Discrete random variables

IB Mathematics AI SL ▪ Section 4.7

50 original questions: distributions, expected value and fair games

Learning goal	Read and construct discrete probability distributions, calculate means and analyse fair games.
Syllabus link	AI SL 4.7: discrete random variables, probability distributions, expected value and applications.
Structure	Q1–15 short response; Q16–30 extended response; Q31–50 AI SL exam-style. Worked solutions begin on page 14.
Technology	A GDC is allowed throughout, as in AI SL examinations. The IB AI formula booklet may be used.

Instructions. Work on separate paper and show your reasoning. Give exact answers or answers to three significant figures, unless stated otherwise. Give money to the nearest cent when asked. Keep unrounded values for later calculations. Define each random variable, including its units, when asked to do so. For games, distinguish prize money from net gain after the entry fee. Show which values you include in an event and the value-times-probability sum for each mean. A calculator answer alone is not enough.

1 Short response

Read carefully, select the correct values and show a short calculation where needed.

1. Define the random variable

A class has 28 students. Attendance is recorded at the beginning of one lesson.

(a) Define a discrete random variable X for this experiment.

(b) For your definition, list the possible values of X.

2. Values and probabilities

A player's net gain G, in euros, on one play has the distribution

g	-1.5	0	2
$\mathrm{P}(G = g)$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{5}$

(a) Explain why G is discrete even though one value is not an integer.

(b) Find $\mathrm{P}(G = 1)$ and $\mathrm{P}(G \geq 0)$.

3. Select the correct values

Let X be the number of messages received in one five-minute interval, with

x	0	1	2	3
$\mathrm{P}(X = x)$	0.10	0.25	0.40	0.25

(a) Find $\mathrm{P}(X > 1)$, listing the values you include.

(b) Find $\mathrm{P}(0 < X \leq 2)$.

(c) Use a complement to find the probability of at least one message.

4. Check a proposed probability distribution

Each row is proposed as the complete list of probabilities for three distinct values of a random variable. Explain why each row is invalid.

(a) $0.20,\ 0.40,\ 0.50$.

(b) $-0.05,\ 0.25,\ 0.80$.

5. A missing constant

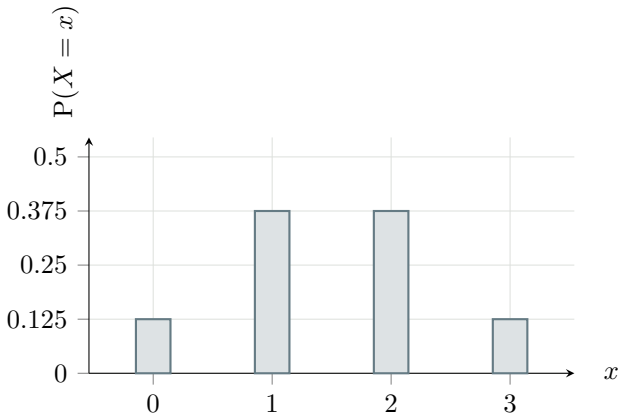
The number X of books borrowed on one visit has the distribution

x	0	1	3
$\mathrm{P}(X = x)$	$\frac{1}{5}$	k	$3k$

Find k and check that the completed distribution is valid.

6. Read the bar heights

The chart gives the distribution of X, the number of damaged items in one small package.



(a) Write down $\mathrm{P}(X = 1)$.

(b) Find $\mathrm{P}(X \geq 2)$.

7. Use a probability formula

On one spin, the score X has probability function $\mathrm{P}(X = x) = k(5 - x)$ for $x \in \{1, 2, 3, 4\}$.

(a) Find k.

(b) Find $\mathrm{P}(X = 4)$.

8. A probability-weighted mean

The number X of customers arriving in one minute has the distribution

x	0	1	4
$\mathrm{P}(X = x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

Calculate $\mathrm{E}(X)$.

9. Retain negative signs

The score X on one round of a game has the distribution

x	-3	0	2
$\mathrm{P}(X = x)$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{3}{10}$

Calculate $\mathrm{E}(X)$.

10. An expected total

The mean number of bicycles repaired per day is 2.4. Assume the same model applies on each of 20 working days.

(a) Find the expected total number repaired.

(b) Explain why this is not a prediction of the exact total.

11. Must the mean be possible?

Let X be the number of goals scored by a team in one match. A model gives $\mathrm{E}(X) = 1.8$. A student says, "This is impossible because a team cannot score 1.8 goals. Explain the error.

12. Prize or net gain?

A ticket costs €1. The prize R, in euros, is 6 with probability $\frac{1}{3}$ and 0 otherwise. Let G be the player's net gain on one ticket.

(a) Write down the two possible values of G.

(b) Calculate $\mathrm{E}(G)$ and decide whether the game is fair.

13. Fairness is about the mean

The net gain G, in euros, on one play is 8 with probability $\frac{1}{5}$ and -2 with probability $\frac{4}{5}$.

(a) Show that the game is fair.

(b) Find the probability of making a profit.

14. Use the wording carefully

The number X of parcels delivered to a home in one day has distribution

x	0	1	2	3
$\mathrm{P}(X = x)$	0.12	0.28	0.35	0.25

(a) Find the probability of at least one parcel, using a complement.

(b) Find the probability of fewer than two parcels.

15. The mean and the most likely value

The number X of goals scored by a team in one match has distribution

x	0	1	2
$\mathrm{P}(X = x)$	0.35	0.40	0.25

(a) Write down the most likely number of goals.

(b) Calculate $\mathrm{E}(X)$ and explain why it differs from your answer to part (a).

2 Extended response

Connect representations and explain your results in context.

16. Repeated labels on a spinner

A spinner has four equally likely sectors labelled $0, 0, 2, 5$. Let X be the score on one spin.

(a) Construct the probability distribution of X.

(b) Find $\mathrm{P}(X \geq 2)$.

(c) Calculate and interpret $\mathrm{E}(X)$.

