



Topic 2: Functions

IB Mathematics: Analysis and Approaches HL

100-question complete revision package - SL 2.1–2.11 and AHL 2.12–2.16

Learning goal

Consolidate the full Topic 2 Functions syllabus at HL level, moving from core fluency to mixed and exam-style problem solving.

Syllabus link

IB Mathematics: Analysis and Approaches - SL 2.1–2.11 and AHL 2.12–2.16.

Structure

30 short-response + 30 extended-response + 40 original IB-style questions (20 Paper 1 style, 20 technology-enabled).

Calculator use

Questions marked  are designed for graphing technology/GDC use. Unmarked questions should be attempted analytically unless instructed otherwise.

Instructions

Show clear working. Give exact answers unless a decimal approximation is requested. For technology questions, give answers to the stated accuracy and record enough calculator evidence to make your method clear. For sketches, label axes, intercepts, turning points and asymptotes where relevant.

How to use this revision package

Use Questions 1–30 as a rapid diagnostic. Then target weak areas in Questions 31–60 before moving to the IB-style set. Questions 61–80 are Paper 1 style; Questions 81–100 are technology-enabled and are especially useful for graph features, numerical intersections and non-analytic equations.

Coverage snapshot

2.1–2.5	Lines; functions; domain/range; graphs; key features; composite and inverse functions	2.6–2.11	Quadratics; rational, exponential and logarithmic functions; equations; graph transformations
2.12	Polynomial functions; factor/remainder theorems; root relationships	2.13	HL rational functions and horizontal/vertical/oblique asymptotes
2.14	Odd/even; inverses with restriction; self-inverse functions	2.15–2.16	Function inequalities; modulus equations/inequalities; $ f(x) $, $f(x)$, $1/f(x)$, $f(ax+b)$ and $[f(x)]^2$

Section 1: Short response

Questions 1–30

1. Find the gradient and equation of the straight line through $A(-3, 5)$ and $B(5, -7)$. Give the equation in the form $y = mx + c$.

2. The line L has equation $4x - 3y + 12 = 0$. Find the equation of the line perpendicular to L that passes through $(8, -1)$.

3. Determine the domain of $f(x) = \frac{\sqrt{5-2x}}{x+1}$.

4. Let $f(x) = x^2 - 4x + 7$ with domain $x \geq 2$. State the range of f and find $f^{-1}(x)$, including the domain of f^{-1} .

5. For $p(x) = -(x-2)^2 + 9$, state the vertex, axis of symmetry, x -intercepts, y -intercept and maximum value.

6. Let $f(x) = 2x - 3$ and $g(x) = x^2 + 1$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

7. Find the inverse of $f(x) = \frac{3x-5}{2}$.

8. For $q(x) = -2(x+1)(x-4)$, find the zeros, the y -intercept, the axis of symmetry and the coordinates of the vertex.

9. For $q(x) = 3(x-2)^2 - 12$, state the vertex and axis of symmetry, then find the zeros and the y -intercept.

10. Write $x^2 - 6x + 5$ in vertex form and hence state its vertex.

11. Find the values of k for which $2x^2 + kx + 8 = 0$ has two equal real roots.

12. For $r(x) = \frac{2x-3}{x+4}$, state the vertical and horizontal asymptotes and both axis intercepts.

13. Rewrite $5^x = 12$ in logarithmic form and hence express x exactly using natural logarithms. State the inverse function of $y = 5^x$.

14. Solve $e^{2x} - 5e^x + 4 = 0$ exactly.

15. Describe a valid sequence of transformations that maps $y = f(x)$ to $y = -3f(2(x-1)) + 4$.

16. Find the remainder when $P(x) = 2x^3 - 5x^2 + x + 7$ is divided by $x - 2$.

17. Given $P(x) = x^3 + ax^2 - 5x + 6$ and that $x - 2$ is a factor of P , find a .

18. Without solving the equation, state the sum and product of the roots of $2x^3 - 7x^2 + 4x - 5 = 0$.

19. Find the vertical and oblique asymptotes of $R(x) = \frac{2x^2 + 3x + 1}{x - 1}$.

20. Classify each function as odd, even or neither: $f(x) = x^4 - 3x^2 + 1$ and $g(x) = x^5 - 2x$.

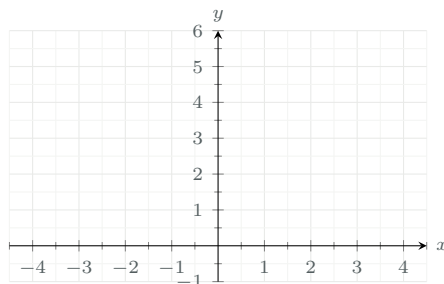
21. Let $f(x) = (x - 1)^2 + 2$ with domain $x \geq 1$. Find $f^{-1}(x)$ and state its domain.

