



Sequences & Series: Complete Practice

IB Mathematics: Analysis and Approaches SL & HL

100 shared questions on SL 1.2, SL 1.3 and SL 1.8, followed by 20 additional AA HL-only challenges.

Learning goal	Master arithmetic sequences and series, finite geometric sequences and series, and infinite convergent geometric series; move confidently between terms, sums, sigma notation, models and exam-style contexts.
Syllabus link	IB Mathematics AA SL/HL: SL 1.2 arithmetic sequences and series; SL 1.3 geometric sequences and finite geometric series; SL 1.8 sum of infinite convergent geometric sequences, including $ r < 1$.
Structure	Section 1: 30 short-response questions. Section 2: 30 extended-response questions. Section 3: 40 original IB-style questions. Section 4: 20 additional AA HL-exclusive challenge questions.
Calculator	Primarily Paper 1 / analytical practice. Questions marked GDC are designed for technology. Give exact answers unless a decimal approximation is requested.

Instructions

Show clear working. Identify whether a pattern is arithmetic or geometric before selecting a formula. For an infinite geometric series, check $|r| < 1$ before using $S_{\infty} = \frac{u_1}{1-r}$. Unless another accuracy is stated, give non-exact numerical answers to **3 significant figures**.

1. Short response — core fluency (30 questions)

1. An arithmetic sequence begins 7, 12, 17, ... State the common difference.

2. For an arithmetic sequence, $u_1 = 4$ and $d = 3$. Find u_{20} .

3. An arithmetic sequence has $u_8 = 31$ and $d = 4$. Find u_1 .

4. An arithmetic sequence has $u_5 = 18$ and $u_{12} = 46$. Find the common difference.



5. For an arithmetic sequence, $u_1 = 9$ and $d = 7$. Find n if $u_n = 128$.

6. Find the sum of the first 20 terms of $3, 7, 11, \dots$

7. An arithmetic sequence has $u_1 = 12$ and $d = -2$. Find S_{15} .

8. Evaluate $\sum_{k=1}^{20} (3k + 2)$.

9. The three expressions $2x - 1$, $5x + 2$, $9x - 1$ are consecutive terms of an arithmetic sequence. Find x .

10. The sum of the first n terms of a sequence is $S_n = 3n^2 + 2n$. Find an expression for u_n .

11. A geometric sequence begins $2, 6, 18, \dots$. State the common ratio.

12. For a geometric sequence, $u_1 = 5$ and $r = 3$. Find u_7 .

13. A geometric sequence has $u_4 = 54$ and $r = \frac{1}{3}$. Find u_7 using a flexible term formula.

14. A geometric sequence has $u_2 = 12$ and $u_5 = 96$. Given $r > 0$, find r and u_1 .

15. The three positive numbers $4, x, 36$ are consecutive terms of a geometric sequence. Find x .



16. Find the sum of the first 8 terms of a geometric sequence with $u_1 = 7$ and $r = 2$.

17. Find the sum of the first 5 terms of $81, 27, 9, \dots$

18. For a geometric sequence, $u_1 = 3$ and $r = 2$. Find n if $u_n = 384$.

19.

GDC

A geometric sequence is given by $u_n = 2.5(1.4)^{n-1}$. Find the least value of n for which $u_n > 50$.

20. Evaluate $\sum_{k=1}^6 4 \left(\frac{1}{2}\right)^{k-1}$.

21. Determine whether $10 + 8 + 6.4 + \dots$ has a finite sum to infinity.

22. Determine whether $5 - 5.5 + 6.05 - \dots$ has a finite sum to infinity. Give a reason.

23. Find the sum to infinity of $12 + 6 + 3 + \dots$.

24. Find the sum to infinity of $20 - 5 + 1.25 - \dots$.

25. A convergent geometric series has $S_\infty = 50$ and $r = 0.6$. Find u_1 .



26. A convergent geometric series has $u_1 = 18$ and $S_\infty = 30$. Find r .
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27. A geometric series has common ratio $r = \frac{k-2}{3}$. Find the values of k for which the series converges.
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28. A geometric series has $u_1 = 64$ and $r = \frac{1}{2}$. Find $S_\infty - S_6$.
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29. Express $0.272727\dots$ as a fraction using an infinite geometric series.
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30. A ball is dropped from a height of 3 m and each rebound reaches 60% of the previous maximum height. Find the total vertical distance travelled over all time.

