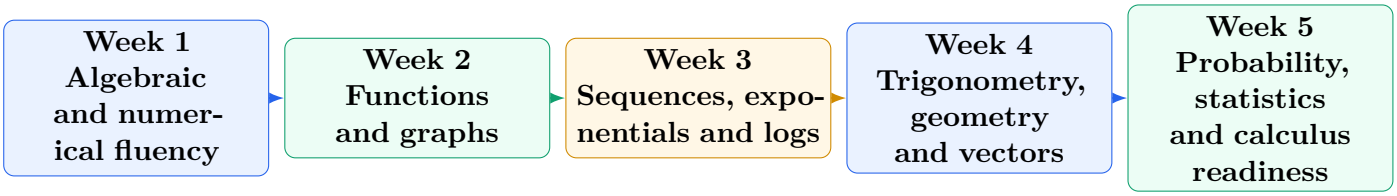
Pre-IB AA Mathematics

Structured Summer Preparation

for Future AA SL and AA HL Mathematics Students

A five-week bridge from IGCSE Extended Mathematics into IB Mathematics: Analysis and Approaches.

Learning goal	Review the most important IGCSE Extended foundations so that students enter IB Mathematics AA with stronger algebraic fluency, graph sense, trigonometric confidence, statistical literacy and problem-solving habits.
Syllabus bridge	Cambridge IGCSE Extended/International Mathematics foundations leading into IB Mathematics AA SL/HL: number and algebra, functions, coordinate geometry, trigonometry, vectors, probability, statistics and early calculus thinking.
Big idea	AA students need to move fluently between algebra, exact values, diagrams, graphs and written mathematical arguments. Future HL students should also begin developing comfort with proof, parameters, abstraction and harder multi-step problems.
How to use this pack	Complete the main work each week. Future HL students should also complete the amber **HL extension** boxes and extension questions. Future SL students may use them as optional challenge tasks.



Pre-IB boundary. This is a preparation pack, not an early IB textbook. The main questions are built from IGCSE Extended ideas. A small number of bridge ideas are introduced from first principles because they prepare students for the style of IB Mathematics AA. The amber HL extension material is deliberately more demanding and is intended to stretch future HL students without assuming that they have already started the IB course.

Contents

Teacher overview: scope and weekly structure 3

Week 1: Algebraic and numerical fluency 4

Week 2: Functions and graphs 9

Week 3: Sequences, exponentials and logarithmic thinking 14

Week 4: Trigonometry, geometry and vectors 20

Week 5: Probability, statistics and calculus readiness 29

Teacher overview: scope and weekly structure

Week	Focus	Main purpose
1	Algebraic and numerical fluency	Secure exact arithmetic, surds, indices, factorising, expanding, rearranging and inequality reasoning.
2	Functions and graphs	Strengthen function notation, coordinate geometry, graph sketching, transformations, inverses and composites.
3	Sequences, exponentials and logarithmic thinking	Bridge IGCSE sequences and compound change into AA sequences, sigma notation, exponentials and logarithms.
4	Trigonometry, geometry and vectors	Review right and non-right triangle methods, exact values, radians, trigonometric graphs and vector diagrams.
5	Probability, statistics and calculus readiness	Review data and probability while preparing for AA-style interpretation, gradients, rates of change and area accumulation.

Suggested pace. Each week is designed as a summer review block. Students should spend about 2–3 focused sessions on each week: one for notes and examples, one for practice, and one for checking corrections and completing the HL extension if appropriate.

Week 1: Algebraic and numerical fluency

Week 1: Algebraic and numerical fluency

Exact arithmetic, surds, indices, factorising, quadratics, rearranging formulae and inequalities.

Key results

- Standard form: $a \times 10^k$, where $1 \le a < 10$.
- Index laws: $a^m a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$.
- Surds: $\sqrt{ab} = \sqrt{a}\sqrt{b}$ for $a, b \ge 0$.
- Rationalising: $\dfrac{1}{\sqrt{a}} = \dfrac{\sqrt{a}}{a}$.
- Factorising quadratics: $ax^2 + bx + c = (px + q)(rx + s)$ when possible.
- Completing the square: $x^2 + bx + c = (x + \frac{b}{2})^2 + c - \left(\frac{b}{2}\right)^2$.
- Linear inequalities behave like equations except that multiplying or dividing by a negative reverses the inequality sign.
- Exact answers are often preferred in AA: leave answers such as $\sqrt{18} = 3\sqrt{2}$ rather than decimals.

Focus for this week. Work carefully with exact values, algebraic structure and clear notation. Several later questions revisit completing the square and discriminants in a more exam-style way, so the emphasis here is on method rather than memorising a picture.

Worked examples

Example 1: simplifying surds and rationalising

Question. Simplify $\sqrt{72} - \sqrt{18} + \dfrac{6}{\sqrt{2}}$.

$$\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2},$$
$$\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2},$$
$$\frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}.$$

Therefore

$$\sqrt{72} - \sqrt{18} + \frac{6}{\sqrt{2}} = 6\sqrt{2} - 3\sqrt{2} + 3\sqrt{2} = 6\sqrt{2}.$$

Answer: $6\sqrt{2}$.

Example 2: factorising and solving a quadratic

Question. Solve $2x^2 - 5x - 3 = 0$.

$$2x^2 - 5x - 3 = (2x + 1)(x - 3).$$

So

$$(2x + 1)(x - 3) = 0 \quad \Rightarrow \quad 2x + 1 = 0 \text{ or } x - 3 = 0.$$

Hence

$$x = -\frac{1}{2} \quad \text{or} \quad x = 3.$$

Answer: $x = -\frac{1}{2}$ or $x = 3$.

Example 3: completing the square

Question. Write $x^2 - 6x + 11$ in completed-square form and state its minimum value.

$$x^2 - 6x + 11 = (x-3)^2 - 9 + 11 = (x-3)^2 + 2.$$

Since $(x-3)^2 \geq 0$, the smallest possible value is 2, when $x = 3$. **Answer:** $x^2 - 6x + 11 = (x-3)^2 + 2$ and the minimum value is 2.

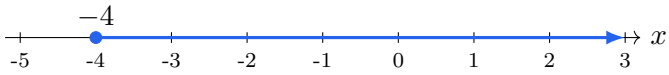
Example 4: inequalities on a number line

Question. Solve $3 - 2x \leq 11$.

$$3 - 2x \leq 11 \quad \Rightarrow \quad -2x \leq 8.$$

Now divide by -2. Because we divide by a negative number, the sign reverses:

$$x \geq -4.$$



Answer: $x \geq -4$.

HL extension: parameters and discriminants

Future HL students should be comfortable with a letter standing for an unknown constant. The discriminant

$$\Delta = b^2 - 4ac$$

tells us about the roots of $ax^2 + bx + c = 0$:

$$\Delta > 0 \Rightarrow \text{two distinct real roots}, \quad \Delta = 0 \Rightarrow \text{one repeated real root}, \quad \Delta < 0 \Rightarrow \text{no real roots}.$$

Worked extension. Find the values of k for which $x^2 + kx + 9 = 0$ has no real roots.

$$\Delta = k^2 - 4(1)(9) = k^2 - 36.$$

No real roots means $\Delta < 0$:

$$k^2 - 36 < 0 \quad \Rightarrow \quad k^2 < 36 \quad \Rightarrow \quad -6 < k < 6.$$

Answer: $-6 < k < 6$.

Practice questions: Week 1

1. Write 0.000072 in standard form.
2. Write 6.31×10^5 as an ordinary number.
3. Write 4.8×10^{-3} as an ordinary number.
4. Write 75 000 000 in standard form.
5. Simplify $3^4 \times 3^{-7}$.
6. Simplify $\dfrac{(2^3)^4}{2^5}$.
7. Simplify $\dfrac{12a^5b^{-2}}{3a^{-1}b^3}$.
8. Simplify $(x^2y^{-1})^3$, giving your answer with positive indices.
9. Simplify $\sqrt{98}$.
10. Simplify $\sqrt{45} + \sqrt{20}$.
11. Simplify $\sqrt{50} - \sqrt{8}$.
12. Rationalise $\dfrac{5}{\sqrt{3}}$.
13. Rationalise $\dfrac{3}{2-\sqrt{5}}$.
14. Expand and simplify $(2x-3)(x+5)$.
15. Expand and simplify $(x-4)^2$.
16. Factorise $x^2 + 7x + 12$.
17. Factorise $3x^2 - 11x - 4$.
18. Factorise $2x^2 + x - 15$.
19. Solve $x^2 + 7x + 12 = 0$.
20. Solve $3x^2 - 11x - 4 = 0$.
21. Solve $2x^2 + x - 15 = 0$.
22. Rearrange $A = \dfrac{1}{2}h(a+b)$ to make h the subject.
23. Rearrange $v^2 = u^2 + 2as$ to make s the subject.
24. Rearrange $y = \dfrac{3x-5}{2}$ to make x the subject.
25. Solve $5x - 7 > 18$.
26. Solve $4 - 3x \leq 19$.
27. Solve $-2(3x-4) < 10$ and show the solution on a number line.
28. Solve $\dfrac{x-2}{3} + \dfrac{x+1}{2} = 5$.
29. Write $x^2 + 8x + 10$ in completed-square form.
30. State the minimum value of $x^2 + 8x + 10$.
31. Write $x^2 - 10x + 29$ in completed-square form.