17. Equally likely cards

Five cards are labelled $-2, -2, 1, 3, 3$. One card is chosen at random, each card being equally likely. Let X be its number.

(a) Construct the probability distribution of X.

(b) Find $\mathrm{P}(X \leq 1)$ and $\mathrm{E}(X)$.

(c) Explain why giving each distinct value probability $\frac{1}{3}$ would be incorrect.

18. Construct a distribution from outcomes

Two spinners are spun independently. Each has two equally likely sectors, labelled 1 and 3. Let S be the sum of the two scores.

	Second spinner	
First spinner	1	3
1		
3		

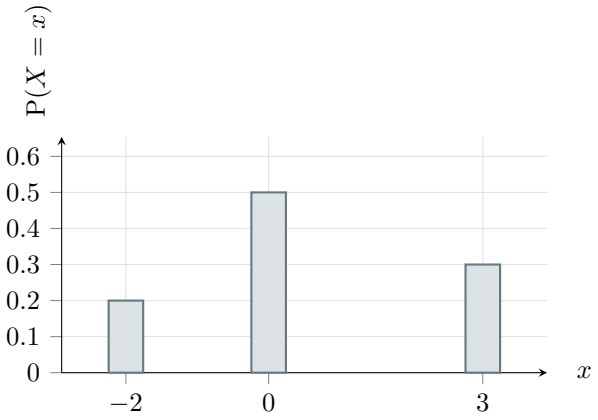
(a) Complete the outcome table, then construct the distribution of S.

(b) Find $\mathrm{P}(S \geq 4)$.

(c) Calculate $\mathrm{E}(S)$.

19. A chart with negative values

Let X be the score on one attempt. Its distribution is shown below.



(a) Construct a probability table.

(b) Find $\mathrm{P}(-2 < X \leq 3)$.

(c) Calculate $\mathrm{E}(X)$.

20. Use a given mean

The number X of parcels collected by a customer is modelled by

x	0	2	5
$\mathrm{P}(X = x)$	a	b	0.20

It is given that $\mathrm{E}(X) = 2$.

(a) Use the total probability to write an equation in a and b.

(b) Use the mean to find b, then find a. Check that the probabilities are valid.

(c) Find the probability that the customer collects at least one parcel.

21. From a formula to a mean

The number X of files sent in one transaction has distribution $\mathrm{P}(X = x) = k(x + 2)$ for $x \in \{1, 2, 3\}$.

(a) Find k and write the distribution as a table.

(b) Find $\mathrm{P}(X > 1)$.

(c) Calculate $\mathrm{E}(X)$.

22. An unknown value

The score X on one round has distribution

x	0	2	v
$\mathrm{P}(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

where $v > 2$ and $\mathrm{E}(X) = \frac{5}{2}$.

(a) Find v.

(b) Find $\mathrm{P}(X > \mathrm{E}(X))$.

23. Equal means, different chances

Two machines produce a score on one use. Machine A gives $0, 2, 4$ with probabilities $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$. Machine B gives 0 or 4, each with probability $\frac{1}{2}$.

(a) Calculate the expected score for each machine.

(b) For each machine, find the probability of a score of at least 3.

(c) Explain why the mean alone does not answer the question in part (b).

24. Keep the unrounded mean

Let X be the number of donations received in one hour. A model gives

x	0	1	2	3
$\mathrm{P}(X = x)$	0.173	0.298	0.361	0.168

(a) Calculate $\mathrm{E}(X)$ and give it to three significant figures. Interpret this mean.

(b) Using the unrounded mean, find the expected total over 75 hours, to one decimal place. Assume the same model applies each hour.

(c) Explain why the answer to part (b) does not need to be a whole number.

25. Build the net-gain distribution

A game costs €3 per play. The prize R, in euros, has distribution

r	0	4	10
$\mathrm{P}(R = r)$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{10}$

Let G be the player's net gain on one play.

(a) Construct the distribution of G.

(b) Calculate $\mathrm{E}(G)$ and the probability of a profit.

(c) Find the entry fee that would make the game fair.

26. The organiser's target

The prize R on one play is €0, €5 or €20, with probabilities 0.70, 0.20 and 0.10. The entry fee is c euros. The organiser pays only the prize and has no other costs.

(a) Find the mean prize.

(b) Find c if the organiser wants an expected profit of €1 per play.

(c) At this fee, find the player's probability of a profit.

27. Diagnose an incorrect mean

A model for the number X of visitors in one interval is

x	1	3	6
$\mathrm{P}(X = x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

A student calculates $\mathrm{E}(X) = \frac{1+3+6}{3} = \frac{10}{3}$.

(a) Explain the mistake.

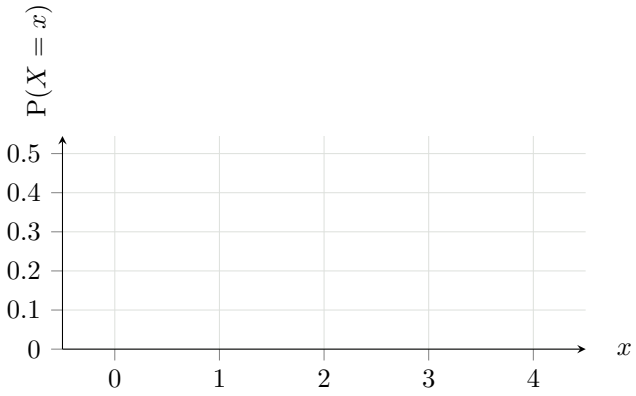
(b) Calculate the correct mean and interpret it.

28. Draw a probability bar chart

The number X of items sold in one interval has distribution

x	0	1	2	4
$\mathrm{P}(X = x)$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{5}$

(a) Draw a probability bar chart on the supplied axes. Keep the numerical spacing of the values correct.



(b) Find $\mathrm{P}(X = 3)$ and $\mathrm{E}(X)$.

29. A missing probability from a mean

The score X on one attempt has distribution

x	0	1	3
$\mathrm{P}(X = x)$	0.20	t	$0.80 - t$

It is given that $\mathrm{E}(X) = 1.10$.

