allowed values.
30. $2n + 3$.
31. $y = 3n + 4$.
32. $n = 16$.
33. $B \approx 22.2$.
34. 291.60.
35. A closed circle at -1 with shading to the right.
36. $x = -14$.
37. $x = 12$.
38. $x - 4$, with $xe - 4$.
39. $4x^2 - 12x + 9$.
40. $(x + 3)^2 + 5$.
41. 5.
42. $x = -1, -5$.
43. $\{2, 4\}$.
44. $\{1, 2, 3, 4, 6, 8\}$.
45. 3.
46. 125.
47. 12.8.
48. Extrapolation may be unreliable because the same pattern may not continue outside the observed range.



Week 3: Coordinate geometry and functions as models

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Gradients, straight lines, transformations, functions, simple regression language and interpretation of models.

Key results

- Gradient between two points: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- A straight-line model $y = mx + c$ has gradient m and intercept c .
- Interpolation means predicting inside the data range.
- Extrapolation means predicting outside the data range.
- A residual is often calculated as actual minus predicted.
- Graphs help us decide whether a model is reasonable.

interpreting a linear model

Suppose $C = 0.8n + 15$, where C is the cost in euros and n is the number of items. The number 15 is the fixed cost. The number 0.8 is the extra cost per item. For $n = 20$,

$$C = 0.8(20) + 15 = 31.$$

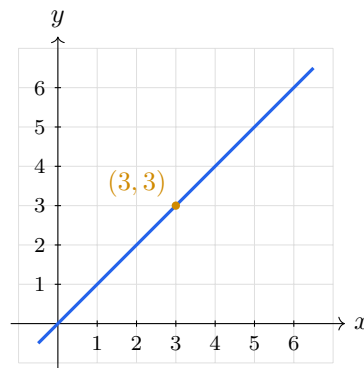
Answer: The cost is €31.

Practice questions: Week 3

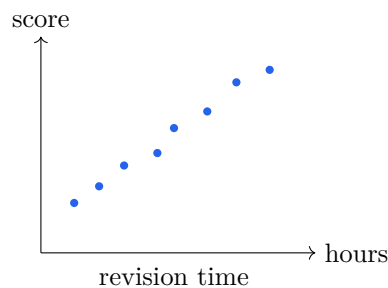
There are 48 varied questions in this section. Show working where appropriate and write interpretations in context.

1. Find the gradient of the line through (2, 5) and (6, 17).
2. Find the distance between (1, 2) and (7, 10).
3. Find the midpoint of (-4, 3) and (6, -1).
4. Find the equation of the line with gradient 3 passing through (2, 7).
5. Write down the y -intercept of $y = -2x + 5$.
6. Find the x -intercept of $3x + 2y = 12$.
7. Find the equation of the line through (0, 4) and (5, -6).
8. Find the equation of the line parallel to $y = \frac{1}{2}x + 3$ that passes through (4, 1).
9. Find the gradient of a line perpendicular to a line with gradient -3.
10. Find the point of intersection of $y = 2x + 1$ and $y = -x + 7$.
11. Complete the table for $f(x) = x^2 - 4$ when $x = -2, -1, 0, 1, 2$.
12. Find the roots of $f(x) = x^2 - 4$.
13. State the coordinates of the vertex of $y = x^2 - 4$.
14. For $f(x) = x^2 - 4x + 1$, find $f(3)$.
15. Solve $f(x) = 1$ for $f(x) = x^2 - 4x + 1$.
16. A model is $h(t) = 2t + 5$ for $0 \leq t \leq 10$. State the range of possible values of h .
17. In $T = 18 - 0.6t$, interpret the gradient -0.6 in context.
18. A model is $y = 1.5x + 12$. Find y when $x = 8$.
19. For the same model, find x when $y = 30$.

20. A population model is $N = 300(1.12)^t$. Find N after 4 years.
21. For $N = 300(1.12)^t$, state the initial value and annual growth factor.
22. Let $f(x) = 2x + 3$ and $g(x) = x^2$. Find $f(g(4))$.
23. Let $f(x) = 2x + 3$ and $g(x) = x^2$. Find $g(f(4))$.
24. Find the inverse of $f(x) = 2x - 5$.
25. Write the equation of $y = x^2$ translated 3 units right and 2 units up.
26. Describe the transformation from $y = x^2$ to $y = (x + 1)^2 - 4$.
27. Use the graph to state the value of y when $x = 3$.