32. Simplify $\dfrac{x^2-9}{x^2+6x+9}$.
33. Complete the table above for $y = 2x^2 - 3x + 1$.
34. Estimate $\dfrac{49.7 \times 3.02}{0.198}$ by rounding each number to 1 significant figure.
35. Simplify $(2x^2y)(-3xy^3)$.
36. Factorise by grouping: $xy + 3x + 2y + 6$.
37. Solve the simultaneous equations $2x + y = 7$ and $x - y = 2$.
38. Solve $3(x - 2) = 2(x + 5)$.
39. A length is recorded as 7.4 cm to the nearest 0.1 cm. Write the lower and upper bounds.
40. A rectangle has width x cm and length $(x + 5)$ cm. Its area is 84 cm^2. Find the positive value of x.

HL extension questions

41. Find the values of k for which $x^2 - 4x + k = 0$ has two distinct real roots.
42. Find p if $x^2 + px + 16 = 0$ has a repeated root.
43. Prove that $(n + 1)^2 - n^2 = 2n + 1$ for all integers n.
44. Show that $x^2 - 10x + 29 > 0$ for all real x.
45. Find the values of k for which $2x^2 + kx + 8 = 0$ has equal roots.
46. Find the values of m for which $x^2 - 2mx + 9 = 0$ has no real roots.
47. Prove that $(n + 2)^2 - n^2 = 4n + 4$ for all integers n.
48. Find the values of k for which both roots of $x^2 - 5x + k = 0$ are real and positive.

Answer key: Week 1

1. 7.2×10^{-5}.
2. 631000.
3. 0.0048.
4. 7.5×10^{7}.
5. $3^{-3} = \dfrac{1}{27}$.
6. $2^7 = 128$.
7. $\dfrac{4a^6}{b^5}$.
8. $\dfrac{x^6}{y^3}$.
9. $7\sqrt{2}$.
10. $5\sqrt{5}$.
11. $3\sqrt{2}$.
12. $\dfrac{5\sqrt{3}}{3}$.
13. $-6 - 3\sqrt{5}$.
14. $2x^2 + 7x - 15$.
15. $x^2 - 8x + 16$.
16. $(x + 3)(x + 4)$.
17. $(3x + 1)(x - 4)$.
18. $(2x - 5)(x + 3)$.
19. $x = -3, -4$.
20. $x = 4, -\dfrac{1}{3}$.
21. $x = \dfrac{5}{2}, -3$.
22. $h = \dfrac{2A}{a+b}$.
23. $s = \dfrac{v^2 - u^2}{2a}$.
24. $x = \dfrac{2y+5}{3}$.

25. $x > 5$.
26. $x \geq -5$.
27. $x > -\dfrac{1}{3}$.
28. $x = \dfrac{31}{5}$.
29. $(x+4)^2 - 6$.
30. -6.
31. $(x-5)^2 + 4$.
32. $\dfrac{x-3}{x+3}$, $x \neq -3$.
33. $6, 1, 0, 3$.
34. About 750.
35. $-6x^3y^4$.
36. $(x+2)(y+3)$.
37. $x = 3$, $y = 1$.
38. $x = 16$.
39. $7.35 \leq L < 7.45$.
40. $x = 7$.
41. $k < 4$.
42. $p = 8$ or $p = -8$.
43. $(n+1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1$.
44. $(x-5)^2 + 4 > 0$.
45. $k = 8$ or $k = -8$.
46. $-3 < m < 3$.
47. $(n+2)^2 - n^2 = n^2 + 4n + 4 - n^2 = 4n + 4$.
48. $0 < k \leq \dfrac{25}{4}$.

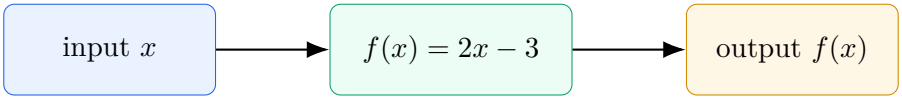
Week 2: Functions and graphs

Week 2: Functions and graphs

Coordinate geometry, function notation, domain and range, graph sketching, transformations, inverse functions and composites.

Key results

- Gradient between two points: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.
- Straight line forms: $y = mx + c$, $y - y_1 = m(x - x_1)$, $ax + by + d = 0$.
- Parallel lines have equal gradients.
- Perpendicular lines satisfy $m_1 m_2 = -1$.
- A function assigns each input exactly one output.
- $f(a)$ means the output when the input is a.
- $fg(x)$ means $f(g(x))$.
- f^{-1} reverses f, when the inverse exists.
- Quadratics have a vertex and an axis of symmetry.
- Transformations: $f(x)+a$, $f(x-a)$, $af(x)$ and $f(ax)$ each move or stretch a graph in a predictable way.



A function is a rule, but the graph shows the whole rule visually.

Worked examples

Example 1: equation of a straight line

Question. Find the equation of the line through $A(-2, 5)$ and $B(4, -1)$.

$$m = \frac{-1 - 5}{4 - (-2)} = \frac{-6}{6} = -1.$$

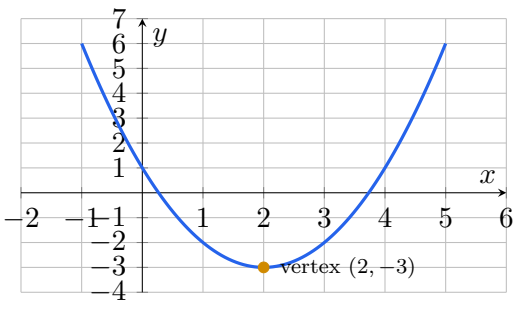
Using $y - y_1 = m(x - x_1)$ with $(-2, 5)$:

$$y - 5 = -1(x + 2) \quad \Rightarrow \quad y = -x + 3.$$

Answer: $y = -x + 3$.

Example 2: sketching a quadratic from completed-square form

Question. Sketch $y = (x - 2)^2 - 3$ and state the vertex.
The graph of $y = x^2$ is translated 2 units right and 3 units down. The vertex is therefore $(2, -3)$.



Answer: The vertex is $(2, -3)$ and the axis of symmetry is $x = 2$.

Example 3: composite functions

Question. Let $f(x) = 3x - 1$ and $g(x) = x^2 + 2$. Find $fg(x)$ and $gf(x)$.

$$fg(x) = f(g(x)) = 3(x^2 + 2) - 1 = 3x^2 + 5.$$

$$gf(x) = g(f(x)) = (3x - 1)^2 + 2 = 9x^2 - 6x + 3.$$

Answer: $fg(x) = 3x^2 + 5$ and $gf(x) = 9x^2 - 6x + 3$. Notice that usually $fg(x) \neq gf(x)$.

Example 4: inverse functions

Question. Find the inverse of $f(x) = \dfrac{x-4}{3}$.

Write $y = \dfrac{x-4}{3}$, then swap x and y:

$$x = \frac{y-4}{3}.$$

Solve for y:

$$3x = y - 4 \quad \Rightarrow \quad y = 3x + 4.$$

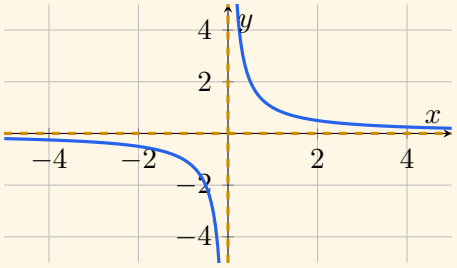
Answer: $f^{-1}(x) = 3x + 4$.

HL extension: rational functions and asymptotes

The reciprocal graph $y = \dfrac{1}{x}$ is a basic example of a rational function. It has two asymptotes:

$$x = 0 \quad \text{and} \quad y = 0.$$

These are lines that the graph approaches but does not touch.



Extension skill. For $y = \dfrac{2}{x-3} + 1$, the graph of $y = \dfrac{2}{x}$ is translated 3 units right and 1 unit up, so the asymptotes are $x = 3$ and $y = 1$.

