Parent functions and vertical transformations

IB Mathematics: Analysis and Approaches | Higher Level

First set ▪ 20 questions ▪ Answer key at the back

Learning goal	Use parent graphs to sketch and analyse $g(x) = af(x) + d$.
Syllabus link	AA 2.11; supporting skills 2.2–2.4, 2.8–2.9 (shared SL/HL content).
Structure	1–8: single transformations; 9–16: combinations; 17–20: parameters and reasoning.
Technology	Work by hand. Use a graphing calculator afterwards to check sketches.

The vertical transformation rule

A point on $y = f(x)$ has coordinates (x, y). Under $g(x) = af(x) + d$, with $a \neq 0$,

$$\boxed{(x, y) \longmapsto (x, ay + d).}$$

Change	Effect on the graph
$y = f(x) + d$	Translate up d units if $d > 0$, or down $\lvert d \rvert$ units if $d < 0$.
$y = pf(x)$, $p > 0$	Vertical stretch with scale factor p. Every y-coordinate is multiplied by p, including when $0 < p < 1$.
$y = -f(x)$	Reflect in the x-axis. Every y-coordinate changes sign.

For $g(x) = af(x) + d$: stretch by $\lvert a \rvert$, reflect in the x-axis if $a < 0$, then translate vertically by d. The stretch and reflection can be interchanged; use the translation last.

What happens to the features?

- The **domain stays the same**: the x-coordinates do not change.
- Transform every value in the **range** by $y \mapsto ay + d$. A negative a reverses the direction of an inequality.
- A vertical asymptote $x = c$ stays $x = c$. A horizontal asymptote $y = L$ becomes $y = aL + d$.

For every question: the feature check

1. State the parent function's **domain, range and vertical/horizontal asymptotes**.
2. Describe the **ordered vertical transformations**.
3. State the transformed function's **domain, range and vertical/horizontal asymptotes**.

Write "none" when appropriate. Use exact values and interval or set notation. Work in your exercise book and on the supplied axes. Sketches need labelled key points and dashed asymptotes, where relevant.

Notation: $\log x = \log_{10} x$; $\ln x = \log_e x$. General bases satisfy $b > 1$. The accompanying parent-function chart gives the starting graphs.

1. Core fluency: one transformation

Every question: parent features; ordered transformations; transformed features. State domain, range, and all vertical and horizontal asymptotes.

1. Linear function

Let $f(x) = x$ and $g(x) = x + 3$.

(a) Find the images of $(0, 0)$ and $(1, 1)$ on the graph of g.

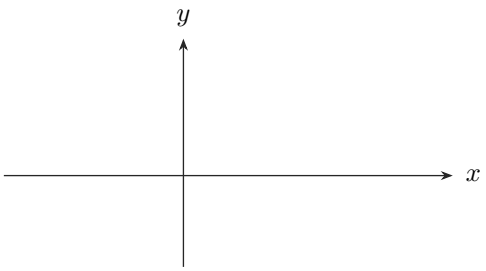
(b) Find the coordinates of both axis intercepts of g.

2. Quadratic function

Let $f(x) = x^2$ and $g(x) = x^2 - 4$.

(a) Find the vertex and all axis intercepts of g.

(b) Sketch f and g on the same axes. Label the vertices and intercepts.



Choose a suitable scale.

3. Cubic function

Let $f(x) = x^3$ and $g(x) = -x^3$.

(a) Find the images of $(-1, -1)$, $(0, 0)$ and $(1, 1)$.

(b) Explain why the reflection leaves the range equal to $\mathbb{R}$.

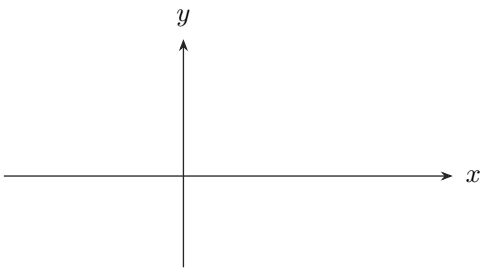
4. Square-root function

Let $f(x) = \sqrt{x}$ and $g(x) = 2\sqrt{x}$.

(a) Find the images of $(0, 0)$, $(1, 1)$ and $(4, 2)$.

(b) Sketch f and g on the same axes. Label the endpoint and mapped points.