2. Extended response — mixed sequences and series (30 questions)

31. An arithmetic sequence has $u_4 = 17$ and $u_{10} = 41$. Find u_1 , d and S_{25} .
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32. The finite arithmetic sequence $14, 19, 24, \dots, 204$ has n terms. Find n and the sum of all its terms.
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33. The sum of the first n terms of a sequence is $S_n = 2n^2 + 3n$. Find u_n , and hence state u_1 , d and u_{10} .
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34. The three expressions $x + 1$, $3x - 2$ and $7x - 11$ are consecutive terms of an arithmetic sequence. Find x and the three terms.
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35. A starting salary is \$48 000 and increases by \$1800 each year. Find the salary in year 15 and the total of the first 15 annual salaries.
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36. A decreasing arithmetic sequence begins $100, 94, 88, \dots$. Find the first negative term and determine the maximum value of S_n .



37. For the arithmetic sequence with $u_1 = 7$ and $d = 3$, find $u_{20} + u_{21} + \cdots + u_{35}$.

38. Sequence A has $u_1 = 12$, $d = 4$. Sequence B has $v_1 = 50$, $d = 1$. Find the first value of n for which $u_n > v_n$.

39. The data 52.3, 55.0, 57.5, 60.4, 62.9 are approximately arithmetic. Estimate a common difference using the first and fifth values, then predict the tenth value.

40. Show that $\sum_{k=1}^n (5k - 2) = \frac{n(5n + 1)}{2}$. Hence find n if the sum is 429.

41. A geometric sequence has $u_3 = 20$ and $u_6 = 160$, with $r > 0$. Find u_1 , r and S_8 .

42. A geometric sequence has $u_2 = 24$ and $u_5 = 3$, with $r > 0$. Find u_1 , r and S_7 .

43. The three positive numbers $x, 12, 3$ are consecutive terms of a geometric sequence. Find x , r and S_6 .

44.

GDC

The sequence $u_n = 320(0.75)^{n-1}$ is decreasing. Find the largest value of n for which $u_n > 20$.

45.

GDC

A geometric series has $u_1 = 5$ and $r = 1.3$. Find the least value of n for which $S_n > 500$.



46. A geometric sequence has $u_4 = 54$ and $S_4 = 80$. Find u_1 and r , and hence find u_7 .

47. The partial sums of a sequence satisfy $S_n = 3(2^n - 1)$. Find u_n and identify the first term and common ratio.

48. A population is 1200 in year 1 and grows by 6% per year. Find the population in year 12 and the sum of the yearly populations for the first 12 years.

49. A machine is worth \$80 000 and loses 12% of its value each year. Find its value after 8 years and the first whole number of years after which its value is below \$30 000.

50. A ball is dropped from 5 m and each rebound reaches 70% of the previous maximum height. Find the total distance travelled after the ball has completed four full rebounds.

51. A geometric sequence has $u_4 = 16$ and $r = \frac{1}{2}$. Find u_1 and S_∞ .

52. A geometric series has common ratio $r = \frac{2k-1}{3}$. Find the values of k for which the series converges.

53. A convergent geometric series has $S_\infty = 36$, $r = \frac{1}{3}$. Find u_1 and u_3 .

54. For the alternating geometric series with $u_1 = 50$ and $r = -0.4$, find S_∞ and S_5 .

55. For a convergent geometric series with $r = \frac{1}{2}$, the tail after the first four terms is 6. Find u_1 and S_∞ .

56. Express $0.184184184\dots$ as a fraction using an infinite geometric series.



57. A self-similar design contains squares whose perimeters form a geometric sequence. The first perimeter is 40 cm and each new perimeter is 80% of the previous one. Find the total of all perimeters.

58. A convergent geometric sequence has $u_1 = 12$ and $r = \frac{1}{2}$. Find $u_1 + u_3 + u_5 + \dots$.

59. For a convergent geometric series, the sum of the odd-indexed terms is 15 and the sum of the even-indexed terms is 6. Find r , u_1 and S_∞ .