(a) Write an equation using the expected-value formula.

(b) Solve for t and check the three probabilities.

(c) Find $\mathrm{P}(X > 1)$.

30. Two fair games

Both games cost €2 per play. Game A pays a prize of €12 with probability $\frac{1}{6}$ and nothing otherwise. Game B pays €4 with probability $\frac{1}{2}$ and nothing otherwise.

(a) Define a net-gain random variable for each game and show that both games are fair.

(b) Which game gives a greater probability of a profit?

(c) Does being fair guarantee that a player breaks even after ten plays? Explain.

3 AI SL exam-style / Paper 1

Original calculator-allowed questions. Show your setup and use the mark allocations to guide the depth of your working.

31. Daily bookings

[Maximum mark: 9]

Let X be the number of bookings made by one customer. Its distribution is

x	0	1	2	3
$\mathrm{P}(X = x)$	0.19	0.27	k	0.14

(a) Find k. [2 marks]

(b) Find the probability of at least two bookings. [2 marks]

(c) Calculate and interpret $\mathrm{E}(X)$. [3 marks]

(d) Assuming the same model applies to each customer, find the expected total number of bookings from 20 customers. [2 marks]

32. Two missing probabilities

[Maximum mark: 7]

The number X of items collected on one visit has distribution

x	0	2	3
$\mathrm{P}(X = x)$	a	0.25	b

It is given that $\mathrm{E}(X) = 1.75$.

(a) Write an equation using the total probability. [1 mark]

(b) Write an equation using the mean, and hence find a and b. [4 marks]

(c) Find $\mathrm{P}(X \geq 2)$. [2 marks]

33. A decreasing probability function

[Maximum mark: 7]

The score X on one attempt has probability function $\mathrm{P}(X = x) = k(6 - x)$ for $x \in \{1, 2, 3, 4\}$.

(a) Find k. [2 marks]

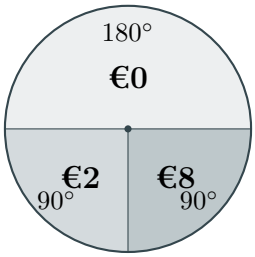
(b) Find $\mathrm{P}(2 \leq X < 4)$. [2 marks]

(c) Calculate $\mathrm{E}(X)$. [3 marks]

34. A spinner game

[Maximum mark: 8]

The spinner below is equally likely to stop in any direction. The three sectors show the prize amounts in euros; their angles are labelled. It costs €3 to spin once.



Let R be the prize, in euros, and G the player's net gain, in euros, on one spin.

(a) Construct the distribution of R. [2 marks]

(b) Construct the distribution of G. [2 marks]

(c) Calculate $\mathrm{E}(G)$ and state whether the game is fair. [3 marks]

(d) Find $\mathrm{P}(G > 0)$. [1 mark]

35. The larger of two numbers

[Maximum mark: 7]

A card numbered 0, 1 or 2 is chosen at random from three cards, replaced, and a second card is chosen independently. Let X be the larger of the two numbers (or their common number if they are equal).

(a) Using a table of the nine equally likely ordered outcomes, find the distribution of X. [4 marks]

(b) Find $\mathrm{P}(X = 1)$. [1 mark]

(c) Calculate $\mathrm{E}(X)$. [2 marks]

36. A raffle

[Maximum mark: 7]

A raffle sells all 100 tickets for €1 each. Exactly one ticket wins €40, exactly four other tickets win €10 each, and all remaining tickets win nothing. Each ticket is equally likely to be selected by a buyer. Let R be the prize on one ticket, in euros.

(a) Construct the distribution of R. [2 marks]

(b) Calculate the player's expected net gain on one ticket. [2 marks]

(c) Find the organiser's total profit after all prizes are paid, ignoring other costs. [2 marks]

(d) Find the probability that one ticket makes a profit. [1 mark]

37. A delivery protection fee

[Maximum mark: 8]

For one delivery, a company pays compensation R of €0, €7.50 or €40 with probabilities 0.82, 0.14 and 0.04. It charges a separate €3.20 protection fee for each delivery. Ignore other income and costs.

(a) Calculate the expected compensation per delivery. [3 marks]

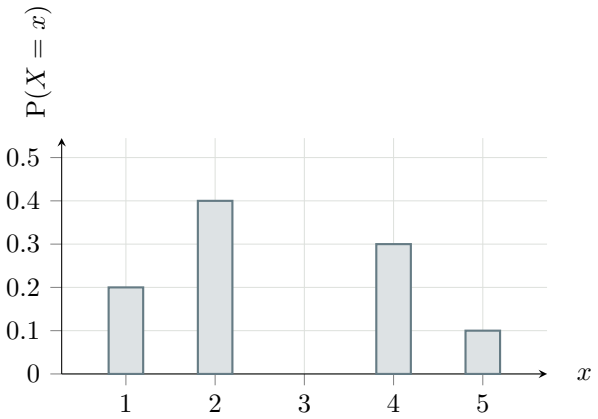
(b) Let C be the company's net income, in euros, from the protection fee and compensation on one delivery. Find $\mathrm{E}(C)$. [3 marks]

(c) Assuming the model remains unchanged, find the expected total compensation for 250 deliveries. [2 marks]

38. A probability chart

[Maximum mark: 6]

The chart shows the number X of orders received in one interval.



(a) Write down $\mathrm{P}(X = 3)$. [1 mark]

(b) Find $\mathrm{P}(1 < X \leq 4)$. [2 marks]

(c) Calculate $\mathrm{E}(X)$. [2 marks]

(d) Find the expected total over 50 intervals governed by this model. [1 mark]

39. Use the mean to find a probability

[Maximum mark: 7]

The player's net gain G, in euros, on one play is distributed as

g	-2	1	7
$\mathrm{P}(G = g)$	0.50	p	$0.50 - p$

It is given that $\mathrm{E}(G) = 0.40$.