(c) Explain why $x = 0$ is not a vertical asymptote.



Choose a suitable scale.

1. Core fluency: one transformation (continued)

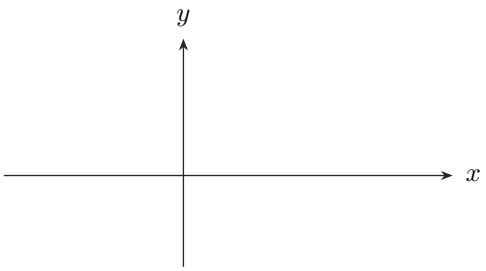
Every question: parent features; ordered transformations; transformed features. State domain, range, and all vertical and horizontal asymptotes.

5. Reciprocal function

Let $f(x) = \dfrac{1}{x}$ and $g(x) = -\dfrac{1}{x}$.

(a) Find the images of $(-1, -1)$ and $(1, 1)$. State whether g has any axis intercepts.

(b) Sketch both graphs. Label the mapped points and show the asymptotes.



Choose a suitable scale.

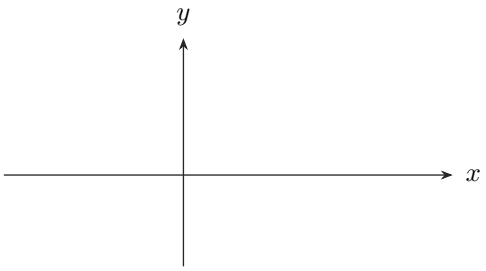
6. Exponential function

Let $f(x) = 2^x$ and $g(x) = 2^x + 3$.

(a) Find the images of $(0, 1)$, $(1, 2)$ and $(2, 4)$.

(b) Sketch both graphs and show their horizontal asymptotes.

(c) Can $g(x) = 3$? Explain your answer.



Choose a suitable scale.

7. Logarithm to base 2

Let $f(x) = \log_2 x$ and $g(x) = 3\log_2 x$.

(a) Find the images of $\left(\frac{1}{2}, -1\right)$, $(1, 0)$ and $(2, 1)$.

(b) State the coordinates of any axis intercepts of g.

8. Natural logarithm

Let $f(x) = \ln x$ and $g(x) = \frac{1}{2}\ln x$.

(a) Find the images of $(1, 0)$, $(e, 1)$ and $(e^2, 2)$.

(b) Explain why the range of g is still $\mathbb{R}$, although every output of f is multiplied by $\frac{1}{2}$.

2. Combined vertical transformations

Every question: parent features; ordered transformations; transformed features. State domain, range, and all vertical and horizontal asymptotes.

9. Linear function

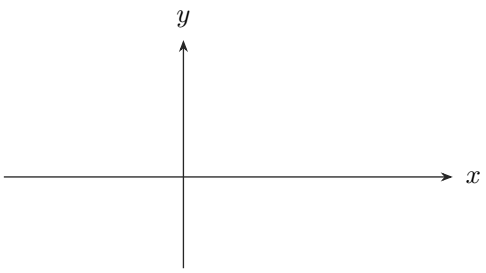
Let $f(x) = x$ and $g(x) = -2x + 5$.

(a) Find the image of $(1, 1)$.
(b) Find the coordinates of both axis intercepts of g.

10. Quadratic function

Let $f(x) = x^2$ and $g(x) = -\frac{1}{2}x^2 + 2$.

(a) Find the vertex and all axis intercepts of g.
(b) Sketch both graphs on the same axes. Label the vertices and intercepts.



Choose a suitable scale.

11. Cubic function

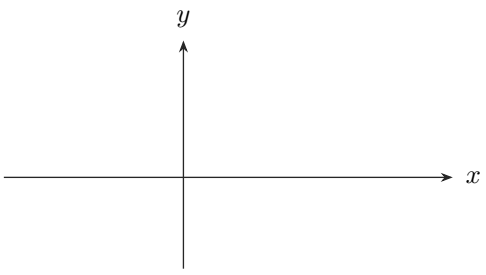
Let $f(x) = x^3$ and $g(x) = \frac{1}{2}x^3 - 4$.

(a) Find the images of $(-2, -8)$, $(0, 0)$ and $(2, 8)$.
(b) Find the coordinates of both axis intercepts of g.