22. Show by composition that $h(x) = \frac{5 - x}{x + 1}$ is self-inverse.

23. Solve $(x - 2)(x + 1)(x - 4) \geq 0$.

24. Sketch $y = |x^2 - 4|$. State its zeros, y -intercept and the coordinates of all local turning points visible from the transformation.

Use the supplied axes if helpful.



25. Solve $|2x - 5| = 7$.

26. The point $(5, -2)$ lies on $y = f(x)$. Find the corresponding point on $y = f(2x - 3)$.

27. Given $f(x) = \frac{(x - 3)(x + 2)}{x - 1}$, state the domain of $y = \frac{1}{f(x)}$.

28. Let $f(x) = x + 2$ and $g(x) = 3x - 1$. Solve $f(g(x)) = 13$.

29. Solve $x^2 - 4x + 1 \geq 2x - 5$ exactly.

30.  Using technology, solve $e^x = 5 - x$ correct to three significant figures.

31. The line L_1 passes through $A(2, -3)$ and $B(8, 9)$. Find its equation. The line L_2 is perpendicular to L_1 and passes through $(4, 5)$. Find the coordinates of the intersection of L_1 and L_2 .

32. A straight access ramp rises 1.2 m over a horizontal distance of 15 m. Model the ramp by a linear function $h(x)$, where x is the horizontal distance in metres from the start. State the gradient as a decimal and as a percentage, and find $h(22.5)$.

33. Let $f(x) = \sqrt{x+4}$ and $g(x) = \frac{1}{x-1}$. Determine the domain of $(g \circ f)(x)$.

34. For $f(x) = 2^x + 1$, state the domain and range, find $f^{-1}(x)$, and state the domain and range of the inverse. Explain the graphical relationship between f and f^{-1} .

35. Let $f(x) = x^2 - 4x$ and $g(x) = 2x - 3$. Find $(f + g)(x)$ and $(f - g)(x)$. Hence find the zeros of $f + g$.

36.  Use technology to analyse $p(x) = x^4 - 6x^2 + 5$. State all real zeros, all local maximum/minimum points, the symmetry of the graph and its range.

37. Let $f(x) = \frac{x-4}{3}$ and $g(x) = 3x + 4$. Verify that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$. What does this show about the two functions?

38. A quadratic $f(x) = x^2 + px + q$ has vertex $(3, -7)$ and y -intercept 2. Find p and q , then find the zeros of f exactly.

39. Solve $-2(x-1)(x-5) > 0$.

40. Find all values of k for which $kx^2 - 4x + k = 0$ has two distinct real roots.

41. A rational function has the form $f(x) = \frac{ax+b}{x-2}$. Its horizontal asymptote is $y = 3$ and its x -intercept is $(-1, 0)$. Find a and b , then find the y -intercept.

42. The functions $y = 3^x$ and $y = \log_3 x$ are inverses. Use the points $(-1, \frac{1}{3})$, $(0, 1)$ and $(2, 9)$ on $y = 3^x$ to write down three corresponding points on $y = \log_3 x$.

43. Solve $4^x - 5(2^x) + 4 = 0$ exactly.

44. Solve $\log_2(x-1) + \log_2(x-3) = 3$, stating the domain restriction used.

45. A population is modelled by $P(t) = 1200(1.08)^t$, where t is measured in years. State the initial population and annual percentage growth rate. Determine the first integer value of t for which $P(t) > 2000$.

46. The graph of $y = f(x)$ contains the point $A(6, 4)$. The graph is transformed to $y = \frac{1}{2}f(-3(x + 2)) - 5$. Describe the transformations and find the image of A .

47. Starting from $y = x^2$, translate the graph 3 units right, stretch vertically by scale factor 2, reflect it in the x -axis, and translate it 5 units up. Write the final equation, state its vertex and find its x -intercepts.

48. Let $P(x) = x^3 + ax^2 + bx + 6$. Given that $P(1) = 0$ and the remainder when P is divided by $x - 2$ is 8, find a and b . Hence factorize $P(x)$ completely.