Practice questions: Week 2

1. Find the gradient of the line through $(1, 4)$ and $(5, 12)$.
2. Find the gradient of the line through $(-3, 2)$ and $(1, -6)$.
3. Find the equation of the line with gradient 3 through $(2, -1)$.
4. Find the equation of the line through $(0, 7)$ and $(4, -1)$.
5. Find the equation of the line perpendicular to $y = 2x + 5$ through $(4, 1)$.
6. Find the equation of the line parallel to $y = -4x + 1$ through $(2, 5)$.
7. Find the midpoint of $A(-2, 7)$ and $B(6, -1)$.
8. Find the distance between $A(-2, 7)$ and $B(6, -1)$.
9. For $f(x) = x^2 - 4x + 1$, find $f(3)$.
10. For $f(x) = x^2 - 4x + 1$, write it in completed-square form.
11. State the vertex of $y = x^2 - 4x + 1$.
12. State the range of $f(x) = (x + 1)^2 - 5$ for $x \in \mathbb{R}$.
13. State the intercepts of $y = x^2 - 4$.
14. Solve $x^2 - 5x + 6 = 0$.
15. Solve $x^2 - 5x + 6 > 0$.
16. For the graph $y = (x - 1)^2 - 4$, state the vertex and the roots.
17. State the domain of $f(x) = \dfrac{1}{x - 4}$.
18. State the range of $f(x) = \sqrt{x + 3}$.
19. Let $f(x) = 2x + 3$ and $g(x) = x^2 - 1$. Find $fg(x)$.
20. Let $f(x) = 2x + 3$ and $g(x) = x^2 - 1$. Find $gf(x)$.
21. Find the inverse of $f(x) = 5x - 2$.
22. Find the inverse of $g(x) = \dfrac{x + 7}{4}$.
23. Describe the transformation from $y = f(x)$ to $y = f(x) + 6$.
24. Describe the transformation from $y = f(x)$ to $y = f(x - 4)$.
25. Describe the transformation from $y = f(x)$ to $y = 2f(x)$.
26. Describe the transformation from $y = f(x)$ to $y = f(2x)$.
27. Describe the transformation from $y = x^2$ to $y = (x - 3)^2 + 2$.
28. Solve simultaneously $y = x + 1$ and $y = x^2 - 5$.
29. Find the intersections of $y = x^2$ and $y = 2x + 3$.
30. Complete the table for $f(x) = 3x - 2$ when $x = -1, 0, 1, 2$.
31. A straight line passes through $(0, -2)$ and $(3, 4)$. Find its equation.
32. Write $y = x^2 + 6x + 5$ in completed-square form and state its axis of symmetry.
33. Solve $x^2 + 6x + 5 \leq 0$.

34. Translate $(2, -5)$ by the vector $\begin{pmatrix} -5 \\ 4 \end{pmatrix}$.
35. Describe the transformation from $y = f(x)$ to $y = -f(x)$.
36. Describe the transformation from $y = f(x)$ to $y = f(-x)$.
37. Let $f(x) = x + 4$ and $g(x) = 2x$. Solve $fg(x) = 10$.
38. If $f(x) = 3x - 2$, find $f^{-1}(7)$.
39. In the model $C = 15 + 4t$, interpret the gradient 4.
40. Complete the values of $y = x^2 - 2x$ for $x = -1, 0, 1, 2, 3$.

HL extension questions

41. State the asymptotes of $y = \dfrac{3}{x+2} - 4$.
42. Sketch the main features of $y = |x^2 - 4|$.
43. Find the inverse of $f(x) = x^2 + 1$ for the restricted domain $x \geq 0$.
44. Solve $\dfrac{2}{x-1} = x + 1$.
45. Find a and b if $f(x) = ax + b$, $f(2) = 7$ and $f^{-1}(1) = 0$.
46. Find the inverse of $f(x) = (x-2)^2 + 1$ for $x \geq 2$, and state its domain.
47. Find the values of k for which the line $y = kx$ touches the parabola $y = x^2 + 4$ exactly once.
48. For $f(x) = x + 1$ and $g(x) = x^2$, show algebraically that $fg(x) \neq gf(x)$.

Answer key: Week 2

1. 2.
2. -2.
3. $y = 3x - 7$.
4. $y = -2x + 7$.
5. $y = -\dfrac{1}{2}x + 3$.
6. $y = -4x + 13$.
7. $(2, 3)$.
8. $8\sqrt{2}$.
9. -2.
10. $(x-2)^2 - 3$.
11. $(2, -3)$.
12. $f(x) \geq -5$.
13. $(-2, 0)$, $(2, 0)$ and $(0, -4)$.
14. $x = 2$ or $x = 3$.
15. $x < 2$ or $x > 3$.
16. Vertex $(1, -4)$; roots $x = -1$ and $x = 3$.
17. $x \neq 4$.
18. $y \geq 0$.
19. $fg(x) = 2x^2 + 1$.
20. $gf(x) = 4x^2 + 12x + 8$.
21. $f^{-1}(x) = \dfrac{x+2}{5}$.
22. $g^{-1}(x) = 4x - 7$.
23. Translation 6 units up.
24. Translation 4 units right.
25. Vertical stretch, scale factor 2.
26. Horizontal stretch, scale factor $\dfrac{1}{2}$.
27. Right 3, up 2.
28. $(-2, -1)$ and $(3, 4)$.
29. $(-1, 1)$ and $(3, 9)$.
30. $-5, -2, 1, 4$.

31. $y = 2x - 2$.
32. $y = (x + 3)^2 - 4$; axis $x = -3$.
33. $-5 \leq x \leq -1$.
34. $(-3, -1)$.
35. Reflection in the x-axis.
36. Reflection in the y-axis.
37. $x = 3$.
38. 3.
39. Each extra unit of t adds 4 units of cost.
40. $3, 0, -1, 0, 3$.
41. $x = -2$, $y = -4$.
42. Zeros at ± 2; negative part of $x^2 - 4$ reflected upward; point $(0, 4)$.
43. $f^{-1}(x) = \sqrt{x - 1}$, domain $x \geq 1$.
44. $x = \pm\sqrt{3}$.
45. $a = 3$, $b = 1$.
46. $f^{-1}(x) = 2 + \sqrt{x - 1}$, domain $x \geq 1$.
47. $k = 4$ or $k = -4$.
48. $fg(x) = x^2 + 1$, while $gf(x) = (x + 1)^2 = x^2 + 2x + 1$.

Week 3: Sequences, exponentials and logarithmic thinking

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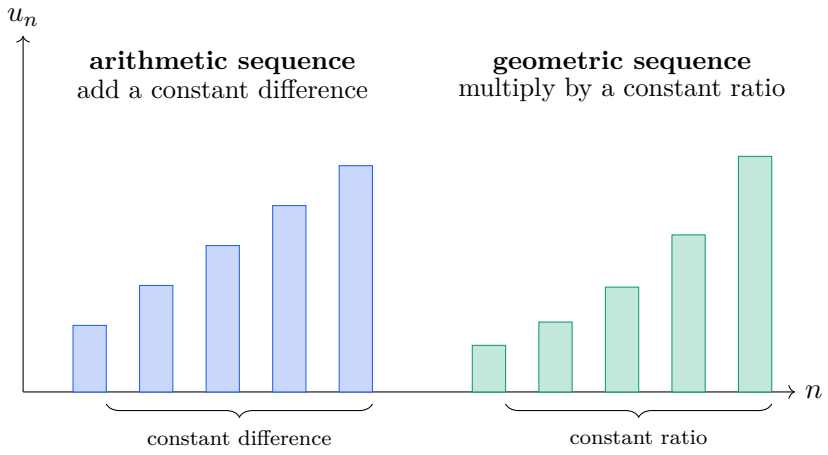
Arithmetic and geometric sequences, sigma notation, compound change, exponential models, logarithms and binomial patterns.

Key results

- Arithmetic sequence: $u_n = u_1 + (n-1)d$.
- Arithmetic sum: $S_n = \frac{n}{2}(2u_1 + (n-1)d)$.
- Sigma notation: $\sum_{k=1}^{n} a_k$ means add the terms $a_1 + a_2 + \cdots + a_n$.
- Geometric sequence: $u_n = u_1 r^{n-1}$.
- Geometric sum: $S_n = \frac{u_1(1-r^n)}{1-r}$, $r \neq 1$.
- Compound change: $A = P(1+r)^n$ for growth.
- Depreciation: $A = P(1-r)^n$.
- Exponential equations can often be solved by matching bases or using logarithms.
- $a^x = b$ is equivalent to $x = \log_a b$.
- $\ln x$ means the logarithm base e.



Worked examples

Example 1: arithmetic sequence and sum

Question. An arithmetic sequence has first term 7 and common difference 4. Find u_{20} and S_{20}.

$$u_{20} = 7 + (20-1)4 = 7 + 76 = 83.$$

$$S_{20} = \frac{20}{2}(2(7) + (20-1)4) = 10(14 + 76) = 900.$$

Answer: $u_{20} = 83$ and $S_{20} = 900$.

Example 2: reading and writing sigma notation

Question.

(i) Write $5+8+11+\cdots+32$ using sigma notation.

(ii) Expand $\sum_{k=1}^{5}(2k^2-1)$ by writing out every term.

(iii) Evaluate $\sum_{r=3}^{7}(4r-1)$.

Solution. The symbol Σ is the Greek capital letter sigma. In mathematics it tells us to **sum** terms.

$$\sum_{k=1}^{5} a_k = a_1+a_2+a_3+a_4+a_5.$$

The lower value tells us where the counter starts, the upper value tells us where it stops, and the expression tells us what each term is.
For (i), the sequence $5, 8, 11, \ldots, 32$ is arithmetic with first term 5 and common difference 3, so its nth term is

$$u_n = 5+3(n-1) = 3n+2.$$

To find the final value of n, solve $3n+2=32$, giving $n=10$. Therefore

$$5+8+11+\cdots+32 = \sum_{n=1}^{10}(3n+2).$$

For (ii), substitute $k=1,2,3,4,5$:

$$\sum_{k=1}^{5}(2k^2-1) = 1+7+17+31+49.$$

For (iii), substitute $r=3,4,5,6,7$ and add:

$$\sum_{r=3}^{7}(4r-1) = 11+15+19+23+27 = 95.$$

Answer: (i) $\sum_{n=1}^{10}(3n+2)$; (ii) $1+7+17+31+49$; (iii) 95.