28. State the type of correlation shown in the scatter diagram.



29. A regression model is $p = 1.2h + 35$. Predict p when $h = 10$.
30. If the actual value is 50 when the predicted value is 47, find the residual actual minus predicted.
31. Data were collected for $0 \leq h \leq 12$. Is predicting at $h = 25$ interpolation or extrapolation?
32. The profit model is $P = -2x^2 + 40x - 120$. Find the axis of symmetry.
33. Find the maximum value of $P = -2x^2 + 40x - 120$.
34. Find the break-even values of x for $P = -2x^2 + 40x - 120$.
35. Choose a suitable model type, linear or exponential, for a population increasing by a fixed percentage each year.
36. A taxi model is $C = 4 + 1.6d$ for $d \leq 10$ and $C = 20 + 2(d - 10)$ for $d > 10$. Find $C(12)$.
37. Find the average rate of change of $f(x) = x^2$ from $x = 1$ to $x = 5$.
38. A tangent line passes through $(2, 3)$ and $(6, 11)$. Estimate its gradient.
39. Reflect $(3, -5)$ in the x -axis.
40. Rotate $(2, 1)$ by 90° anticlockwise about the origin.
41. Enlarge $(-3, 4)$ by scale factor 2 about the origin.
42. Translate $(2, 3)$ by the vector $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$.



43. For $C = 0.8n + 15$, interpret the numbers 15 and 0.8.
44. A relation contains the pairs $(1, 2), (1, 3), (2, 4)$. Is it a function? Explain.
45. State the domain restriction for $y = \frac{1}{x - 3}$.
46. State the vertical asymptote of $y = \frac{5}{x + 2}$.
47. Solve $x^2 = 2x + 3$.
48. Explain why a graph can be useful before choosing which model to fit to data.

Answer key: Week 3

1. 3.
2. 10.
3. $(1, 1)$.
4. $y = 3x + 1$.
5. 5.
6. 4.
7. $y = -2x + 4$.
8. $y = \frac{1}{2}x - 1$.
9. $\frac{1}{3}$.
10. $(2, 5)$.
11. $0, -3, -4, -3, 0$.
12. $x = -2, 2$.
13. $(0, -4)$.
14. -2 .
15. $x = 0, 4$.
16. $5 \leq h \leq 25$.
17. The temperature decreases by 0.6 units per unit increase in t .
18. 24.
19. 12.
20. 472 approximately.
21. Initial value 300; growth factor 1.12.
22. 35.
23. 121.
24. $f^{-1}(x) = \frac{x + 5}{2}$.
25. $y = (x - 3)^2 + 2$.
26. Translation 1 left and 4 down.
27. 3.
28. Positive correlation.
29. 47.
30. 3.
31. Extrapolation.
32. $x = 10$.
33. 80.
34. $x = 10 \pm 2\sqrt{10}$, approximately 3.68 and 16.32.
35. Exponential.
36. 24.
37. 6.
38. 2.
39. $(3, 5)$.
40. $(-1, 2)$.
41. $(-6, 8)$.
42. $(7, 1)$.
43. 15 is the fixed cost; 0.8 is the cost per item.
44. No: input 1 has two different outputs.
45. $xe3$.
46. $x = -2$.
47. $x = -1, 3$.
48. It shows shape, outliers and whether a linear or non-linear model is more appropriate.



Week 4: Geometry, mensuration, trigonometry and transformations

Week 4: Geometry, mensuration, trigonometry and transformations

Area, volume, scale, Pythagoras, trigonometry, bearings, transformations and simple vector reasoning.