49. The roots of $2x^3 - 3x^2 - 11x + 6 = 0$ are α, β, γ . State $\alpha + \beta + \gamma$, $\alpha\beta + \alpha\gamma + \beta\gamma$ and $\alpha\beta\gamma$. Given that two roots are 3 and $\frac{1}{2}$, find the third root.

50. For $f(x) = \frac{x^2 + 2x + 5}{x - 1}$, use polynomial division to write $f(x)$ in the form $mx + c + \frac{k}{x - 1}$. Hence state all asymptotes and all axis intercepts.

51. For $f(x) = \frac{x + 1}{x^2 - 4}$, state the domain, all vertical/horizontal asymptotes, and both axis intercepts.

52. The polynomial $p(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$ is odd. State the values of a , c and e that are forced by this information.

53. Let $f(x) = x^2 - 6x + 11$. Restrict the domain so that f is one-to-one with an increasing branch, then find $f^{-1}(x)$ and state its domain.

54. Show that $h(x) = \frac{2x + 3}{x - 2}$ is self-inverse. State the domain and range of h .

55. Let $f(x) = x^3 - 4x$ and $g(x) = x$. Solve $g(x) \geq f(x)$ exactly.

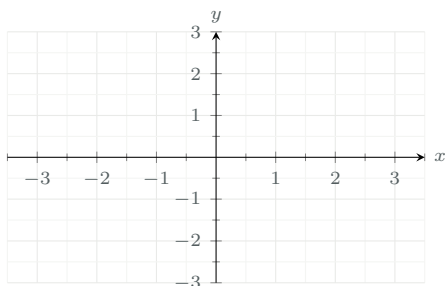
56. Let $f(x) = x^2 - 6x + 5$. Sketch $y = |f(x)|$ using key features and solve $|f(x)| = 3$ exactly.
Use the supplied axes if helpful.



57. Let $f(x) = x^3 - 4x$. Describe how to obtain $y = f(|x|)$ from $y = f(x)$. Show that the transformed function is even and state all of its zeros.

58. For $f(x) = x^2 - 1$, sketch the main features of $y = \frac{1}{f(x)}$. State all asymptotes and intercepts and identify the symmetry.

Use the supplied axes if helpful.



59. Let $f(x) = x^2 - 4$ and $g(x) = [f(x)]^2$. State the zeros and y -intercept of g , and solve $g(x) = 9$ exactly.

60. Solve $|x^2 - 5x + 4| \leq 2$ exactly.

Paper 1 style - analytical / non-calculator

61. A line L passes through $P(-2, 7)$ and $Q(4, -5)$.

- (a) Find the gradient of L . [1 mark]
- (b) Find the equation of L . [2 marks]
- (c) A line M passes through $(3, 1)$ and is perpendicular to L . Find the x -intercept of M . [3 marks]

62. Let $f(x) = \sqrt{6-x}$, with domain $x \leq 6$.

- (a) State the range of f . [1 mark]
- (b) Find $f^{-1}(x)$ and state its domain. [3 marks]
- (c) Evaluate $f^{-1}(f(2))$. [1 mark]

63. Let $f(x) = 3x - 2$ and $h(x) = x^2 + 1$.

- (a) Find $(f \circ h)(x)$. [2 marks]
- (b) Find $(h \circ f)(x)$. [2 marks]
- (c) Solve $(f \circ h)(x) = 10$. [2 marks]

64. The quadratic equation $x^2 + (k-4)x + k = 0$ has equal roots.

- (a) Find the possible values of k . [3 marks]
- (b) For each value of k , find the repeated root. [2 marks]

65. Let $f(x) = 2x^2 - 8x + 3$.

- (a) Write $f(x)$ in vertex form. [2 marks]
- (b) State the minimum value of f . [1 mark]
- (c) Solve $f(x) \leq 3$. [3 marks]

66. Let $f(x) = \frac{3x-1}{x+2}$.

- (a) State the vertical and horizontal asymptotes. [2 marks]
- (b) Find the two axis intercepts. [2 marks]
- (c) Solve $f(x) = 2$. [2 marks]

67. The functions $f(x) = 4^x$ and $g(x) = \log_4 x$ are inverses.

- (a) State the graphical relationship between the graphs of f and g . [1 mark]
- (b) Solve $4^x = 8$ exactly. [2 marks]
- (c) Solve $\log_4 x = -\frac{1}{2}$ exactly. [1 mark]

68. The graph of $y = f(x)$ contains the point $A(-4, 6)$. Let $g(x) = -2f(3x + 6) + 1$.

(a) Describe a valid sequence of transformations from $y = f(x)$ to $y = g(x)$. [4 marks]