60. A geometric series has $u_1 = 100$ and $r = \frac{1}{2}$. Find the least n such that $S_\infty - S_n < 0.1$.

3. IB exam-style / Paper 1 — original sister questions (40 questions)

61. An arithmetic sequence has $u_1 = 36$ and $u_5 = 12$.

(a) Find u_{13} . [2 marks]

(b) The sum of the first n terms is zero. Find n . [3 marks]

62. For an arithmetic sequence, $u_7 = 6$ and $u_6 + u_{12} = 24$. Find u_1 and the common difference. [5 marks]

63. The first three terms of a sequence are $k + 5$, $-2k - 4$, $3k + 4$.

(a) Find k if the terms are arithmetic. [2 marks]

(b) Find all values of k if the terms are geometric. [4 marks]

64. A geometric sequence has first term 50 and fourth term 0.4.

(a) Find the common ratio. [2 marks]

(b) Find S_∞ . [2 marks]

(c) Find the least n for which $S_n > 62.4$. [2 marks]

65. A tennis ball is dropped from a height of 8 m. Each rebound reaches 60% of the previous maximum height.

- (a) Find the maximum height after the fourth bounce. [2 marks]
- (b) Find the total distance travelled immediately before the fifth impact with the ground. [3 marks]
- (c) Find the total distance travelled over all time. [2 marks]

66. Helen and Jane each start on an annual salary of \$50 000. Helen's salary rises by \$2000 each year. Jane's salary rises by 4% each year.

- (a) Write formulae for their salaries in year n . [2 marks]
- (b) Find the first year in which Jane's salary is greater than Helen's. [3 marks]
- (c) Find Jane's total earnings over the first 10 years. [3 marks]

67. Use an infinite geometric series to express $0.4181818\dots$ as a fraction in lowest terms. [5 marks]

68. Let $S_n = \sum_{j=0}^{n-1} 6c^j$.

- (a) Find the values of c for which S_n converges as $n \rightarrow \infty$. [2 marks]
- (b) For $c = 0.75$, find S_∞ . [2 marks]
- (c) GDC

Find the least n such that $|S_\infty - S_n| < 0.05$. [3 marks]

69. The sum of the first n terms of a sequence is $S_n = n(3n + 5)$.

- (a) Show that the sequence is arithmetic. [3 marks]
- (b) Find u_{20} and S_{20} . [3 marks]

70. The partial sums of a geometric sequence are $S_n = \frac{4(1 - (1/2)^n)}{1 - 1/2}$.

- (a) State u_1 and r . [2 marks]
- (b) Find S_∞ . [1 marks]
- (c) Find the least n such that $S_\infty - S_n < 0.01$. [3 marks]



71. An arithmetic sequence and a geometric sequence both begin 6, 12, ...

- (a) Find the tenth term of each sequence. [3 marks]
- (b) Find the sum of the first eight terms of each sequence. [4 marks]
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72. A sequence of picture frames has first area 24 cm^2 . Each new frame has both width and height increased by 20% from the preceding frame.

- (a) Show that the areas form a geometric sequence and state its ratio. [2 marks]
- (b) Find the area of the sixth frame. [2 marks]
- (c) Find the total area of the first six frames. [2 marks]
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73. In a staircase pattern, row n contains $3n + 2$ tiles.

- (a) Explain why the row counts form an arithmetic sequence. [1 marks]
- (b) Find the total number of tiles in the first 20 rows. [2 marks]
- (c) Find the least number of rows needed for the total to exceed 2000. [3 marks]
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74. A wildlife population is 2400 in year 1 and grows by 8% per year.

- (a) Find the population in year 10. [2 marks]
- (b) Find the first year in which the population exceeds 5000. [3 marks]
- (c) Find the sum of the yearly populations over the first 10 years. [2 marks]
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75. A car costs \$42 000. Its value falls by 12% during the first year and then by 9% per year thereafter.

- (a) Find its value after the first year. [1 marks]
- (b) Find its value after 8 years. [3 marks]
- (c) Find the first whole number of years after which its value is below half its original value. [3 marks]
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76. A donor gives \$1200 in the first year and half as much in each subsequent year.