Example 3: geometric sequence and compound change

Question. A population of 1200 grows by 6% each year. Estimate the population after 5 years.

$$A = 1200(1.06)^5 \approx 1605.87.$$

Answer: About 1606 people.

Example 4: solving an exponential equation by matching bases

Question. Solve $2^{x+1} = 32$.

$$32 = 2^5,$$

so

$$2^{x+1} = 2^5 \quad \Rightarrow \quad x+1=5 \quad \Rightarrow \quad x=4.$$

Answer: $x=4$.

Example 5: solving an exponential equation using logarithms

Question. Solve $3^x = 20$.

$$x = \log_3 20 = \frac{\ln 20}{\ln 3} \approx 2.727.$$

Answer: $x \approx 2.727$.

HL extension: infinite geometric series and binomial patterns

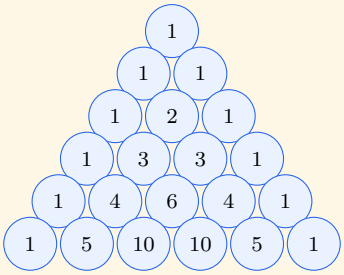
If $|r| < 1$, the terms of a geometric sequence get smaller and smaller. The infinite sum is

$$S_\infty = \frac{u_1}{1-r}.$$

For example,

$$3 + 1.5 + 0.75 + \cdots = \frac{3}{1-0.5} = 6.$$

The binomial coefficients in $(a+b)^n$ follow Pascal's triangle:



So

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5.$$

Practice questions: Week 3

1. Find the next three terms: $5, 8, 11, 14, \ldots$.
2. Find the 30th term of $5, 8, 11, 14, \ldots$.
3. Find the sum of the first 30 terms of $5, 8, 11, 14, \ldots$.
4. An arithmetic sequence has $u_5 = 18$ and $u_{12} = 46$. Find the common difference.
5. For the sequence in question 4, find u_{20}.
6. For the sequence in question 4, find S_{20}.
7. Find the next three terms: $3, 6, 12, 24, \ldots$.
8. Find the 10th term of $3, 6, 12, 24, \ldots$.
9. Find the sum of the first 10 terms of $3, 6, 12, 24, \ldots$.
10. A geometric sequence has $u_1 = 81$ and $r = \frac{1}{3}$. Find u_5.
11. Expand $\sum_{r=1}^{5}(2r + 1)$ and evaluate it.
12. Evaluate $\sum_{r=1}^{4} r^2$.
13. Write $4 + 8 + 12 + \cdots + 40$ using sigma notation.
14. A savings account contains €900 and grows by 3.5% per year. Find its value after 6 years.
15. A car worth €18000 depreciates by 12% per year. Estimate its value after 4 years.
16. Use the model $A = 1200(1.06)^n$ to estimate the amount after 5 years.
17. Solve $5^x = 125$.
18. Solve $4^{x-1} = 64$.
19. Solve $2^x = 13$, giving your answer to three decimal places.
20. Solve $e^{2x} = 7$, giving your answer to three decimal places.
21. Write $\log_2 32 = 5$ in exponential form.
22. Write $10^3 = 1000$ in logarithmic form.
23. Find $\log_4 64$.
24. Find $\ln(e^5)$.
25. Find $\log_{10}(0.01)$.
26. Solve $\log_2 x = 6$.
27. Solve $\ln x = 2$.
28. Find a formula for the arithmetic sequence $6, 10, 14, 18, 22, \ldots$.
29. Explain why the sequence $100, 80, 64, 51.2, 40.96, \ldots$ is geometric and find its common ratio.
30. For the sequence $100, 80, 64, 51.2, 40.96, \ldots$, predict the sixth term.
31. For an exponential decay graph of the form $y = 5(0.7)^x$, state the horizontal asymptote.
32. A linear model is $L(n) = 50 + 20n$. An exponential model is $E(n) = 50(1.2)^n$. Find $L(3)$ and $E(3)$.
33. Find the smallest integer n such that $600(1.04)^n > 800$.
34. An investment grows from 1000 to 1331 in 3 years. Find the annual percentage growth rate.

35. Find the infinite sum $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$.
36. A geometric sequence has $u_3 = 20$ and $u_6 = 160$. Find the positive common ratio.
37. Solve $7 \cdot 2^{n-1} = 224$.
38. Find the sum $12 + 18 + 24 + \cdots + 96$.
39. A sequence has first differences $5, 5, 5, 5, \ldots$. Is it arithmetic or geometric?
40. A sequence has ratios $1.5, 1.5, 1.5, \ldots$. Is it arithmetic or geometric?

HL extension questions

41. Find S_∞ for $12 + 6 + 3 + \cdots$.
42. Find the values of x for which the infinite series $4 + 4x + 4x^2 + \cdots$ converges.
43. Expand $(1 + 2x)^5$ up to and including the term in x^3.
44. Use logarithms to solve $5(1.08)^n > 10$ for the smallest integer n.
45. Expand $(2 - x)^4$.
46. Find the coefficient of x^3 in $(1 + 3x)^6$.
47. Solve $3e^{0.2t} = 15$, giving an exact answer.
48. Use the first three terms of $(1 + x)^5$ to approximate $(1.02)^5$.

Answer key: Week 3

1. $17, 20, 23$.
2. 92.
3. 1455.
4. $d = 4$.
5. 78.
6. 800.
7. $48, 96, 192$.
8. 1536.
9. 3069.
10. 1.
11. $3 + 5 + 7 + 9 + 11 = 35$.
12. $1 + 4 + 9 + 16 = 30$.
13. $\sum_{r=1}^{10} 4r$.
14. $900(1.035)^6 \approx €1106.33$.
15. $18000(0.88)^4 \approx €10791.16$.
16. About 1606.
17. $x = 3$.
18. $x = 4$.
19. $x \approx 3.700$.
20. $x \approx 0.973$.
21. $2^5 = 32$.
22. $\log_{10} 1000 = 3$.
23. 3.
24. 5.
25. -2.
26. $x = 64$.
27. $x = e^2$.
28. $u_n = 4n + 2$.
29. It has constant ratio 0.8.
30. 32.768.
31. $y = 0$.
32. $L(3) = 110$, $E(3) = 86.4$.
33. $n = 8$.
34. 10%.

35. 1.
36. $r = 2$.
37. $n = 6$.
38. 810.
39. Arithmetic.
40. Geometric.
41. 24.
42. $-1 < x < 1$.
43. $1 + 10x + 40x^2 + 80x^3$.
44. $n = 10$.
45. $16 - 32x + 24x^2 - 8x^3 + x^4$.
46. $\binom{6}{3}3^3 = 540$.
47. $t = 5 \ln 5$.
48. $1 + 5(0.02) + 10(0.02)^2 = 1.104$.

Week 4: Trigonometry, geometry and vectors

Week 4: Trigonometry, geometry and vectors

Right and non-right triangles, exact trig values, radians, trigonometric graphs, bearings and vector diagrams.

Key results

- Pythagoras: $a^2 + b^2 = c^2$ in a right-angled triangle.
- SOHCAHTOA: $\sin\theta = \dfrac{O}{H}$, $\cos\theta = \dfrac{A}{H}$, $\tan\theta = \dfrac{O}{A}$.
- Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.
- Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$.
- Area of triangle: $\dfrac{1}{2}ab\sin C$.
- Radians: $180° = \pi$ radians.
- Arc length: $s = r\theta$ when θ is in radians.
- Sector area: $A = \dfrac{1}{2}r^2\theta$ when θ is in radians.
- Vector magnitude: $\left|\begin{pmatrix} x \\ y \end{pmatrix}\right| = \sqrt{x^2 + y^2}$.

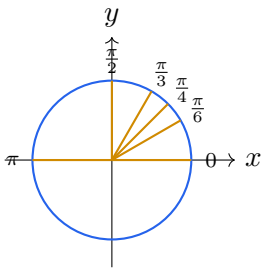
A first look at radians. Degrees divide a full turn into 360 equal parts. Radians measure angles in a more natural way: an angle of 1 radian cuts off an arc whose length is equal to the radius. Since a full circle has circumference $2\pi r$, one full turn is 2π radians, so $180° = \pi$ radians and $90° = \dfrac{\pi}{2}$ radians.

Exact trig values in degrees

x	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Exact trig values in radians

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

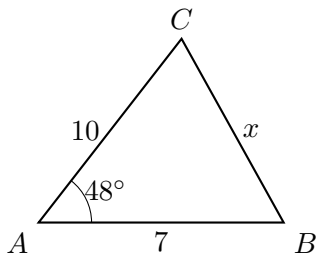


AA bridge. Trigonometry in IB AA quickly moves from degrees to radians. The unit circle helps connect exact values, graphs and identities.

Worked examples

Example 1: cosine rule

Question. In triangle ABC, $AB = 7$, $AC = 10$ and $\angle BAC = 48°$. Find BC.



$$x^2 = 7^2 + 10^2 - 2(7)(10)\cos 48°.$$

$$x \approx \sqrt{55.32} \approx 7.44.$$

Answer: $BC \approx 7.44$.