Key results

- Pythagoras: $a^2 + b^2 = c^2$ in a right-angled triangle.
- Right-triangle trigonometry: $\sin \theta = \frac{O}{H}$, $\cos \theta = \frac{A}{H}$, $\tan \theta = \frac{O}{A}$.
- Sector arc length: $l = r\theta$ when θ is in radians.
- Sector area: $A = \frac{1}{2}r^2\theta$ when θ is in radians.
- Bearings are measured clockwise from north.
- Area scale factor is the square of the linear scale factor.

using a diagram before calculating

For a right-angled triangle with shorter sides 8 cm and 15 cm,

$$c^2 = 8^2 + 15^2 = 64 + 225 = 289,$$

so

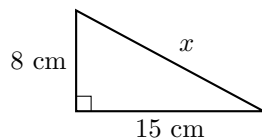
$$c = 17.$$

Answer: The hypotenuse is 17 cm.

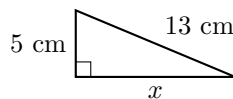
Practice questions: Week 4

There are 48 varied questions in this section. Show working where appropriate and write interpretations in context.

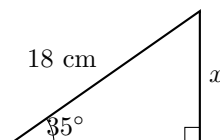
1. Find the hypotenuse of the right-angled triangle.



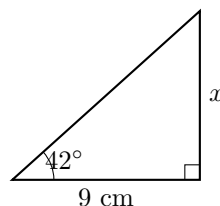
2. Find the missing side of the right-angled triangle.



3. Find the opposite side x , correct to 3 significant figures.

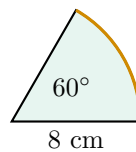


4. Find the opposite side x , correct to 3 significant figures.

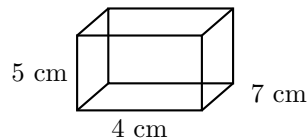


5. In a right-angled triangle, the adjacent side to angle θ is 7 cm and the hypotenuse is 12 cm. Find θ .

6. In triangle ABC , $A = 55^\circ$ and $B = 68^\circ$. Find C .
7. Use the sine rule to find a when $A = 35^\circ$, $B = 80^\circ$ and $b = 12$.
8. Use the cosine rule to find the third side when two sides are 7 and 10 with included angle 60° .
9. Find the area of a triangle with sides 9 cm and 14 cm enclosing an angle of 40° .
10. A point is 4 km east and 3 km north of an observer. Find the bearing of the point from the observer.
11. The angle of elevation to the top of a building is 32° from a point 24 m away. Find the height of the building.
12. On a scale drawing, 1 cm represents 2 m. A garden path is 7.5 cm long on the drawing. Find its real length.
13. Find the circumference of a circle with radius 6 cm.
14. Find the area of a circle with diameter 10 cm.
15. Find the arc length of the sector.



16. Find the area of the same sector.
17. A prism has cross-sectional area 24 cm^2 and length 10 cm. Find its volume.
18. Find the volume of a cylinder with radius 3 cm and height 8 cm.
19. Find the surface area of the cuboid.



20. A cuboid has volume 420 cm^3 , length 7 cm and width 6 cm. Find its height.
21. Two similar shapes have scale factor 1.5 from the smaller to the larger. The smaller area is 20 cm^2 . Find the larger area.
22. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Find V when $r = 4 \text{ cm}$ and $h = 9 \text{ cm}$.
23. Translate the point $(-2, 5)$ by the vector $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$.
24. Reflect $(3, -4)$ in the x -axis.
25. Reflect $(6, 2)$ in the y -axis.
26. Rotate $(5, -3)$ by 180° about the origin.
27. Enlarge $(2, -1)$ by scale factor 3 about the origin.
28. Find $\overrightarrow{AB}$ for $A(1, 4)$ and $B(6, -2)$.
29. Find the magnitude of $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$.
30. Find $\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix}$.
31. The bearing from A to B is 070° . Find the bearing from B to A .
32. Find the area of a trapezium with parallel sides 8 cm and 14 cm and height 5 cm.
33. An L-shape is made from a 7 cm by 6 cm rectangle with a 3 cm by 2 cm rectangle removed. Find its area.