(a) Write an equation in p using the mean. [2 marks]

(b) Find p and check that the completed distribution is valid. [3 marks]

(c) Find $\mathrm{P}(G > 0)$. [1 mark]

(d) State whether the game is fair, giving a reason. [1 mark]

40. Set an unknown prize [Maximum mark: 8]

A game costs €2.40 per play. Its prize R, in euros, is 0, 5 or w, with probabilities 0.72, 0.20 and 0.08, where $w > 5$. Let G be the player's net gain, in euros, on one play.

(a) Write down the three values of G. [1 mark]

(b) Find w so that the game is fair. [4 marks]

(c) For this value of w, find the probability of a profit. [1 mark]

(d) Find the expected total prize money over 150 plays with the same model. [2 marks]

41. Compare a value with the mean [Maximum mark: 5]

The score X on one attempt has distribution

x	-2	1	m
$\mathrm{P}(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

where $m > 1$ and $\mathrm{E}(X) = 1$.

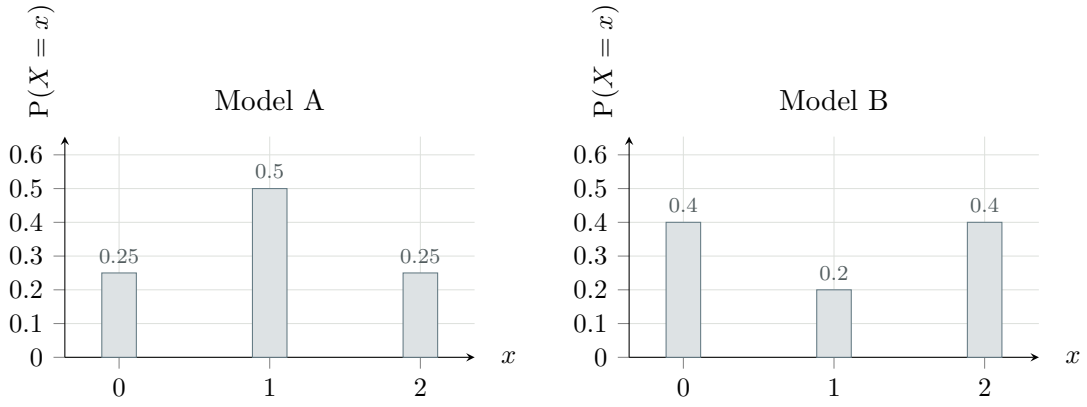
(a) Find m. [2 marks]

(b) Find $\mathrm{P}(X \geq \mathrm{E}(X))$. [2 marks]

(c) Find $\mathrm{P}(X = \mathrm{E}(X))$. [1 mark]

42. Compare two graphical models [Maximum mark: 6]

The charts give the number of seats booked in one transaction under two models, A and B.



(a) Calculate the expected number of seats under each model. [2 marks]

(b) Find the probability of at least one seat being booked under each model. [2 marks]

(c) A student claims that equal expected values imply identical distributions. Use these models to explain why this is false. [2 marks]

43. A die-based game

[Maximum mark: 8]

A fair six-sided die is rolled once. The prize is €0 for a score of 1 or 2, €2 for a score of 3, 4 or 5, and €6 for a score of 6. The entry fee is €2. Let R be the prize and G the player's net gain, both in euros.

(a) Construct the distribution of R. [2 marks]

(b) Calculate $\mathrm{E}(R)$. [2 marks]

(c) Construct the distribution of G and decide whether the game is fair. [3 marks]

(d) Find $\mathrm{P}(G > 0)$. [1 mark]

44. Two equations for a probability table

[Maximum mark: 6]

The number X of documents printed in one job has distribution

x	0	1	2	3
$\mathrm{P}(X = x)$	a	b	$2b$	a

It is given that $\mathrm{E}(X) = 1.60$.

(a) Show that $2a + 3b = 1$. [1 mark]

(b) Use the mean to write a second equation in a and b. [2 marks]

(c) Solve the equations and find $\mathrm{P}(X \leq 1)$. [3 marks]

45. Choose between two bonuses

[Maximum mark: 8]

For each completed delivery, a rider chooses either a guaranteed €2.50 bonus or a random bonus R. The random bonus is €0, €3 or €8 with probabilities 0.55, 0.30 and 0.15.

(a) Calculate $\mathrm{E}(R)$. [3 marks]

(b) Which option gives the higher expected bonus? [1 mark]

(c) Find the probability that the random bonus is strictly greater than the fixed bonus. [2 marks]

(d) Assuming the same model applies throughout, find the expected total bonus from each option over 120 deliveries. [2 marks]

46. Construct a model from two draws

[Maximum mark: 8]

A bag contains three blue counters and two yellow counters. A counter is drawn at random, replaced, and then a second counter is drawn independently. Let X be the number of yellow counters drawn.

(a) Find the probability of drawing no yellow counters. [2 marks]

(b) Find the probability of exactly one yellow counter, showing both possible orders. [3 marks]

(c) Complete the probability distribution of X and calculate $\mathrm{E}(X)$. [3 marks]

47. A fair entry fee and a practical price

[Maximum mark: 10]

A spinner has eight equal sectors. Two sectors pay €10, one pays €4, and five pay nothing. Let R be the prize in euros on one spin.

(a) Construct the distribution of R. [2 marks]

(b) Calculate $\mathrm{E}(R)$ and hence state the fair entry fee. [3 marks]

(c) The organiser charges €3.50. Define the player's net gain G and find $\mathrm{E}(G)$. [3 marks]

(d) At this fee, find the probabilities of a profit and of breaking even. [2 marks]

48. A mean determines a prize

[Maximum mark: 7]

A prize R, in euros, has the distribution

r	0	4	w
$\mathrm{P}(R = r)$	0.55	0.30	0.15

where $w > 4$ and $\mathrm{E}(R) = 3$.