Example 2: exact trigonometry

Question. Find the exact value of $\sin 60° + \cos 30°$.

$$\sin 60° = \frac{\sqrt{3}}{2}, \qquad \cos 30° = \frac{\sqrt{3}}{2}.$$

Therefore

$$\sin 60° + \cos 30° = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}.$$

Answer: $\sqrt{3}$.

Example 3: reading and converting radians

Question. Convert $120°$ to radians and convert $\dfrac{3\pi}{4}$ radians to degrees.

Reminder. Use the facts $180° = \pi$ radians and $1° = \dfrac{\pi}{180}$ radians.

$$120° = 120 \times \frac{\pi}{180} = \frac{2\pi}{3} \text{ radians.}$$

To convert the other way, multiply by $\dfrac{180}{\pi}$:

$$\frac{3\pi}{4} \times \frac{180}{\pi} = 135°.$$

Answer: $120° = \dfrac{2\pi}{3}$ radians and $\dfrac{3\pi}{4}$ radians $= 135°$.

Example 4: radians, arc length and sector area

Question. A sector has radius 6 cm and angle $\dfrac{2\pi}{3}$ radians. Find its arc length and area.

$$s = r\theta = 6 \cdot \frac{2\pi}{3} = 4\pi \text{ cm}.$$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(36) \cdot \frac{2\pi}{3} = 12\pi \text{ cm}^2.$$

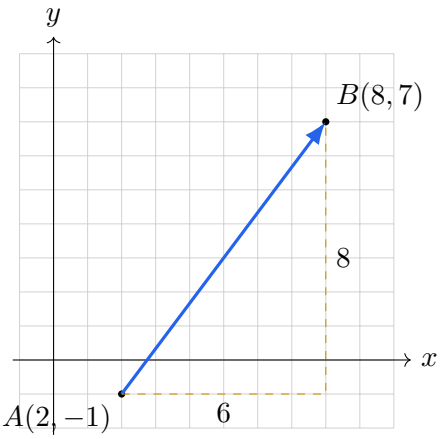
Answer: Arc length 4π cm; sector area 12π cm^2.

Example 5: vectors as movement

Question. Points $A(2,-1)$ and $B(8,7)$ are given. Find $\overrightarrow{AB}$ and $|\overrightarrow{AB}|$.

$$\overrightarrow{AB} = \begin{pmatrix} 8-2 \\ 7-(-1) \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}.$$

$$|\overrightarrow{AB}| = \sqrt{6^2+8^2} = \sqrt{100} = 10.$$



Answer: $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$ and $|\overrightarrow{AB}| = 10$.

HL extension: trigonometric identities and proof habits

The identity

$$\sin^2\theta + \cos^2\theta = 1$$

comes from the unit circle. Future HL students should get used to proving or transforming one side of an identity into the other.

Worked extension. Show that $1 - \cos^2\theta = \sin^2\theta$.

$$\sin^2\theta + \cos^2\theta = 1.$$

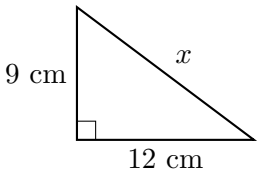
Subtract $\cos^2\theta$ from both sides:

$$\sin^2\theta = 1 - \cos^2\theta.$$

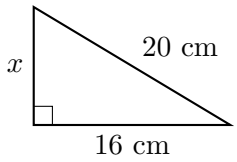
This is the required result.

Practice questions: Week 4

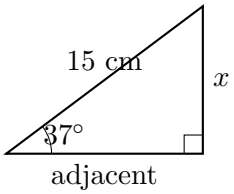
1. Find the hypotenuse of a right-angled triangle with shorter sides 9 cm and 12 cm.



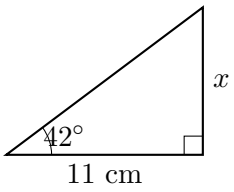
2. A right-angled triangle has hypotenuse 20 cm and one shorter side 16 cm. Find the other shorter side.



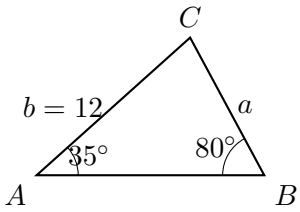
3. In a right-angled triangle, $\theta = 37°$ and the hypotenuse is 15 cm. Find the opposite side.



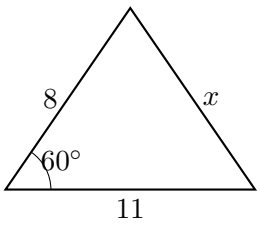
4. In a right-angled triangle, $\theta = 42°$ and the adjacent side is 11 cm. Find the opposite side.



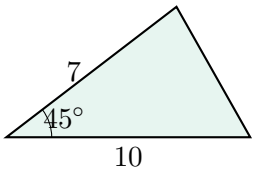
5. In triangle ABC, $A = 50°$, $B = 64°$. Find C.
6. Use the sine rule to find a if $A = 35°$, $B = 80°$ and $b = 12$.



7. Use the cosine rule to find the third side when two sides are 8 and 11 with included angle 60°.



8. Find the area of a triangle with sides 7 and 10 enclosing an angle of 45°.

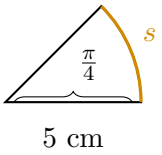


9. Convert 60° to radians.
10. Convert $\frac{5\pi}{6}$ radians to degrees.

11. Find the exact value of $\sin 30° + \cos 60°$.

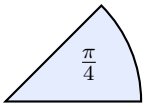
12. Find the exact value of $\tan 45° + \cos 0°$.

13. A sector has radius 5 cm and angle $\frac{\pi}{4}$. Find its arc length.



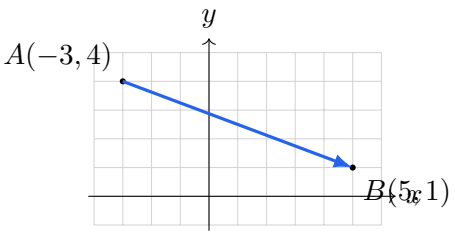
5 cm

14. A sector has radius 5 cm and angle $\frac{\pi}{4}$. Find its area.



5 cm

15. Find $\overrightarrow{AB}$ for $A(-3, 4)$ and $B(5, 1)$.

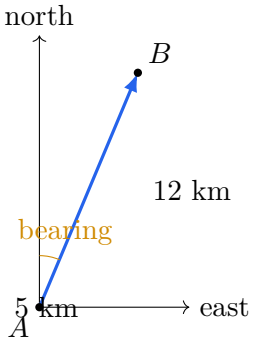


16. Find the magnitude of $\begin{pmatrix} -6 \\ 8 \end{pmatrix}$.

17. Translate $(2, -5)$ by $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$.

18. Reflect $(4, -2)$ in the y-axis.

19. From point A, point B is 5 km east and 12 km north. Find the bearing of B from A.



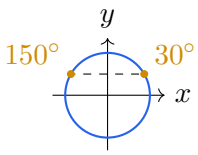
20. State the maximum value of $y = \sin x$ for $0 \leq x \leq 2\pi$.

21. Find $\sin \frac{\pi}{6}$.

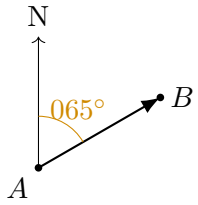
22. Find $\cos \frac{2\pi}{3}$.

23. Find $\tan \frac{3\pi}{4}$.

24. Solve $\sin\theta = \frac{1}{2}$ for $0° \leq \theta \leq 360°$.



25. The bearing of B from A is 065°. Find the bearing of A from B.



26. Find $\begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} -5 \\ 7 \end{pmatrix}$.

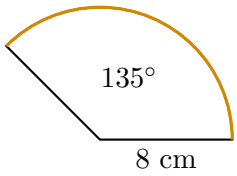
27. If $\mathbf{a} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, find $2\mathbf{a} - \mathbf{b}$.

28. Find the midpoint of $A(2, 5)$ and $B(8, -1)$.

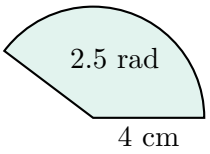
29. Find the area of the parallelogram formed by $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$.

30. Convert $\frac{\pi}{12}$ radians to degrees.

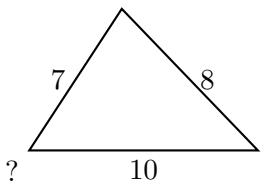
31. A sector has radius 8 cm and angle 135°. Find the arc length exactly.



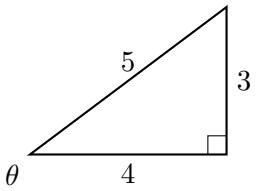
32. A sector has radius 4 cm and angle 2.5 radians. Find its area.



33. A triangle has sides 7, 8 and 10. Find the angle opposite the side of length 10.



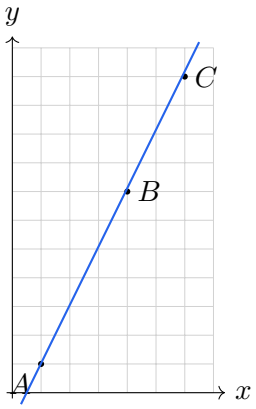
34. Find the exact values of $\sin\theta$ and $\cos\theta$ if $\tan\theta = \frac{4}{3}$ and θ is acute.