- (a) Find the total donated in the first six years. [2 marks]
- (b) Find the total amount donated over all time. [2 marks]
- (c) Find how much remains to be donated after year 6. [2 marks]



77. Use an infinite geometric series to express $0.2474747\dots$ as a fraction in lowest terms. [5 marks]

78. An infinite geometric series has first term $a > 0$ and second term $\frac{a-2}{2}$.

(a) Write the common ratio in terms of a . [1 marks]

(b) Determine the values of a for which the infinite sum exists. [3 marks]

(c) Given $S_\infty = 12$, find a . [4 marks]

79. A family of balls all rebound to $\frac{2}{3}$ of the previous maximum height. Ball n is dropped from a height $8 - 0.5(n-1)$ metres.

(a) Show that the total distance travelled by a ball dropped from height H is $5H$. [3 marks]

(b) Find the first ball whose total distance is less than 20 m. [3 marks]

80. A moving point travels successive distances 10, 7, 4.9, ... metres.

(a) Find the total distance travelled in the first five stages. [2 marks]

(b) Find the total distance travelled if the motion continues indefinitely. [2 marks]

81. Plan A pays \$400 in week 1 and increases by \$25 each week. Plan B pays \$400 in week 1 and increases by 5% each week.

(a) Write formulae for the weekly payments. [2 marks]

(b) GDC

Find the first week in which Plan B pays more than Plan A. [3 marks]

82. A convergent geometric series has $S_\infty = 40$ and $S_4 = 37.5$. Find all possible real values of r and the corresponding first terms. [6 marks]

83. For a convergent geometric series, the sum of the odd-indexed terms is 15 and the sum of the even-indexed terms is 6. Find r , u_1 and S_∞ . [6 marks]



84. A geometric series has $u_1 = 64$ and $r = 0.75$.

GDC

Find the least value of n for which $S_\infty - S_n < 1$.

[5 marks]

85. Sequence A is arithmetic with $u_1 = 8$, $d = 3$. Sequence B is geometric with $v_1 = 8$, $r = 1.15$.

(a) Find S_{10} for each sequence.

[4 marks]

(b) **GDC**

Find the first n for which $v_n > u_n$.

[3 marks]

86. A convergent geometric series has $S_\infty = 54$ and $r = \frac{1}{3}$. Find u_1 , u_3 and the tail beginning with u_5 . [5 marks]

87. The three expressions $2x$, $x + 3$, 8 are consecutive terms of a geometric sequence.

(a) Find all possible values of x .

[3 marks]

(b) For each value, decide whether the corresponding infinite series converges.

[2 marks]

(c) Find S_∞ for the convergent case.

[2 marks]

88. A geometric sequence has $u_3 = 12$ and $u_7 = \frac{3}{4}$. Find all possible real values of r , and the corresponding sums to infinity.

[7 marks]

89. A decreasing arithmetic sequence begins $80, 77, 74, \dots$

(a) Find the first negative term.

[2 marks]

(b) Find the maximum value of S_n .

[3 marks]

90. An annual bonus is \$1500 in year 1 and increases by \$250 per year. Find the total bonus paid over the first 12 years.

[4 marks]

91. Two data sets are shown. A: 80, 84.1, 88.0, 92.2, 96.1. B: 50, 60.5, 72.4, 86.9, 104.2.

- (a) State which set is better modelled arithmetically and which geometrically, with a brief justification. [3 marks]
- (b) Use a reasonable model to predict the next value of each set. [3 marks]
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92. Runner A runs 5 km in week 1 and adds 1.5 km each week. Runner B runs 5 km in week 1 and increases the weekly distance by 12%.

GDC

Find the first week in which Runner B's weekly distance exceeds Runner A's. [5 marks]

93. The perimeters of successive shapes in a fractal are 12, 7.2, 4.32, ... cm. Find the total of all perimeters. [3 marks]

94. A nested-square pattern starts with area 100 cm^2 . Each new square has side length 70% of the preceding square. Find the total area of all squares. [4 marks]

95. Use an infinite geometric series to express $0.1252525 \dots$ as a fraction in lowest terms. [5 marks]

96. A geometric series has ratio $r = \frac{3k - 2}{4}$.

- (a) Find the values of k for which it converges. [3 marks]
- (b) Given $u_1 = 5$ and $S_\infty = 20$, find k . [3 marks]
-