34. Two parallel lines are crossed by a transversal. One corresponding angle is 65° . Find the corresponding angle on the other line.
35. A triangle has sides 7, 8 and 10. Find the angle opposite the side of length 10.
36. Find the space diagonal of a cuboid with side lengths 3 cm, 4 cm and 12 cm.
37. The volume of a sphere is $V = \frac{4}{3}\pi r^3$. Find the volume when $r = 5$ cm.
38. A roof rises 2.4 m over a horizontal run of 7.2 m. Find the angle the roof makes with the horizontal.
39. Find the perimeter of a semicircle with diameter 14 cm, including the diameter.
40. Find the area of an annulus with outer radius 6 cm and inner radius 4 cm.
41. A regular hexagon has interior angle 120° . How many regular hexagons meet exactly around a point?
42. A walker travels 10 km east and then 24 km north. Find the displacement from the start.
43. Find the bearing of the displacement in the previous question.
44. Triangle $A(1, 1)$, $B(3, 1)$, $C(1, 4)$ maps to $A'(4, -1)$, $B'(6, -1)$, $C'(4, 2)$. Describe the transformation.
45. A length changes from 12 cm to 18 cm under an enlargement. Find the linear scale factor.
46. Using the previous scale factor, find the area scale factor.
47. A rectangle has length 8.0 cm and width 3.0 cm, each correct to the nearest 0.1 cm. Find the upper bound for its perimeter.
48. Convert 0.035 m^2 to cm^2 .

Answer key: Week 4

- | | |
|--------------------------------------|---|
| 1. 17 cm. | 22. $48\pi \text{ cm}^3$. |
| 2. 12 cm. | 23. $(2, -2)$. |
| 3. 10.3 cm. | 24. $(3, 4)$. |
| 4. 8.10 cm. | 25. $(-6, 2)$. |
| 5. 54.3° . | 26. $(-5, 3)$. |
| 6. 57° . | 27. $(6, -3)$. |
| 7. 7.0 approximately. | 28. $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$. |
| 8. $\sqrt{79} \approx 8.89$. | 29. $\sqrt{61}$. |
| 9. 40.5 cm^2 . | 30. $\begin{pmatrix} -2 \\ 6 \end{pmatrix}$. |
| 10. 053.1° . | 31. 250° . |
| 11. 15.0 m. | 32. 55 cm^2 . |
| 12. 15 m. | 33. 36 cm^2 . |
| 13. $12\pi \text{ cm}$. | 34. 65° . |
| 14. $25\pi \text{ cm}^2$. | 35. 83.3° . |
| 15. $\frac{8\pi}{3} \text{ cm}$. | 36. 13 cm. |
| 16. $\frac{32\pi}{3} \text{ cm}^2$. | 37. $\frac{500\pi}{3} \approx 523.6 \text{ cm}^3$. |
| 17. 240 cm^3 . | 38. 18.4° . |
| 18. $72\pi \text{ cm}^3$. | 39. $14 + 7\pi \text{ cm}$. |
| 19. 166 cm^2 . | 40. $20\pi \text{ cm}^2$. |
| 20. 10 cm. | |
| 21. 45 cm^2 . | |



41. 3.

42. 26 km.

43. 022.6° .

44. Translation by $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

45. 1.5.

46. 2.25.

47. 22.2 cm.

48. 350 cm^2 .

Week 5: Statistics, histograms and probability

Week 5: Statistics, histograms and probability

Averages, spread, grouped data, histograms, box plots, scatter diagrams, regression interpretation, Venn diagrams, tree diagrams and probability.

Key results

- Mean = $\frac{\text{total}}{\text{number of values}}$.
- IQR = $Q_3 - Q_1$.
- Frequency density = $\frac{\text{frequency}}{\text{class width}}$.
- $P(A') = 1 - P(A)$.
- If independent, $P(A \cap B) = P(A)P(B)$.
- Correlation does not automatically imply causation.

probability from a tree diagram

If a bag contains 3 red and 5 blue counters and the counter is replaced after each draw, then

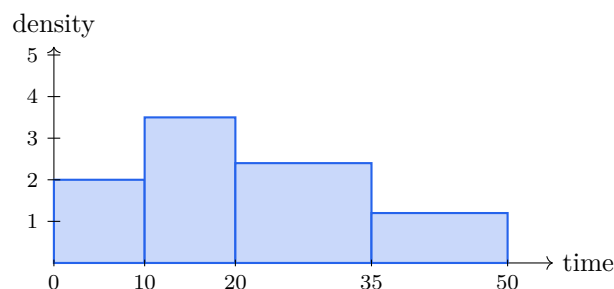
$$P(R \text{ then } B) = \frac{3}{8} \times \frac{5}{8} = \frac{15}{64}.$$

Answer: $\frac{15}{64}$.