12. Square-root function

Let $f(x) = \sqrt{x}$ and $g(x) = 6 - 3\sqrt{x}$.

(a) Find the endpoint and all axis intercepts of g.
(b) Sketch both graphs on the same axes. Label the endpoints and intercepts.



Choose a suitable scale.

2. Combined vertical transformations (continued)

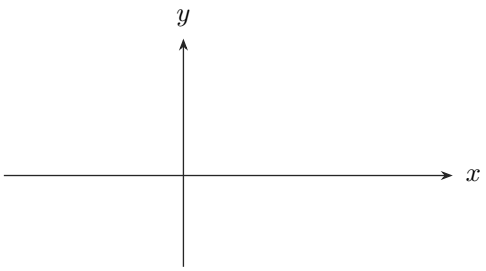
Every question: parent features; ordered transformations; transformed features. State domain, range, and all vertical and horizontal asymptotes.

13. Reciprocal function

Let $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{3}{x} - 2$.

(a) Find the coordinates of any axis intercepts of g.

(b) Sketch both graphs. Label the intercepts and show all asymptotes.



Choose a suitable scale.

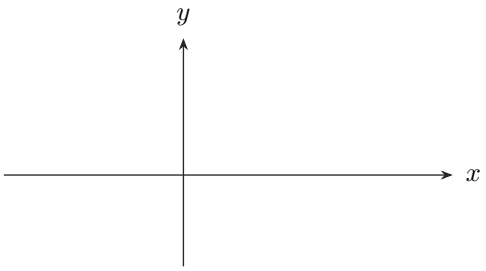
14. Exponential function

Let $f(x) = 3^x$ and $g(x) = 5 - 2 \cdot 3^x$.

(a) Find the images of $(-1, \frac{1}{3})$, $(0, 1)$ and $(1, 3)$.

(b) Find the exact x-intercept of g.

(c) Sketch both graphs and show their horizontal asymptotes.



Choose a suitable scale.

15. Common logarithm

Let $f(x) = \log_{10} x$ and $g(x) = 4 - 2\log_{10} x$.

(a) Find the images of $(1, 0)$, $(10, 1)$ and $(100, 2)$.

(b) Find the x-intercept of g, and explain why there is no y-intercept.

16. A logarithm with an unspecified base

Let $f(x) = \log_b x$ and $g(x) = -2\log_b x + 3$, where $b > 1$.

(a) Find the images of $(1, 0)$, $(b, 1)$ and $(b^2, 2)$.

(b) Find the exact x-intercept of g, in terms of b.

(c) Explain why its x-coordinate lies between b and b^2.

3. Read graphs and explain your reasoning

Every question: parent features; ordered transformations; transformed features. State domain, range, and all vertical and horizontal asymptotes.

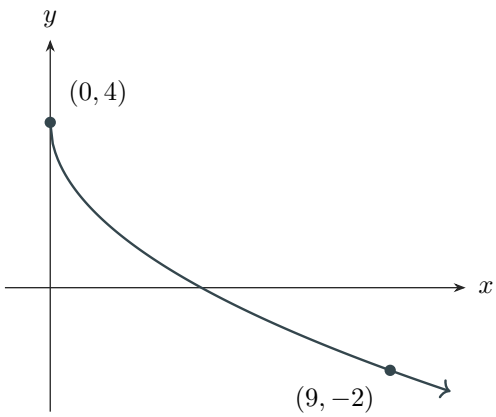
17. Recover a square-root function

The graph of $g(x) = a\sqrt{x} + d$ is shown. Its endpoint is $(0, 4)$, and it passes through $(9, -2)$. Use $f(x) = \sqrt{x}$ as the parent function.

(a) Determine a and d, and write an equation for g.

(b) Find the x-intercept of g.

(c) Explain why the vertical line $x = 0$ is not an asymptote of this graph.