(a) Write an equation using the mean and find w. [3 marks]

(b) State the fair entry fee. [1 mark]

(c) If the fee is €3.50, find the player's expected net gain. [2 marks]

(d) At this fee, find the probability of a profit. [1 mark]

49. Build a model from frequencies

[Maximum mark: 8]

Over 80 days, a repair shop records the number of returned items each day.

Returns in one day	0	1	2	3
Number of days	18	28	24	10

Let X be the number of returns on a future day. Use relative frequencies as a probability model.

(a) Construct the proposed probability distribution of X. [2 marks]

(b) Calculate and interpret $\mathrm{E}(X)$. [3 marks]

(c) Find the expected total over 200 future days. [2 marks]

(d) State one assumption needed for this forecast. [1 mark]

50. A claim about fairness

[Maximum mark: 9]

A game has prize R of €0, €5 or €15 with probabilities $\frac{1}{2}$, $\frac{2}{5}$ and $\frac{1}{10}$.

(a) Calculate $\mathrm{E}(R)$. [2 marks]

(b) Find the fair entry fee. [2 marks]

(c) When the fee is €4, find the player's expected net gain and probability of a profit. [3 marks]

(d) A student says, "A game is fair whenever the probability of a profit is one half. Evaluate this claim using your results. [2 marks]

Optional tutorial support

Distribution tables and bar charts
youtu.be/pC2p2yXxgYQ

Mean and mode of a discrete random variable
youtu.be/63glO_YdrYo

Scope and provenance. Syllabus: IB *Mathematics: applications and interpretation guide*, SL 4.7, p. 55. All questions and solutions are original. The project AI SL Paper 1 reference was consulted for command terms, subpart structure and mark-allocation style. The worksheet is limited to SL 4.7; it does not require binomial distribution formulas, variance or continuous random variables.

Answer key and worked solutions

Try the questions before reading the solutions. The question numbers and sections below match the worksheet.

Section 1: Short response — Questions 1–15

1. Solution

(a) For example, let X be the number of students absent at the beginning of this lesson.

(b) $X \in \{0, 1, 2, \ldots, 28\}$. An equally clear definition using the number present is also valid, with the same set of possible values.

2. Solution

(a) The possible values form a finite list. Discrete values need not be whole numbers.

(b) 1 is not a possible gain, so $\mathrm{P}(G = 1) = 0$. Also, $\mathrm{P}(G \geq 0) = \frac{3}{10} + \frac{1}{5} = \frac{1}{2}$.

3. Solution

(a) Include 2, 3: $\mathrm{P}(X > 1) = 0.40 + 0.25 = 0.65$.

(b) Include 1, 2: $\mathrm{P}(0 < X \leq 2) = 0.25 + 0.40 = 0.65$.

(c) At least one means $X \geq 1$. Thus $\mathrm{P}(X \geq 1) = 1 - \mathrm{P}(X = 0) = 1 - 0.10 = 0.90$.

4. Solution

(a) The sum is 1.10, not 1, although every entry lies in $[0, 1]$.

(b) The sum is 1, but -0.05 is negative. A valid distribution needs **both** a total probability of 1 and every probability between 0 and 1, inclusive.

5. Solution

$\frac{1}{5} + k + 3k = 1$, so $4k = \frac{4}{5}$ and $\boxed{k = \frac{1}{5}}$. The probabilities are $\frac{1}{5}, \frac{1}{5}, \frac{3}{5}$: each lies in $[0, 1]$ and their sum is 1.

6. Solution

(a) $\mathrm{P}(X = 1) = 0.375 = \frac{3}{8}$.

(b) $\mathrm{P}(X \geq 2) = 0.375 + 0.125 = 0.5 = \frac{1}{2}$.

7. Solution

(a) The probabilities are $4k, 3k, 2k, k$. Thus $10k = 1$ and $k = \frac{1}{10}$.

(b) $\mathrm{P}(X = 4) = k(5 - 4) = \frac{1}{10}$.

8. Solution

$\mathrm{E}(X) = 0(\frac{1}{2}) + 1(\frac{1}{4}) + 4(\frac{1}{4}) = \boxed{\frac{5}{4}}$ customers per minute.

9. Solution

$\mathrm{E}(X) = (-3)(\frac{1}{5}) + 0(\frac{1}{2}) + 2(\frac{3}{10}) = -\frac{3}{5} + \frac{3}{5} = \boxed{0}$.

10. Solution

(a) The expected total is $20(2.4) = \boxed{48}$ bicycles.

(b) An expected total is an average across repeated 20-day periods under the model. Individual totals can be different.

Section 1: Short response — Questions 1–15 (continued)

11. Solution

The mean is a long-run average, not a required outcome of one match. Whole-number goal counts can have a mean of 1.8.

12. Solution

(a) $G = R - 1$, so the net gains are 5 and -1 euros.

(b) $\mathrm{E}(G) = 5(\frac{1}{3}) + (-1)(\frac{2}{3}) = \boxed{€1}$. It is not fair: the player has a positive expected net gain.

13. Solution

(a) $\mathrm{E}(G) = 8(\frac{1}{5}) - 2(\frac{4}{5}) = 0$, so the game is fair.

(b) A profit means $G > 0$, so its probability is $\frac{1}{5}$, not $\frac{1}{2}$.

14. Solution

(a) $\mathrm{P}(X \geq 1) = 1 - \mathrm{P}(X = 0) = 1 - 0.12 = 0.88$.

(b) Fewer than two includes 0 and 1: $\mathrm{P}(X < 2) = 0.12 + 0.28 = 0.40$.

15. Solution

(a) The most likely value is 1, because it has the greatest probability, 0.40.

(b) $\mathrm{E}(X) = 0(0.35) + 1(0.40) + 2(0.25) = 0.90$ goals per match. The mean uses all values and their probabilities; the most likely value uses only the greatest probability.