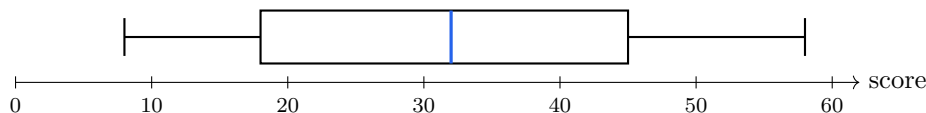
Practice questions: Week 5

There are 48 varied questions in this section. Show working where appropriate and write interpretations in context.

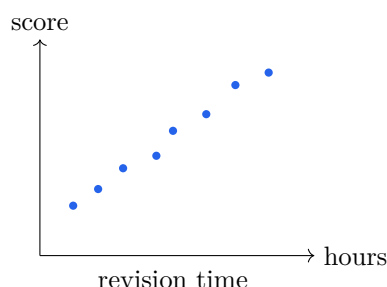
1. Find the mean of 4, 7, 8, 11, 15.
2. Find the median of 3, 5, 5, 9, 12, 14.
3. Find the mode and range of 2, 6, 6, 7, 10, 10, 10, 12.
4. For the data 4, 6, 8, 9, 11, 15, 18, find Q_1 , the median, Q_3 and the IQR.
5. A data set has $Q_1 = 12$ and $Q_3 = 20$. Use the $1.5 \times IQR$ rule to decide whether 35 is an outlier.
6. A frequency table has values 1, 2, 3, 4 with frequencies 3, 5, 6, 2. Find the mean.
7. A cumulative frequency table has $< 10 : 4$, $< 20 : 12$, $< 30 : 25$, $< 40 : 30$. State the median class.
8. Estimate the grouped mean for intervals 0–10, 10–20, 20–30 with frequencies 5, 8, 7.
9. In a histogram, a class has width 10 and frequency 30. Find its frequency density.
10. In a histogram, a class from 20 to 25 has frequency density 4. Find its frequency.
11. Use the histogram to identify the class with the greatest frequency density.



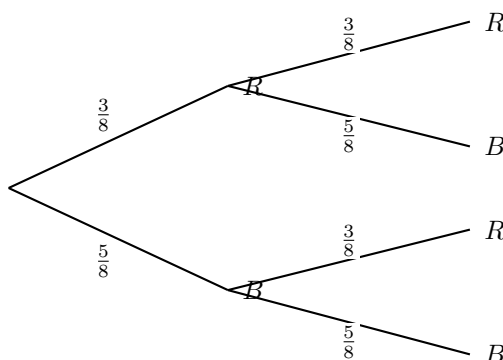
12. Use the box plot to state the median.



13. Use the box plot to find the interquartile range.
14. A fair die is rolled. Find the probability of an even number.
15. A fair die is rolled. Find the probability of not rolling a prime number.
16. If $P(A) = 0.65$, find $P(A')$.
17. If $P(A) = 0.6$ and $P(B) = 0.25$ and A and B are independent, find $P(A \cap B)$.
18. If A and B are mutually exclusive, $P(A) = 0.30$ and $P(B) = 0.45$, find $P(A \cup B)$.
19. If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$, find $P(A \cup B)$.
20. In a group of 40, 22 play football, 18 play tennis and 10 play both. How many play neither?
21. For the same group, find the probability that a randomly chosen person plays football only.
22. A product has probability 0.04 of being defective. Two products are chosen independently. Find the probability both are good.
23. Find the probability exactly one of the two products is defective.
24. The regression model $y = 2.4x + 15$ is used to predict y . Find the predicted value when $x = 8$.
25. A correlation coefficient is $r = -0.82$. Describe the correlation.
26. Does correlation prove causation? Answer briefly.
27. State the type of correlation shown.