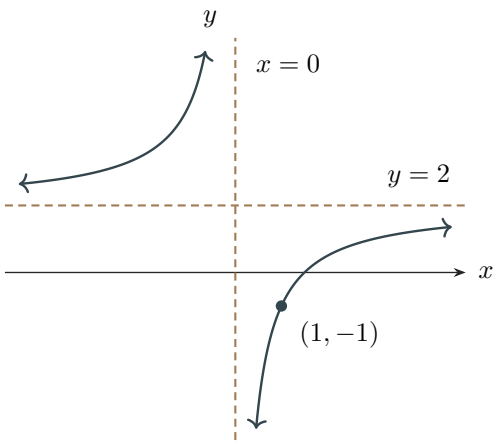
18. Recover a reciprocal function

The graph of $g(x) = \dfrac{a}{x} + d$, where $a \neq 0$, is shown. The dashed lines are asymptotes, and the graph passes through $(1, -1)$. Use $f(x) = 1/x$ as the parent function.

(a) Determine a and d, and write an equation for g.

(b) Find the x-intercept of g.

(c) Explain why this graph has no y-intercept.



3. Read graphs and explain your reasoning (continued)

Every question: parent features; ordered transformations; transformed features. State domain, range, and all vertical and horizontal asymptotes.

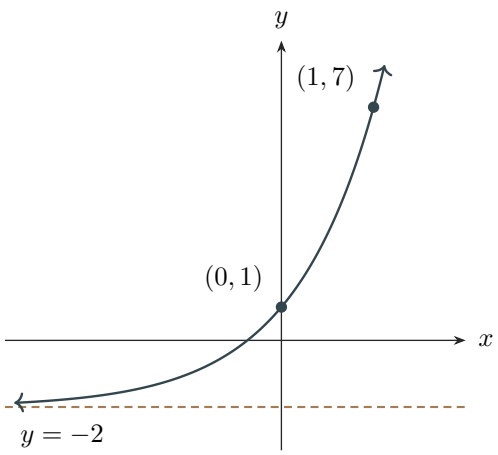
19. Recover an exponential function and its base

The graph of $g(x) = a\,b^x + d$, where $a \neq 0$ and $b > 1$, is shown. It passes through $(0, 1)$ and $(1, 7)$. The dashed line is a horizontal asymptote.

(a) Determine d, then a, then b.

(b) Write the equation of g and identify its parent function. Complete the feature check using this parent.

(c) Find the exact x-intercept of g.



20. Does the order matter?

Starting from $f(x) = \sqrt{x}$, a student wants to obtain

$$g(x) = -2\sqrt{x} + 3.$$

The student suggests: *translate up 3, then stretch vertically by scale factor 2, then reflect in the x-axis.*

(a) Show the equation after each suggested transformation, and hence find the function h that this sequence actually produces.

(b) Give a correct ordered sequence of transformations that produces g. Complete the feature check for the parent and for both g and h.

(c) Find all axis intercepts of g and h. Can the two transformed graphs intersect? Justify your answer by comparing $g(x)$ and $h(x)$.

Answer key and solution outlines

In every feature table, compare the parent f with the transformed function g.

Section 1: single transformations | Questions 1–4

1. $g(x) = x + 3$

Translate up 3 units.

Feature	**Parent** f	**Transformed** g
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$\mathbb{R}$	$\mathbb{R}$
Vertical asymptote	none	none
Horizontal asymptote	none	none

$(0, 0) \mapsto (0, 3)$ and $(1, 1) \mapsto (1, 4)$. Intercepts: $(-3, 0)$ and $(0, 3)$.

2. $g(x) = x^2 - 4$

Translate down 4 units.

Feature	**Parent** f	**Transformed** g
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$[0, \infty)$	$[-4, \infty)$
Vertical asymptote	none	none
Horizontal asymptote	none	none

Vertex and y-intercept: $(0, -4)$. Since $x^2 - 4 = 0$, the x-intercepts are $(-2, 0)$ and $(2, 0)$. The parabola opens upwards.

3. $g(x) = -x^3$

Reflect in the x-axis.

Feature	**Parent** f	**Transformed** g
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$\mathbb{R}$	$\mathbb{R}$
Vertical asymptote	none	none
Horizontal asymptote	none	none

$(-1, -1) \mapsto (-1, 1)$, $(0, 0) \mapsto (0, 0)$, $(1, 1) \mapsto (1, -1)$. Negating all real outputs still gives all real numbers.

4. $g(x) = 2\sqrt{x}$

Vertical stretch with scale factor 2.