Section 2: Extended response — Questions 16–30

16. Solution

(a)

x	0	2	5
$\mathrm{P}(X = x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

(b) $\mathrm{P}(X \geq 2) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

(c) $\mathrm{E}(X) = 0(\frac{1}{2}) + 2(\frac{1}{4}) + 5(\frac{1}{4}) = \frac{7}{4}$. The long-run mean score is 1.75 per spin.

17. Solution

(a)

x	-2	1	3
$\mathrm{P}(X = x)$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{2}{5}$

(b) $\mathrm{P}(X \leq 1) = \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$. Also $\mathrm{E}(X) = (-2)(\frac{2}{5}) + 1(\frac{1}{5}) + 3(\frac{2}{5}) = \frac{3}{5}$.

(c) The cards are equally likely, but the values are not: two cards show -2, one shows 1 and two show 3.

18. Solution

(a) The four equally likely sums are $2, 4, 4, 6$.

s	2	4	6
$\mathrm{P}(S = s)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(b) $\mathrm{P}(S \geq 4) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.

(c) $\mathrm{E}(S) = 2(\frac{1}{4}) + 4(\frac{1}{2}) + 6(\frac{1}{4}) = 4$.

Section 2: Extended response — Questions 16–30 (continued)

19. Solution

(a)

x	-2	0	3
$\mathrm{P}(X = x)$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{3}{10}$

(b) Include $0, 3$ but not -2: $\mathrm{P}(-2 < X \leq 3) = \frac{1}{2} + \frac{3}{10} = \frac{4}{5}$.

(c) $\mathrm{E}(X) = (-2)(\frac{1}{5}) + 0(\frac{1}{2}) + 3(\frac{3}{10}) = \frac{1}{2}$.

20. Solution

(a) $a + b + 0.20 = 1$, so $a + b = 0.80$.

(b) $0a + 2b + 5(0.20) = 2$, so $2b + 1 = 2$ and $b = 0.50$. Then $a = 0.80 - 0.50 = 0.30$. The entries $0.30, 0.50, 0.20$ lie in $[0, 1]$ and sum to 1.

(c) Include 2 and 5: $\mathrm{P}(X \geq 1) = 0.50 + 0.20 = 0.70$.

21. Solution

(a) $3k + 4k + 5k = 1$, so $k = \frac{1}{12}$.

x	1	2	3
$\mathrm{P}(X = x)$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{5}{12}$

(b) $\mathrm{P}(X > 1) = \frac{1}{3} + \frac{5}{12} = \frac{3}{4}$.

(c) $\mathrm{E}(X) = 1(\frac{1}{4}) + 2(\frac{1}{3}) + 3(\frac{5}{12}) = \frac{3+8+15}{12} = \frac{13}{6}$.

22. Solution

(a) $0(\frac{1}{4}) + 2(\frac{1}{2}) + v(\frac{1}{4}) = \frac{5}{2}$, so $1 + \frac{v}{4} = \frac{5}{2}$ and $v = 6$.

(b) Only 6 exceeds $\frac{5}{2}$, so the probability is $\frac{1}{4}$.

23. Solution

(a) A: $0(\frac{1}{4}) + 2(\frac{1}{2}) + 4(\frac{1}{4}) = 2$. B: $0(\frac{1}{2}) + 4(\frac{1}{2}) = 2$.

(b) A: $\frac{1}{4}$. B: $\frac{1}{2}$.

(c) The same probability-weighted mean can arise from different distributions, with different probabilities for a specified event.

24. Solution

(a) $\mathrm{E}(X) = 0(0.173) + 1(0.298) + 2(0.361) + 3(0.168) = 1.524 \approx 1.52$. The long-run mean is about 1.52 donations per hour.

(b) Use 1.524, not its rounded form: $75(1.524) = 114.3$ donations.

(c) This is an average of totals over repeated 75-hour periods. Each actual total is a whole number, but an average need not be.

Section 2: Extended response — Questions 16–30 (continued)

25. Solution

(a) $G = R - 3$.

g	-3	1	7
$\mathrm{P}(G = g)$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{10}$

(b) $\mathrm{E}(G) = -3(\frac{1}{2}) + 1(\frac{2}{5}) + 7(\frac{1}{10}) = -\frac{2}{5}$ euros. Also $\mathrm{P}(G > 0) = \frac{2}{5} + \frac{1}{10} = \frac{1}{2}$.

(c) The mean prize is $4(\frac{2}{5}) + 10(\frac{1}{10}) = \frac{13}{5}$ euros. A fee of €2.60 gives expected net gain 0.

26. Solution

(a) $\mathrm{E}(R) = 0(0.70) + 5(0.20) + 20(0.10) = €3$.

(b) The organiser's gains are $c, c-5, c-20$. Their mean is $0.7c + 0.2(c-5) + 0.1(c-20) = c - 3$. Thus $c - 3 = 1$, so $c = 4$.

(c) Both prizes €5 and €20 exceed €4. The probability is $0.20 + 0.10 = 0.30$.

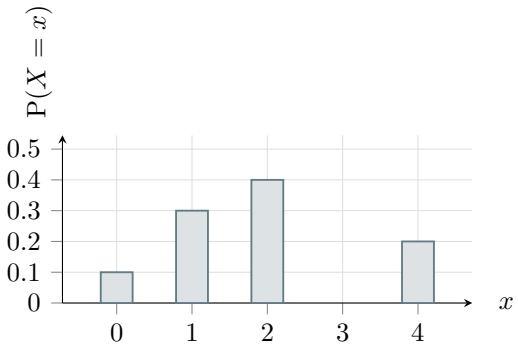
27. Solution

(a) This unweighted average treats the three values as equally likely. The actual probabilities are unequal.

(b) $\mathrm{E}(X) = 1(\frac{1}{2}) + 3(\frac{1}{3}) + 6(\frac{1}{6}) = \frac{5}{2}$. Over many intervals the mean is 2.5 visitors per interval.

28. Solution

(a) Bars have heights $0.1, 0.3, 0.4, 0.2$ at $0, 1, 2, 4$ respectively; there is no positive bar at 3.