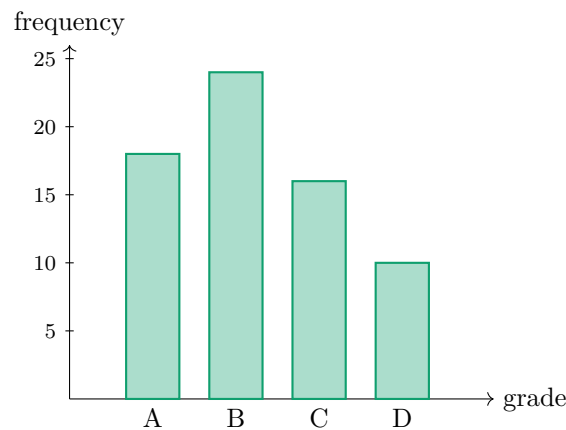


28. A prediction is 47 and the actual value is 51. Find the residual actual minus predicted.
29. For the tree diagram, find $P(R \text{ then } B)$.



30. For the same tree diagram, find $P(\text{same colours})$.
31. A two-way table shows 24 students play sport. Of these, 10 are female. Find $P(\text{female} \mid \text{plays sport})$.
32. The expected value of a game is found from winning €5 with probability 0.2 and losing €1 with probability 0.8. Find the expected value.

33. A bag contains 3 red and 5 blue counters. With replacement, find $P(\text{red then blue})$.
34. Using the same bag but without replacement, find $P(\text{red then blue})$.
35. Use the bar chart to find the total frequency.



36. Use the bar chart to find the modal category.
37. Calculate the weighted mean of grades 2, 3, 4, 5 with frequencies 1, 4, 3, 2.
38. In a capture-recapture estimate, 20 fish are marked. Later, 30 fish are caught and 6 are marked. Estimate the population.
39. A school has 120 girls and 80 boys. A stratified sample of 25 students is chosen. How many girls should be in the sample?
40. A systematic sample of 25 is chosen from a list of 500 names. Find the sampling interval.
41. If the probability of success on one trial is 0.3, find the probability of at least one success in two independent trials.
42. In a pie chart, a sector angle is 72° . What percentage of the data is represented?
43. A data set has mean 50. Each value is increased by 3. Find the new mean.
44. If every value in a data set is multiplied by 2, what happens to the range?
45. A data set is strongly skewed by one very large value. Which average is usually more representative: mean or median?
46. Two events have $P(A) = 0.7$, $P(B) = 0.4$ and $P(A \cap B) = 0.28$. Are they independent?
47. Choose a suitable chart for categorical data such as favourite colour.
48. Give one reason why a larger random sample is usually better than a small biased sample.

Answer key: Week 5

- | | |
|---|---------------------|
| 1. 9. | 10. 20. |
| 2. 7. | 11. 10–20. |
| 3. Mode 10; range 10. | 12. 32. |
| 4. $Q_1 = 6$, median 9, $Q_3 = 15$, IQR 9. | 13. 27. |
| 5. Yes. IQR = 8, upper fence = 32, so 35 is an outlier. | 14. $\frac{1}{2}$. |
| 6. $\frac{39}{16} = 2.4375$. | 15. $\frac{1}{2}$. |
| 7. 20–30. | 16. 0.35. |
| 8. 16. | 17. 0.15. |
| 9. 3. | 18. 0.75. |



19. 0.70.
20. 10.
21. $\frac{12}{40} = 0.3$.
22. 0.9216.
23. 0.0768.
24. 34.2.
25. Strong negative correlation.
26. No. Correlation alone does not prove causation.
27. Positive correlation.
28. 4.
29. $\frac{15}{64}$.
30. $\frac{34}{64} = \frac{17}{32}$.
31. $\frac{10}{24} = \frac{5}{12}$.
32. e0.20.
33. $\frac{15}{64}$.
34. $\frac{15}{56}$.
35. 68.
36. Category B.
37. 3.6.
38. 100 fish.
39. 15 girls.
40. 20.
41. 0.51.
42. 20%.
43. 53.
44. It doubles.
45. Median.
46. Yes, because $0.7 \times 0.4 = 0.28$.
47. A bar chart.
48. It is more likely to represent the population accurately and reduce random variation/bias.



End of Pre-IB AI Mathematics Summer Pack

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