Feature	**Parent** f	**Transformed** g
Domain	$[0, \infty)$	$[0, \infty)$
Range	$[0, \infty)$	$[0, \infty)$
Vertical asymptote	none	none
Horizontal asymptote	none	none

$(0, 0) \mapsto (0, 0)$, $(1, 1) \mapsto (1, 2)$, $(4, 2) \mapsto (4, 4)$. Both graphs start at $(0, 0)$. Their values tend to 0 as x approaches 0 from the right, so $x = 0$ is not a vertical asymptote.

Answer key and solution outlines

A bound is included only if the function actually reaches it.

Section 1: single transformations | Questions 5–8

5. $g(x) = -\dfrac{1}{x}$

Reflect in the x-axis.

Feature	**Parent** f	**Transformed** g
Domain	$\mathbb{R} \setminus \{0\}$	$\mathbb{R} \setminus \{0\}$
Range	$\mathbb{R} \setminus \{0\}$	$\mathbb{R} \setminus \{0\}$
Vertical asymptote	$x = 0$	$x = 0$
Horizontal asymptote	$y = 0$	$y = 0$

$(-1,-1) \mapsto (-1,1)$ and $(1,1) \mapsto (1,-1)$. There are no axis intercepts: $x = 0$ is excluded and $-1/x$ never equals 0.

6. $g(x) = 2^x + 3$

Translate up 3 units.

Feature	**Parent** f	**Transformed** g
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$(0,\infty)$	$(3,\infty)$
Vertical asymptote	none	none
Horizontal asymptote	$y = 0$	$y = 3$

$(0,1) \mapsto (0,4)$, $(1,2) \mapsto (1,5)$, $(2,4) \mapsto (2,7)$. Since $2^x > 0$, $g(x) > 3$ for every real x; the graph never reaches $y = 3$.

7. $g(x) = 3\log_2 x$

Vertical stretch with scale factor 3.

Feature	**Parent** f	**Transformed** g
Domain	$(0,\infty)$	$(0,\infty)$
Range	$\mathbb{R}$	$\mathbb{R}$
Vertical asymptote	$x = 0$	$x = 0$
Horizontal asymptote	none	none

$\left(\frac{1}{2},-1\right) \mapsto \left(\frac{1}{2},-3\right)$, $(1,0) \mapsto (1,0)$, $(2,1) \mapsto (2,3)$. The x-intercept is $(1,0)$; there is no y-intercept.

8. $g(x) = \dfrac{1}{2}\ln x$

Vertical stretch with scale factor $\frac{1}{2}$.

Feature	**Parent** f	**Transformed** g
Domain	$(0,\infty)$	$(0,\infty)$
Range	$\mathbb{R}$	$\mathbb{R}$
Vertical asymptote	$x = 0$	$x = 0$
Horizontal asymptote	none	none

$(1,0) \mapsto (1,0)$, $(e,1) \mapsto \left(e,\frac{1}{2}\right)$, $(e^2,2) \mapsto (e^2,1)$. Halving every real number still produces all real numbers; for any desired output r, use the parent output $2r$.

Answer key and solution outlines

In the sequences below, the translation is last. Stretch and reflection may be interchanged.

Section 2: combined transformations | Questions 9–12

9. $g(x) = -2x + 5$

Vertical stretch, factor 2; reflect in the x-axis; translate up 5.

Feature	**Parent** f	**Transformed** g
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$\mathbb{R}$	$\mathbb{R}$
Vertical asymptote	none	none
Horizontal asymptote	none	none

$(1, 1) \mapsto (1, 3)$. Intercepts: $\left(\frac{5}{2}, 0\right)$ and $(0, 5)$.

10. $g(x) = -\frac{1}{2}x^2 + 2$

Vertical stretch, factor $\frac{1}{2}$; reflect in the x-axis; translate up 2.

Feature	**Parent** f	**Transformed** g
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$[0, \infty)$	$(-\infty, 2]$
Vertical asymptote	none	none
Horizontal asymptote	none	none

Vertex and y-intercept: $(0, 2)$. Solving $-\frac{1}{2}x^2 + 2 = 0$ gives the x-intercepts $(-2, 0)$ and $(2, 0)$. The parabola opens downwards.

11. $g(x) = \frac{1}{2}x^3 - 4$

Vertical stretch, factor $\frac{1}{2}$; translate down 4.