(b) $\mathrm{P}(X = 3) = 0$. Also $\mathrm{E}(X) = 1(\frac{3}{10}) + 2(\frac{2}{5}) + 4(\frac{1}{5}) = \frac{19}{10}$.

29. Solution

(a) $0(0.20) + 1(t) + 3(0.80 - t) = 1.10$.

(b) $t + 2.40 - 3t = 1.10$, so $t = 0.65$. The probabilities are $0.20, 0.65, 0.15$, all in $[0, 1]$, with sum 1.

(c) Only 3 is greater than 1: $\mathrm{P}(X > 1) = 0.15$.

30. Solution

(a) Let G_A, G_B be the respective net gains, in euros, on one play. $\mathrm{E}(G_A) = 10(\frac{1}{6}) - 2(\frac{5}{6}) = 0$ and $\mathrm{E}(G_B) = 2(\frac{1}{2}) - 2(\frac{1}{2}) = 0$.

(b) Game B: probability $\frac{1}{2}$, compared with $\frac{1}{6}$ for A.

(c) No. Fairness concerns expected net gain, not a guaranteed outcome over a finite number of plays.

Section 3: AI SL exam-style / Paper 1 — Questions 31–50

31. Solution

(a) $k = 1 - 0.19 - 0.27 - 0.14 = 0.40$.

(b) $\mathrm{P}(X \geq 2) = 0.40 + 0.14 = 0.54$.

(c) $\mathrm{E}(X) = 0(0.19) + 1(0.27) + 2(0.40) + 3(0.14) = 1.49$. Over many customers, the mean is 1.49 bookings per customer.

(d) $20(1.49) = 29.8$ bookings. This is an expected total, not a guaranteed count.

32. Solution

(a) $a + 0.25 + b = 1$, or $a + b = 0.75$.

(b) $0a + 2(0.25) + 3b = 1.75$. Hence $b = \frac{1.25}{3} = \frac{5}{12}$ and $a = 0.75 - \frac{5}{12} = \frac{1}{3}$. Both are valid probabilities.

(c) $\mathrm{P}(X \geq 2) = \frac{1}{4} + \frac{5}{12} = \frac{2}{3} \approx 0.667$.

33. Solution

(a) $(5 + 4 + 3 + 2)k = 1$, so $k = \frac{1}{14}$.

(b) Include $2, 3$: $\mathrm{P}(2 \leq X < 4) = \frac{4}{14} + \frac{3}{14} = \frac{1}{2}$.

(c) $\mathrm{E}(X) = \frac{1(5)+2(4)+3(3)+4(2)}{14} = \frac{30}{14} = \frac{15}{7}$.

34. Solution

(a) Use each sector angle divided by $360°$.

r	0	2	8
$\mathrm{P}(R = r)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

(b) $G = R - 3$.

g	-3	-1	5
$\mathrm{P}(G = g)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

(c) $\mathrm{E}(G) = -3(\frac{1}{2}) - 1(\frac{1}{4}) + 5(\frac{1}{4}) = -\frac{1}{2}$ euros. The game is not fair; the expected loss is €0.50 per spin.

(d) Only the €8 prize makes a profit, so $\mathrm{P}(G > 0) = \frac{1}{4}$.

Section 3: AI SL exam-style / Paper 1 — Questions 31–50 (continued)

35. Solution

(a) The larger values in the 3×3 outcome table are

	Second card		
First card	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

There is one 0, three 1s and five 2s.

x	0	1	2
$\mathrm{P}(X = x)$	$\frac{1}{9}$	$\frac{3}{9}$	$\frac{5}{9}$

(b) $\mathrm{P}(X = 1) = \frac{3}{9} = \frac{1}{3}$.

(c) $\mathrm{E}(X) = 0(\frac{1}{9}) + 1(\frac{3}{9}) + 2(\frac{5}{9}) = \frac{13}{9}$.

36. Solution

(a)

r	0	10	40
$\mathrm{P}(R = r)$	$\frac{95}{100}$	$\frac{4}{100}$	$\frac{1}{100}$

(b) Net gains are $-1, 9, 39$. Their mean is $\frac{-95+36+39}{100} = -\frac{1}{5}$ euros, an expected loss of €0.20.

(c) Income is €100 and prize cost is $40 + 4(10) =$ €80. Total profit is €20.

(d) All five winning tickets make a profit: $\frac{5}{100} = \frac{1}{20}$.

37. Solution

(a) $\mathrm{E}(R) = 0(0.82) + 7.50(0.14) + 40(0.04) = 1.05 + 1.60 =$ €2.65.

(b) $C = 3.20 - R$, so its values are $3.20, -4.30, -36.80$. Thus $\mathrm{E}(C) = 3.20(0.82) - 4.30(0.14) - 36.80(0.04) =$ €0.55.

(c) $250(2.65) =$ €662.50. This is an expected total, not a guaranteed payment.

38. Solution

(a) $\mathrm{P}(X = 3) = 0$.

(b) $\mathrm{P}(1 < X \leq 4) = 0.40 + 0.30 = 0.70$.

(c) $\mathrm{E}(X) = 1(0.20) + 2(0.40) + 4(0.30) + 5(0.10) = 2.7 = \frac{27}{10}$.

(d) $50(2.7) = 135$ orders.

39. Solution

(a) $(-2)(0.50) + 1p + 7(0.50 - p) = 0.40$.

(b) $2.50 - 6p = 0.40$, so $p = 0.35$. The probabilities $0.50, 0.35, 0.15$ lie in $[0, 1]$ and sum to 1.

(c) $\mathrm{P}(G > 0) = 0.35 + 0.15 = 0.50$.

(d) It is not fair: $\mathrm{E}(G) =$ €$0.40 > 0$. A profit probability of 0.50 does not make a game fair.

Section 3: AI SL exam-style / Paper 1 — Questions 31–50 (continued)

40. Solution

(a) Subtract the fee from each prize: $-2.40, 2.60, w - 2.40$.