(b) Find the image of A on $y = g(x)$. [2 marks]

69. Let $P(x) = x^3 + ax^2 - 4x + 12$. Given that $x - 2$ is a factor of P ,

(a) find a ; [2 marks]

(b) factorize $P(x)$ completely. [3 marks]

70. The polynomial $x^3 - 6x^2 + px + q$ has roots 1, 2 and r .

(a) Find r . [1 mark]

(b) Find p and q . [4 marks]

71. Let $f(x) = \frac{2x^2 - x + 4}{x + 1}$.

(a) Use polynomial division to express $f(x)$ in the form $mx + c + \frac{k}{x + 1}$. [3 marks]

(b) Hence state the vertical and oblique asymptotes. [2 marks]

(c) Find all axis intercepts. [2 marks]

72. The polynomial $p(x) = x^4 + ax^3 + bx^2 + cx + d$ is even. It is also known that $p(0) = 1$ and $p(1) = 6$.

(a) Find a and c . [2 marks]

(b) Find b and d . [2 marks]

73. Let $f(x) = x^2 + 4x + 7$.

(a) Explain why f has no inverse function if its domain is all real numbers. [1 mark]

(b) Restrict the domain to $x \geq -2$ and find $f^{-1}(x)$. [3 marks]

(c) State the domain of f^{-1} . [1 mark]

74. Let $h(x) = \frac{1 - x}{1 + x}$.

(a) Show algebraically that $h(h(x)) = x$. [3 marks]

(b) State the domain and range of h . [2 marks]

75. Solve $x^3 - 4x^2 - x + 4 \geq 0$, showing the factorization and sign analysis used.

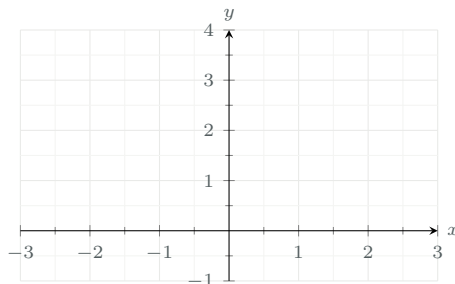
[5 marks]

76. Let $f(x) = |x^2 - 1|$.

(a) Sketch $y = f(x)$, labelling its zeros and y -intercept. [3 marks]

(b) Solve $|x^2 - 1| = 3$. [2 marks]

Use the supplied axes if helpful.



77. Let $f(x) = x^2 - 4x + 3$, $g(x) = |f(x)|$ and $h(x) = f(|x|)$.

(a) State the zeros of g . [1 mark]

(b) State the zeros of h . [2 marks]

(c) Which of g and h is necessarily even? Justify your answer. [2 marks]

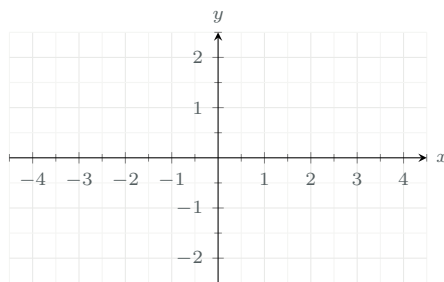
(d) Find $g(2)$ and $h(-2)$. [1 mark]

78. Let $f(x) = x^2 - 4$. Consider $y = \frac{1}{f(x)}$.

(a) State all asymptotes and intercepts. [3 marks]

(b) State the intervals on which the function is positive and the interval on which it is negative. [2 marks]

Use the supplied axes if helpful.



79. Let $f(x) = x^2 - 1$ and $g(x) = [f(x)]^2$.

(a) State the range of g . [1 mark]

(b) State the zeros of g . [1 mark]

(c) Solve $g(x) = 9$. [3 marks]

80. Let $P(x) = x^3 - 2x^2 - 5x + 6$ and $R(x) = \frac{P(x)}{x - 1}$.

(a) Factorize $P(x)$ completely. [3 marks]

(b) Simplify $R(x)$, stating the restriction on x . [2 marks]

(c) Describe the removable discontinuity on the graph of R and find the zeros of R . [3 marks]

81.  A taxi fare C euros is modelled linearly by distance d km. A 4 km journey costs 18.40 euros and an 11 km journey costs 34.50 euros.