Feature	**Parent** f	**Transformed** g
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$\mathbb{R}$	$\mathbb{R}$
Vertical asymptote	none	none
Horizontal asymptote	none	none

$(-2, -8) \mapsto (-2, -8)$, $(0, 0) \mapsto (0, -4)$, $(2, 8) \mapsto (2, 0)$. Intercepts: $(2, 0)$ and $(0, -4)$.

12. $g(x) = 6 - 3\sqrt{x}$

Vertical stretch, factor 3; reflect in the x-axis; translate up 6.

Feature	**Parent** f	**Transformed** g
Domain	$[0, \infty)$	$[0, \infty)$
Range	$[0, \infty)$	$(-\infty, 6]$
Vertical asymptote	none	none
Horizontal asymptote	none	none

Endpoint and y-intercept: $(0, 6)$. For the x-intercept, $6 - 3\sqrt{x} = 0$ gives $\sqrt{x} = 2$, so $(4, 0)$. Other useful points: $(1, 3)$ and $(9, -3)$.

Answer key and solution outlines

A vertical transformation preserves the excluded input values, but can change excluded outputs.

Section 2: combined transformations | Questions 13–16

13. $g(x) = \dfrac{3}{x} - 2$

Vertical stretch, factor 3; translate down 2.

Feature	**Parent** f	**Transformed** g
Domain	$\mathbb{R} \setminus \{0\}$	$\mathbb{R} \setminus \{0\}$
Range	$\mathbb{R} \setminus \{0\}$	$\mathbb{R} \setminus \{-2\}$
Vertical asymptote	$x = 0$	$x = 0$
Horizontal asymptote	$y = 0$	$y = -2$

The x-intercept is $\left(\frac{3}{2}, 0\right)$; no y-intercept. Useful points: $(1, 1)$ and $(-1, -5)$. The graph approaches $y = -2$ but never reaches it.

14. $g(x) = 5 - 2 \cdot 3^x$

Vertical stretch, factor 2; reflect in the x-axis; translate up 5.

Feature	**Parent** f	**Transformed** g
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$(0, \infty)$	$(-\infty, 5)$
Vertical asymptote	none	none
Horizontal asymptote	$y = 0$	$y = 5$

$\left(-1, \frac{1}{3}\right) \mapsto \left(-1, \frac{13}{3}\right)$, $(0, 1) \mapsto (0, 3)$, $(1, 3) \mapsto (1, -1)$. The x-intercept is $\left(\log_3 \frac{5}{2}, 0\right)$. The curve approaches $y = 5$ from below as $x \to -\infty$.

15. $g(x) = 4 - 2\log_{10} x$

Vertical stretch, factor 2; reflect in the x-axis; translate up 4.

Feature	**Parent** f	**Transformed** g
Domain	$(0, \infty)$	$(0, \infty)$
Range	$\mathbb{R}$	$\mathbb{R}$
Vertical asymptote	$x = 0$	$x = 0$
Horizontal asymptote	none	none

$(1, 0) \mapsto (1, 4)$, $(10, 1) \mapsto (10, 2)$, $(100, 2) \mapsto (100, 0)$. The x-intercept is $(100, 0)$; no y-intercept because $x = 0$ is outside the domain.

16. $g(x) = -2\log_b x + 3, \quad b > 1$

Vertical stretch, factor 2; reflect in the x-axis; translate up 3.

Feature	**Parent** f	**Transformed** g
Domain	$(0, \infty)$	$(0, \infty)$
Range	$\mathbb{R}$	$\mathbb{R}$
Vertical asymptote	$x = 0$	$x = 0$
Horizontal asymptote	none	none

$(1, 0) \mapsto (1, 3)$, $(b, 1) \mapsto (b, 1)$, $(b^2, 2) \mapsto (b^2, -1)$. The x-intercept solves $\log_b x = \frac{3}{2}$, so it is $(b^{3/2}, 0)$. Since $b > 1$, $b < b^{3/2} < b^2$.

Answer key and solution outlines

Read the vertical shift from an endpoint or a horizontal asymptote, then use a marked point.

Section 3: parameters and reasoning | Questions 17–19

17. $g(x) = a\sqrt{x} + d$

Solution

Endpoint $(0, 4)$: $d = 4$. Point $(9, -2)$: $3a + 4 = -2$, so $a = -2$.

$$\boxed{g(x) = -2\sqrt{x} + 4.}$$

Parent: $f(x) = \sqrt{x}$. Vertical stretch, factor 2; reflect in the x-axis; translate up 4.