(b) For fairness, $\mathrm{E}(G) = 0$: $(-2.40)(0.72) + 2.60(0.20) + (w - 2.40)(0.08) = 0$. Thus $0.08w - 1.40 = 0$ and $w = 17.50$ euros.

(c) Both €5 and €17.50 exceed the fee, so the probability is $0.20 + 0.08 = 0.28$.

(d) The mean prize in a fair game equals the fee, €2.40. Expected total: $150(2.40) = €360$.

41. Solution

(a) $-2(\frac{1}{4}) + 1(\frac{1}{2}) + \frac{m}{4} = 1$, so $\frac{m}{4} = 1$ and $m = 4$.

(b) Include 1 and 4: the probability is $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.

(c) Only $X = 1$ equals the mean, so the probability is $\frac{1}{2}$.

42. Solution

(a) A: $0(\frac{1}{4}) + 1(\frac{1}{2}) + 2(\frac{1}{4}) = 1$. B: $0(\frac{2}{5}) + 1(\frac{1}{5}) + 2(\frac{2}{5}) = 1$.

(b) A: $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$. B: $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$.

(c) Both means are 1, but, for example, the probabilities of no booking are $\frac{1}{4}$ and $\frac{2}{5}$. The distributions differ.

43. Solution

(a)

r	0	2	6
$\mathrm{P}(R = r)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

(b) $\mathrm{E}(R) = 0(\frac{1}{3}) + 2(\frac{1}{2}) + 6(\frac{1}{6}) = €2$.

(c) $G = R - 2$ has values $-2, 0, 4$ with the same respective probabilities. $\mathrm{E}(G) = -2(\frac{1}{3}) + 4(\frac{1}{6}) = 0$, so the game is fair.

(d) Only a six gives a positive net gain: $\mathrm{P}(G > 0) = \frac{1}{6}$.

44. Solution

(a) $a + b + 2b + a = 1$, so $2a + 3b = 1$.

(b) $0a + 1b + 2(2b) + 3a = 1.60$, so $3a + 5b = 1.60$.

(c) Solving $2a + 3b = 1$ and $3a + 5b = 1.60$ gives $a = b = 0.20$. The completed distribution is $0.20, 0.20, 0.40, 0.20$, so $\mathrm{P}(X \leq 1) = 0.40$.

45. Solution

(a) $\mathrm{E}(R) = 0(0.55) + 3(0.30) + 8(0.15) = €2.10$.

(b) The fixed bonus: €2.50 is greater than €2.10.

(c) Both €3 and €8 exceed €2.50. The probability is $0.30 + 0.15 = 0.45$.

(d) Fixed: $120(2.50) = €300$. Random: $120(2.10) = €252$.

Section 3: AI SL exam-style / Paper 1 — Questions 31–50 (continued)

46. Solution

(a) No yellow means blue then blue: $(\frac{3}{5})(\frac{3}{5}) = \frac{9}{25} = 0.36$.

(b) Blue then yellow or yellow then blue: $(\frac{3}{5})(\frac{2}{5}) + (\frac{2}{5})(\frac{3}{5}) = \frac{12}{25} = 0.48$.

(c) Two yellow counters have probability $(\frac{2}{5})^2 = \frac{4}{25} = 0.16$.

x	0	1	2
$\mathrm{P}(X = x)$	0.36	0.48	0.16

$\mathrm{E}(X) = 0(0.36) + 1(0.48) + 2(0.16) = 0.80$ yellow counters per pair of draws.

47. Solution

(a)

r	0	4	10
$\mathrm{P}(R = r)$	$\frac{5}{8}$	$\frac{1}{8}$	$\frac{2}{8}$

(b) $\mathrm{E}(R) = 0(\frac{5}{8}) + 4(\frac{1}{8}) + 10(\frac{2}{8}) = €3$. The fair fee is €3.

(c) Let $G = R - 3.50$ be the player's net gain, in euros, on one spin. $\mathrm{E}(G) = -3.50(\frac{5}{8}) + 0.50(\frac{1}{8}) + 6.50(\frac{2}{8}) = -€0.50$.

(d) A profit occurs for the €4 and €10 prizes: probability $\frac{3}{8} = 0.375$. No prize equals the fee, so the probability of breaking even is 0.

48. Solution

(a) $0(0.55) + 4(0.30) + 0.15w = 3$, so $0.15w = 1.80$ and $w = 12$ euros.

(b) The fair fee is the mean prize, €3.

(c) Let G be the player's net gain, in euros, on one play. Its values are $-3.50, 0.50, 8.50$. Thus $\mathrm{E}(G) = -3.50(0.55) + 0.50(0.30) + 8.50(0.15) = -€0.50$.

(d) Both €4 and €12 exceed the fee: probability $0.30 + 0.15 = 0.45$.

49. Solution

(a) Divide each frequency by 80.

x	0	1	2	3
$\mathrm{P}(X = x)$	0.225	0.350	0.300	0.125

(b) $\mathrm{E}(X) = 0(0.225) + 1(0.350) + 2(0.300) + 3(0.125) = 1.325$. The model predicts a long-run mean of about 1.33 returns per day.

(c) Keep the unrounded mean: $200(1.325) = 265$ returns.

(d) The observed days should be representative of the future days; for example, operating conditions should remain similar.

50. Solution

(a) $\mathrm{E}(R) = 5(\frac{2}{5}) + 15(\frac{1}{10}) = \frac{7}{2} = €3.50$.

(b) An entry fee of €3.50 makes the expected net gain zero.

(c) Let G be the player's net gain, in euros, on one play. The net gains are $-4, 1, 11$. Thus $\mathrm{E}(G) = -4(\frac{1}{2}) + 1(\frac{2}{5}) + 11(\frac{1}{10}) = -\frac{1}{2}$ euros. The profit probability is $\frac{2}{5} + \frac{1}{10} = \frac{1}{2}$.

(d) The claim is false. Here the profit probability is $\frac{1}{2}$, but the expected net gain is $-€0.50$, not zero. The sizes of gains and losses matter.