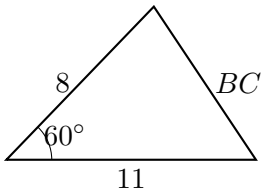
35. Evaluate $\sin^2 30° + \cos^2 30°$.

36. Sketch $y = \sin x$ for $0 \leq x \leq 2\pi$, labelling zeros, maximum and minimum.

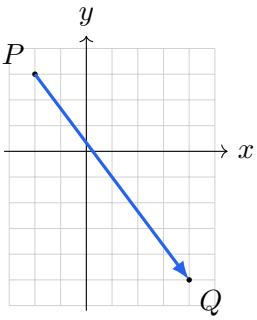
37. Determine whether $A(1,1)$, $B(4,7)$ and $C(6,11)$ are collinear.



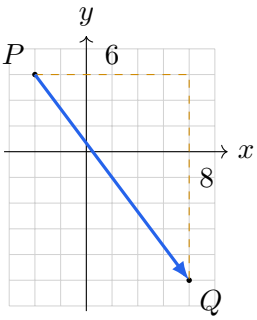
38. In triangle ABC, $AB = 11$, $AC = 8$ and angle $A = 60°$. Find BC using the cosine rule.



39. Find the vector from $P(-2,3)$ to $Q(4,-5)$.



40. Find the length of the vector from $P(-2,3)$ to $Q(4,-5)$.



HL extension questions

41. Show that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ gives $\tan 45° = 1$.
42. Solve $2\sin x = 1$ for $0° \le x \le 360°$.
43. Find the angle between $\begin{pmatrix}1\\2\end{pmatrix}$ and $\begin{pmatrix}4\\-1\end{pmatrix}$ using $\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$.
44. Prove that $\dfrac{1-\cos^2 x}{\sin x} = \sin x$, for $\sin x \neq 0$.
45. Solve $\cos x = -\dfrac{1}{2}$ for $0 \le x \le 2\pi$.
46. If $\mathbf{a} = \begin{pmatrix}2\\k\end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix}3\\-1\end{pmatrix}$ are perpendicular, find k.
47. Solve $\tan^2 x = 3$ for $0 \le x \le 2\pi$.
48. Use $\sin^2 x + \cos^2 x = 1$ to show that $1 + \tan^2 x = \dfrac{1}{\cos^2 x}$.

Answer key: Week 4

1. 15 cm.
2. 12 cm.
3. $15\sin 37° \approx 9.03$ cm.
4. $11\tan 42° \approx 9.90$ cm.
5. $66°$.
6. $a = \dfrac{12\sin 35°}{\sin 80°} \approx 6.99$.
7. $\sqrt{97} \approx 9.85$.
8. $\dfrac{35\sqrt{2}}{2} \approx 24.75$.
9. $\dfrac{\pi}{3}$.
10. $150°$.
11. 1.
12. 2.
13. $\dfrac{5\pi}{4}$ cm.
14. $\dfrac{25\pi}{8}$ cm^2.
15. $\begin{pmatrix}8\\-3\end{pmatrix}$.
16. 10.
17. $(9,-2)$.
18. $(-4,-2)$.
19. $\tan^{-1}(5/12) \approx 22.6°$, so bearing $023°$.
20. 1.
21. $\dfrac{1}{2}$.
22. $-\dfrac{1}{2}$.
23. -1.
24. $30°, 150°$.
25. $245°$.
26. $\begin{pmatrix}-2\\5\end{pmatrix}$.
27. $\begin{pmatrix}5\\6\end{pmatrix}$.
28. $(5,2)$.
29. $|3\cdot 5 - 1\cdot 2| = 13$.
30. $15°$.
31. 6π cm.
32. 20 cm^2.
33. $\cos C = \dfrac{13}{112}$, so $C \approx 83.3°$.
34. $\sin\theta = \dfrac{4}{5}$, $\cos\theta = \dfrac{3}{5}$.
35. 1.

36. Zeros at $0, \pi, 2\pi$; maximum $(\pi/2, 1)$; minimum $(3\pi/2, -1)$.

37. Yes; $\overrightarrow{AB} = (3, 6)$ and $\overrightarrow{BC} = (2, 4)$ are parallel.

38. $x = \sqrt{97} \approx 9.85$.

39. $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$.

40. 10.

41. $\tan 45^\circ = \dfrac{\sqrt{2}/2}{\sqrt{2}/2} = 1$.

42. $x = 30^\circ, 150^\circ$.

43. $\theta = \cos^{-1}\left(\dfrac{2}{\sqrt{85}}\right) \approx 77.5^\circ$.

44. $\dfrac{1 - \cos^2 x}{\sin x} = \dfrac{\sin^2 x}{\sin x} = \sin x$.

45. $x = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$.

46. $k = 6$.

47. $x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$.

48. Divide $\sin^2 x + \cos^2 x = 1$ by $\cos^2 x$ to get $\tan^2 x + 1 = \dfrac{1}{\cos^2 x}$.

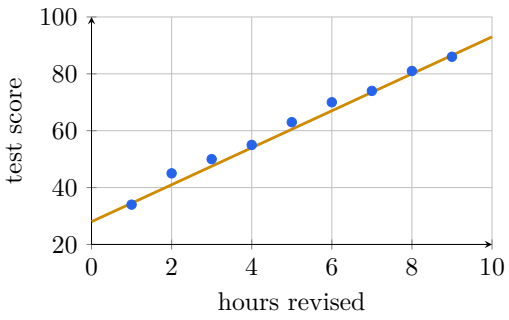
Week 5: Probability, statistics and calculus readiness

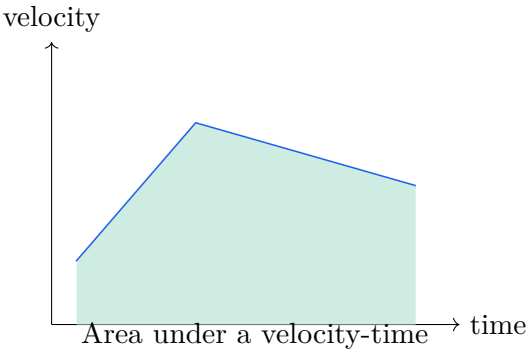
Week 5: Probability, statistics and calculus readiness

Data, probability diagrams, correlation, regression language, gradients, rates of change and area accumulation.

Key results

- Mean: $\bar{x} = \dfrac{\sum x}{n}$.
- Frequency table mean: $\bar{x} = \dfrac{\sum fx}{\sum f}$.
- Probability scale: $0 \le P(A) \le 1$.
- Complement: $P(A') = 1 - P(A)$.
- Tree diagrams multiply along branches and add separate routes.
- Correlation describes association, not proof of causation.
- Gradient of a line: rate of change.
- Average rate of change: $\dfrac{f(b) - f(a)}{b - a}$.
- Area under a rate graph gives accumulated change.





Area under a velocity-time graph gives distance or displacement, depending on signs.

Worked examples

Example 1: mean from a frequency table

Question. The table shows the number of books read by students in a month. Estimate the mean.

Books x	0	1	2	3	4
Frequency f	3	5	9	7	2

$$\sum f = 3 + 5 + 9 + 7 + 2 = 26.$$

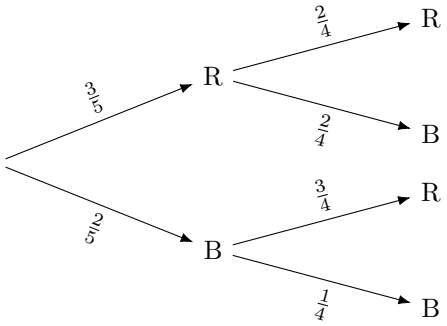
$$\sum fx = 0(3) + 1(5) + 2(9) + 3(7) + 4(2) = 52.$$

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{52}{26} = 2.$$

Answer: The mean is 2 books.

Example 2: probability tree

Question. A bag contains 3 red counters and 2 blue counters. Two counters are chosen without replacement. Find the probability of two red counters.

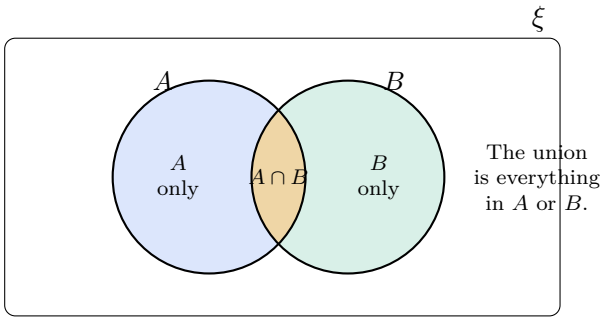


$$P(\text{red then red}) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}.$$

Answer: $\frac{3}{10}$.

Example 3: using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Question. In a survey, $P(A) = 0.55$, $P(B) = 0.40$ and $P(A \cap B) = 0.18$. Find $P(A \cup B)$.