- (a) Find a linear model $C(d)$. [3 marks]
- (b) Estimate the distance of a journey costing 50 euros. Give your answer to three significant figures. [2 marks]

82.  Consider $p(x) = x^4 - 4x^3 - 2x^2 + 12x + 3$. Use technology to determine, correct to three decimal places where necessary,

- (a) all real zeros; [2 marks]
- (b) all local maximum/minimum points; [3 marks]
- (c) the y -intercept. [1 mark]

83.  The curves $y = e^x$ and $y = 4x$ intersect twice. Find the coordinates of both intersection points correct to three significant figures. [4 marks]

84.  The function $f(x) = x^3 + x + 1$ is one-to-one on $\mathbb{R}$. Find $f^{-1}(10)$ correct to three decimal places. [3 marks]

85.  The height of a projectile is modelled by $h(t) = -4.9t^2 + 18t + 1.2$, where t is in seconds and h is in metres.

- (a) Find the maximum height and the time at which it occurs. [3 marks]
- (b) Find the positive time at which the projectile reaches ground level. [2 marks]

86.  Let $f(x) = \frac{x^2 + 1}{x - 2}$.

- (a) State the vertical and oblique asymptotes. [2 marks]
- (b) Use technology to find the local maximum and local minimum points correct to three decimal places. [4 marks]

87.  A radioactive sample is modelled by $A(t) = 800e^{-0.18t}$ mg.

- (a) Find the half-life. [2 marks]
- (b) Find the time when the amount first falls below 100 mg. [2 marks]

88.  Solve $x^4 + 5x - 3 = 0$, giving all real solutions correct to three decimal places. [3 marks]

89.  Solve $\ln(x + 3) = \frac{x}{2}$, giving all real solutions correct to three decimal places. [4 marks]

90.  A quadratic has vertex $(2, -3)$ and passes through $(5, 15)$.

(a) Find its equation in vertex form. [2 marks]

(b) Find its x -intercepts correct to three decimal places. [2 marks]

91.  Use technology to solve $x^4 - 3x^3 - 2x^2 + 5x - 1 = 0$. Record all real roots to three decimal places. Then verify numerically that their sum and product agree with the coefficient relationships for a quartic. [6 marks]

92.  Let $f(x) = \frac{x^2 - 3x + 5}{x^2 - x - 6}$.

(a) State all vertical and horizontal asymptotes. [3 marks]

(b) State the y -intercept and whether any x -intercepts exist. [2 marks]

(c) Solve $f(x) = 2$ correct to three decimal places. [2 marks]

93.  Solve $x^3 - 2x^2 - 5x + 1 > 0$ graphically. Give boundary values to three decimal places. [4 marks]

94.  Solve $|xe^{-x}| = 0.2$ correct to three decimal places. [4 marks]

95.  Let $f(x) = (x - 1)(x - 2)(x + 1)$. Compare $g(x) = |f(x)|$ and $h(x) = f(|x|)$.

(a) State the zeros of g . [1 mark]

(b) State the zeros of h . [2 marks]

(c) State which transformed graph is necessarily even. [1 mark]

96.  On the interval $-5 \leq x \leq 5$, solve

$$\frac{x+2}{x-1} = 2^x.$$

Give all solutions correct to three decimal places. [4 marks]

97.  A cooling object is modelled by $T(t) = 20 + 70e^{-0.12t}$ degrees Celsius. Find the time at which $T = 35^\circ\text{C}$, correct to three significant figures. [3 marks]

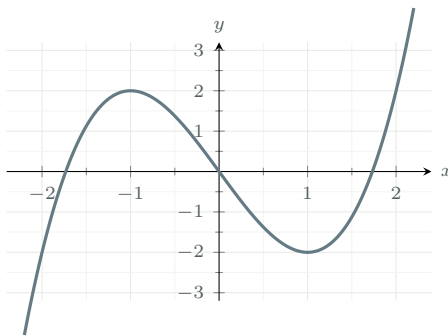
98.  Use technology to analyse $f(x) = x^3 - 6x^2 + 9x + 4$. State the real zero and the two local turning points correct to three decimal places where necessary. [5 marks]

99.  Consider the family of equations $x^3 - 3x = k$. By graphing $y = x^3 - 3x$, determine the values of k for which the equation has

(a) three distinct real solutions; [2 marks]

(b) exactly two distinct real solutions. [2 marks]

A guide graph is provided below.



100.  Let $f(x) = \frac{x^2 - 4}{x - 1}$ and $g(x) = |f(2x - 3)|$.

- (a) State the vertical and oblique asymptotes of f . [2 marks]
- (b) Solve $f(x) = 3$, giving answers to three decimal places. [2 marks]
- (c) State the domain of g , its vertical asymptote and all of its zeros. [4 marks]