Feature	**Parent** f	**Transformed** g
Domain	$[0, \infty)$	$[0, \infty)$
Range	$[0, \infty)$	$(-\infty, 4]$
Vertical asymptote	none	none
Horizontal asymptote	none	none

The x-intercept is $(4, 0)$ and the y-intercept is $(0, 4)$. As $x \to 0^+$, $g(x) \to 4$; the value remains finite, so $x = 0$ is not a vertical asymptote.

18. $g(x) = \dfrac{a}{x} + d$

Solution

Horizontal asymptote $y = 2$: $d = 2$. Point $(1, -1)$: $a + 2 = -1$, so $a = -3$.

$$\boxed{g(x) = -\frac{3}{x} + 2.}$$

Parent: $f(x) = 1/x$. Vertical stretch, factor 3; reflect in the x-axis; translate up 2.

Feature	**Parent** f	**Transformed** g
Domain	$\mathbb{R} \setminus \{0\}$	$\mathbb{R} \setminus \{0\}$
Range	$\mathbb{R} \setminus \{0\}$	$\mathbb{R} \setminus \{2\}$
Vertical asymptote	$x = 0$	$x = 0$
Horizontal asymptote	$y = 0$	$y = 2$

The x-intercept is $\left(\frac{3}{2}, 0\right)$. There is no y-intercept because $x = 0$ is outside the domain.

19. $g(x) = ab^x + d$

Solution

The asymptote gives $d = -2$. From $(0, 1)$, $a - 2 = 1$, so $a = 3$. From $(1, 7)$, $3b - 2 = 7$, so $b = 3$.

$$\boxed{g(x) = 3 \cdot 3^x - 2.}$$

Parent: $f(x) = 3^x$. Vertical stretch, factor 3; translate down 2.

Feature	**Parent** f	**Transformed** g
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$(0, \infty)$	$(-2, \infty)$
Vertical asymptote	none	none
Horizontal asymptote	$y = 0$	$y = -2$

For the x-intercept, $3 \cdot 3^x - 2 = 0$ gives $x = \log_3(2/3)$. The y-intercept is $(0, 1)$.

Answer key and solution outlines

An early translation is also affected by any later stretch or reflection.

Section 3: parameters and reasoning | Question 20

20. $g(x) = -2\sqrt{x} + 3$

Solution

The proposed order gives

$$\sqrt{x} \xrightarrow{\text{up } 3} \sqrt{x} + 3 \xrightarrow{\text{stretch } 2} 2\sqrt{x} + 6 \xrightarrow{\text{reflect}} \boxed{h(x) = -2\sqrt{x} - 6.}$$

The $+3$ has been multiplied by 2 and then negated; it becomes -6.

A correct order for g is:

1. Vertical stretch with scale factor 2.
2. Reflection in the x-axis.
3. Translation up 3 units.

For the actual function h, a sequence with translation last is: stretch by 2, reflect in the x-axis, then translate down 6.

Feature	**Parent** f	**Desired** g	**Actual** h
Domain	$[0, \infty)$	$[0, \infty)$	$[0, \infty)$
Range	$[0, \infty)$	$(-\infty, 3]$	$(-\infty, -6]$
Vertical asymptote	None	None	None
Horizontal asymptote	None	None	None
y-intercept	$(0, 0)$	$(0, 3)$	$(0, -6)$
x-intercept	$(0, 0)$	$\left(\frac{9}{4}, 0\right)$	None

For g, $-2\sqrt{x} + 3 = 0$ gives $\sqrt{x} = 3/2$ and $x = 9/4$. For h, $-2\sqrt{x} - 6 \leq -6$, so there is no x-intercept.

Finally,

$$g(x) - h(x) = (-2\sqrt{x} + 3) - (-2\sqrt{x} - 6) = 9.$$

Thus the two transformed graphs **do not intersect**: g is 9 units above h for every $x \geq 0$.

Three final checks

- A square-root endpoint can be included in the range; an exponential asymptote is not reached by these functions.
- The reciprocal range excludes one output, not an entire half-line.
- Different transformations can leave domain, range and asymptotes unchanged. Check the equation and key points as well.