If we add $P(A) + P(B)$, the overlap $P(A \cap B)$ is included twice, so we subtract it once:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

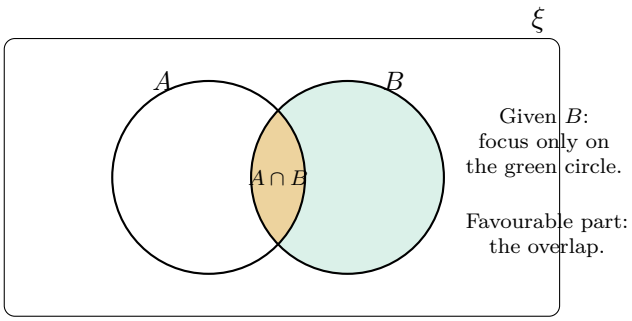
Substitute the values:

$$P(A \cup B) = 0.55 + 0.40 - 0.18 = 0.77.$$

Answer: $P(A \cup B) = 0.77$.

Example 4: conditional probability $P(A \mid B)$

Question. Suppose $P(B) = 0.30$ and $P(A \cap B) = 0.12$. Find $P(A \mid B)$.



Conditional probability means "the probability of A given that B has already happened". So we restrict

our attention to the outcomes inside B only. The favourable part is the overlap $A \cap B$. Therefore

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

Substitute the values:

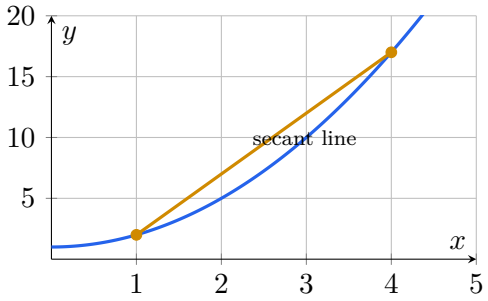
$$P(A \mid B) = \frac{0.12}{0.30} = 0.40.$$

This means that among the outcomes in B, 40% are also in A. **Answer:** $P(A \mid B) = 0.40$.

Example 5: average rate of change

Question. Let $f(x) = x^2 + 1$. Find the average rate of change from $x = 1$ to $x = 4$.

$$\frac{f(4) - f(1)}{4 - 1} = \frac{(17) - (2)}{3} = 5.$$



Answer: The average rate of change is 5.

Example 6: area as accumulation

Question. A cyclist's speed is constant at 6 m/s for 10 seconds. Find the distance travelled.

$$\text{distance} = \text{area under speed-time graph} = 6 \times 10 = 60.$$

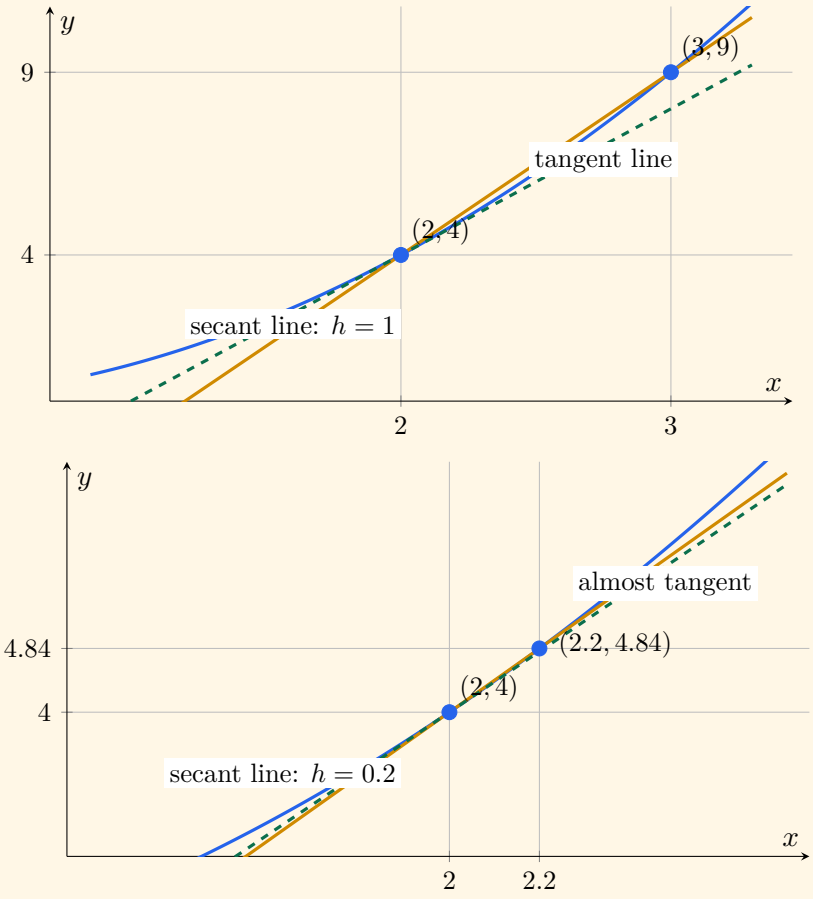
Answer: The cyclist travels 60 m.

HL extension: towards calculus: tangent gradient

The gradient of a curve at one point can be approximated by secant gradients over smaller intervals. For $f(x) = x^2$, the average gradient from $x = 2$ to $x = 2 + h$ is

$$\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^2 - 4}{h} = \frac{4 + 4h + h^2 - 4}{h} = 4 + h.$$

If $h = 1$, the secant gradient is $4 + 1 = 5$. If $h = 0.2$, the secant gradient is $4 + 0.2 = 4.2$. These are getting closer to 4, which is the gradient of the tangent at $x = 2$.



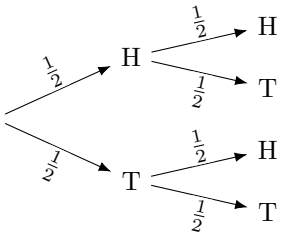
As $h \to 0$, the secant line turns into the tangent line, so the secant gradient approaches the curve's gradient at $x = 2$. This is the central idea of differentiation from first principles.

Practice questions: Week 5

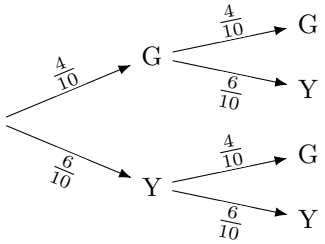
1. Find the mean of $4, 7, 8, 10, 11$.
2. Find the median of $3, 9, 4, 12, 6$.
3. Find the interquartile range of $2, 4, 5, 7, 8, 10, 13$.
4. A frequency table has values $1, 2, 3, 4$ with frequencies $2, 5, 6, 3$. Find the mean.

x	1	2	3	4
f	2	5	6	3

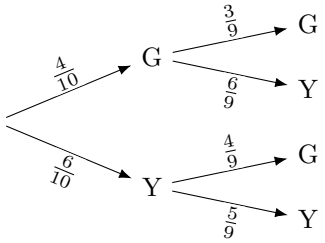
5. A class has 12 boys and 18 girls. One student is chosen at random. Find $P(\text{girl})$.
6. If $P(A) = 0.37$, find $P(A')$.
7. A fair six-sided die is rolled. Find $P(\text{even})$.
8. Two fair coins are tossed. Find $P(\text{two heads})$.



9. A bag has 4 green and 6 yellow counters. One counter is chosen and replaced, then another is chosen. Find $P(\text{two green})$.

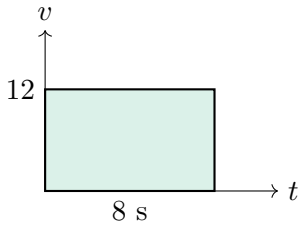


10. A bag has 4 green and 6 yellow counters. Two counters are chosen without replacement. Find $P(\text{two green})$.

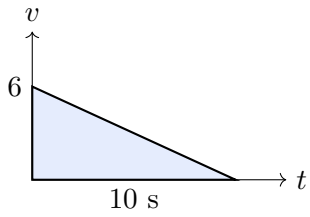


11. Explain why strong positive correlation does not necessarily mean causation.
12. The line of best fit for a data set is $y = 3.2x + 14$. Interpret the gradient.
13. Let $f(x) = 2x^2 - 3$. Find the average rate of change from $x = 1$ to $x = 3$.
14. Let $g(t) = 5t + 2$. Find the average rate of change from $t = 0$ to $t = 6$.

15. A velocity-time graph is a rectangle of height 12 m/s and width 8 s. Find the displacement.



16. A velocity-time graph is a triangle with base 10 s and height 6 m/s. Find the distance travelled.

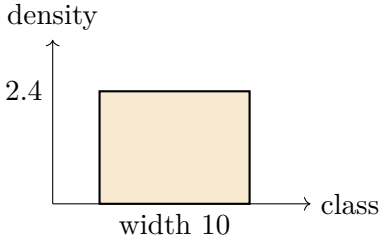


17. Estimate the mean from grouped data with intervals $0 \le x < 10$, $10 \le x < 20$, $20 \le x < 30$ and frequencies $4, 9, 7$.

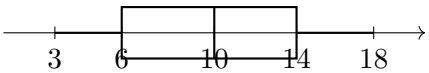
Class	$0 \le x < 10$	$10 \le x < 20$	$20 \le x < 30$
Frequency	4	9	7

18. Find the frequency density for a class of width 5 and frequency 18.

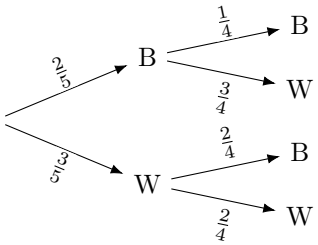
19. A histogram bar has class width 10 and frequency density 2.4. Find the frequency.



20. A box plot has minimum 3 and maximum 18. Find the range.

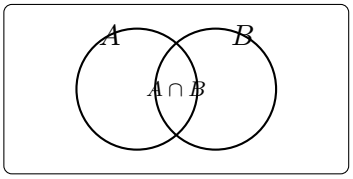


21. A two-stage experiment has $P(B_1) = \frac{2}{5}$ and $P(W_1) = \frac{3}{5}$. If the first result is B, then $P(B_2) = \frac{1}{4}$ and $P(W_2) = \frac{3}{4}$. If the first result is W, then $P(B_2) = \frac{2}{4}$ and $P(W_2) = \frac{2}{4}$. Find the probability that the two results are the same.

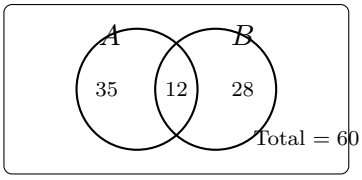


22. If $P(A) = 0.2$ and $P(B) = 0.7$ and A and B are independent, find $P(A \cap B)$.

23. If $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$, find $P(A \cup B)$.



24. In a group of 60, $n(A) = 35$, $n(B) = 28$ and $n(A \cap B) = 12$. Find the number in neither set.



25. A game pays €10 with probability 0.2 and loses €2 with probability 0.8. Find the expected value.

Outcome x	10	-2
Probability $P(X = x)$	0.2	0.8

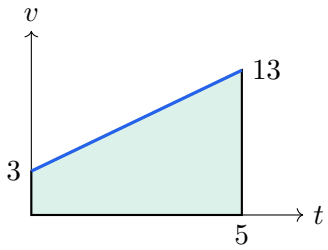
26. A line of best fit is $y = 1.8x + 12$. Predict y when $x = 20$.

27. If the actual value in question 26 is 52, find the residual actual − predicted.

28. A table gives $s(0) = 0$, $s(2) = 5$ and $s(4) = 12$. Find the average speed from $t = 0$ to $t = 4$.

t	0	2	4
s	0	5	12

29. A velocity-time graph is a straight line from $v = 3$ at $t = 0$ to $v = 13$ at $t = 5$. Find the distance travelled.

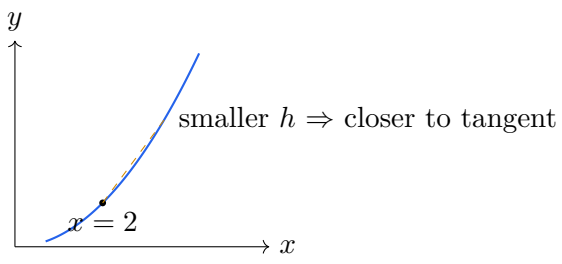


30. For $f(x) = x^2$, find the average gradient from $x = 2$ to $x = 2 + h$.

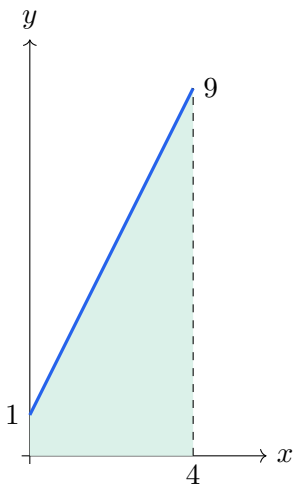
31. For $f(x) = x^2$, find the average gradient from $x = 3$ to $x = 3 + h$.

32. For $f(x) = x^2$, find the average gradient from $x = a$ to $x = a + h$.

33. Use your answer to question 30 to estimate the tangent gradient of $y = x^2$ at $x = 2$.



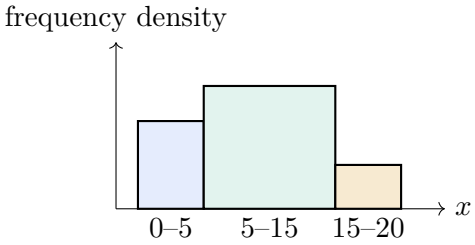
34. Find the area under $y = 2x + 1$ from $x = 0$ to $x = 4$ using a trapezium.



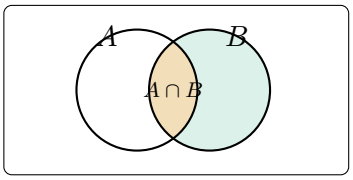
35. The table below gives values of a function. Find the average rate of change of y between $x = 0$ and $x = 3$.

x	0	1	2	3
y	2	4	8	14

36. A histogram has classes $0 \le x < 5$, $5 \le x < 15$, $15 \le x < 20$ with frequencies $10, 30, 5$. Find the three frequency densities.

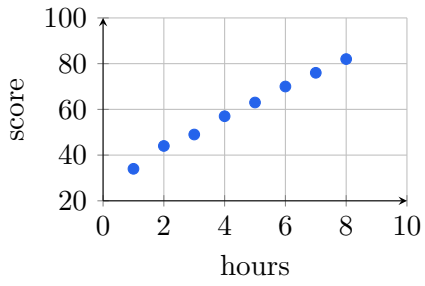


37. If $P(A \cap B) = 0.18$ and $P(B) = 0.6$, find $P(A \mid B)$.

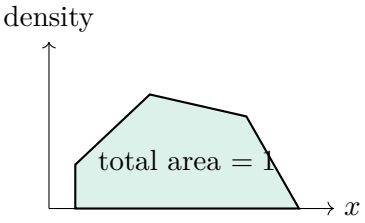


38. Give one reason why an outlier can affect the mean more than the median.

39. A data set comparing hours of revision with test score shows positive correlation. Describe the trend in words.



40. In probability density graphs, why should the total area be 1?



HL extension questions

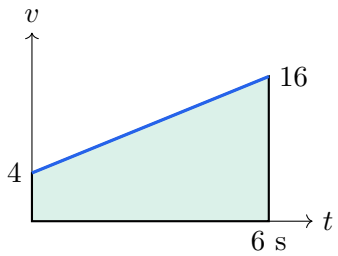
41. For $f(x) = x^3$, find the average gradient from $x = 1$ to $x = 1 + h$.

42. Hence find the limiting gradient of $y = x^3$ at $x = 1$ as $h \to 0$.

43. For $f(x) = x^2$, use first principles to show that the derivative at $x = a$ is $2a$.

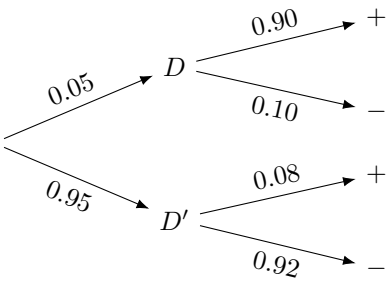
44. Events A and B have $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cap B) = 0.12$. Are they independent?

45. A velocity graph is a straight line from $v = 4$ to $v = 16$ over 6 seconds. Find the distance travelled.



46. A continuous model has $f(x) = kx$ on $0 \le x \le 2$. Find k if the total area under the graph is 1.

47. A medical test gives a positive result for 90% of people who have a condition. It also gives a positive result for 8% of people who do not have the condition. In a population, 5% of people have the condition. Use the tree diagram to find the probability that a randomly chosen person tests positive.



48. Using the information in question 47, find the probability that a person actually has the condition, given that they tested positive.

Answer key: Week 5

1. 8.
2. 6.
3. 6.
4. $\dfrac{42}{16} = 2.625$.
5. $\dfrac{3}{5}$.

6. 0.63.
7. $\frac{1}{2}$.
8. $\frac{1}{4}$.
9. $\frac{4}{25}$.
10. $\frac{2}{15}$.
11. A third variable or coincidence may explain the pattern; correlation alone does not prove cause.
12. For each increase of 1 in x, predicted y increases by 3.2.
13. 8.
14. 5.
15. 96 m.
16. 30 m.
17. 16.5.
18. 3.6.
19. 24.
20. 15.
21. $\frac{2}{5}$.
22. 0.14.
23. 0.9.
24. 9.
25. €0.40.
26. 48.
27. 4.
28. 3 units/s.
29. 40 m.
30. $4 + h$.
31. $6 + h$.
32. $2a + h$.
33. 4.
34. 20.
35. $\frac{14-2}{3-0} = 4$.
36. $2, 3, 1$.
37. 0.3.
38. The mean uses every value numerically, so an extreme value pulls it; the median depends mainly on order.
39. As hours increase, scores tend to increase.
40. Total area 1 represents total probability 1.
41. $\frac{(1+h)^3 - 1}{h} = 3 + 3h + h^2$.
42. 3.
43. $\frac{(a+h)^2 - a^2}{h} = 2a + h \to 2a$.
44. Yes, since $0.3 \times 0.4 = 0.12$.
45. 60 m.
46. $k = \frac{1}{2}$.
47. $P(+) = 0.05(0.90) + 0.95(0.08) = 0.121$.
48. $P(D \mid +) = \frac{0.05(0.90)}{0.121} = \frac{45}{121} \approx 0.372$.