97. For a convergent geometric series, let $R_n = S_\infty - S_n$.

- (a) Show that $R_n = r^n S_\infty$. [3 marks]
- (b) For $u_1 = 9$ and $r = \frac{2}{3}$, find the least n for which $R_n < 0.1$. [3 marks]
-



98. An arithmetic sequence and a geometric sequence both begin $4, 10, \dots$. Find the fifth term and the sum of the first five terms of each sequence. [6 marks]

99. A sequence satisfies $u_1 = 150$ and $u_{n+1} = 0.7u_n$.

(a) Find u_6 . [2 marks]

(b) Find S_{10} . [2 marks]

(c) Find S_∞ . [2 marks]

100. A robot travels 12 m on its first leg and 75% as far on each subsequent leg.

(a) Find the total distance in the first eight legs. [2 marks]

(b) Find the total distance if it continues indefinitely. [2 marks]

(c) **GDC**

Find the least n such that the remaining distance after n legs is less than 0.1 m. [3 marks]

(d) A second robot travels 12, 11.5, 11.0, $\dots$ m. Find its total distance in the first eight legs and compare. [3 marks]

4. AA HL exclusive challenge — Questions 101–120

AA HL students

Questions 1–100 form the shared AA SL/HL worksheet. Questions 101–120 are additional AA HL challenges: they remain grounded in SL 1.2, SL 1.3 and SL 1.8, but are less scaffolded and more algebraically demanding.

101. The first three terms are $k + 5$, $-2k - 4$, $3k + 4$.

(a) Find the two values of k for which the terms are geometric.

(b) For each value, state the common ratio and determine whether the corresponding infinite series converges.

(c) Find S_∞ for the convergent case.



102. Let $S_n = \sum_{j=0}^{n-1} 5(\log_2 c)^j$, where $c > 0$.

- (a) Find the range of c for which the sequence of partial sums converges.
(b) For $c = 1.5$,

GDC

find the least n such that $|S_\infty - S_n| < 0.1$.

103. Express $0.4171717\ldots$ as a fraction by decomposing it into a terminating part and an infinite geometric series.

104. For a geometric sequence with first term a and ratio $r \neq 1$, let $T_n = S_1 + S_2 + \cdots + S_n$. Derive a closed expression for T_n in terms of a, r, n . Hence find T_{10} for $a = 3, r = \frac{1}{2}$.

105. For a convergent geometric series, the sum of the odd-indexed terms is 20 and the total sum is 32. Find r and u_1 .

106. A geometric series has $u_1 = 2k + 1$ and $r = \frac{k-1}{k+2}$. Determine the values of k for which the series converges and write S_∞ in terms of k .

107. A convergent geometric series has $S_\infty = 18$ and $S_2 = 13.5$. Find all possible real values of r and the corresponding values of u_1 .

108. Geometric sequence A has common ratio $r \neq 0$. Geometric sequence B has common ratio $\frac{1}{r}$. Prove that the two corresponding infinite series cannot both converge.

109. The first three terms of a series are $\ln x, p \ln x, \frac{1}{4} \ln x$, where $x > 1$.

- (a) Find the possible values of p if the series is geometric.
(b) For each value, show that the infinite series converges and find its sum in terms of $\ln x$.



110. A geometric sequence is $u_n = 8\left(\frac{1}{3}\right)^{n-1}$. Find $\sum_{n=1}^{\infty} u_n^2$.

111. A geometric sequence has $u_1 = 6$ and $r = \frac{1}{2}$. Find $\sum_{n=1}^{\infty} u_n u_{n+1}$.

112. For a convergent geometric series, the tail $R_n = S_{\infty} - S_n$ satisfies $R_n = 4u_{n+1}$ for every n . Find the common ratio.

113. A geometric series has common ratio $r = \frac{2k+3}{k-4}$. Find the complete set of real values of k for which the series converges.

114. A non-zero real sequence is both arithmetic and geometric. Prove that it must be constant.

115. Use an infinite geometric series to express $0.2686868\dots$ as a fraction in lowest terms.

116. A convergent alternating geometric series has $u_1 = 50$ and $r = -0.6$.

GDC

Find the least n for which $|S_{\infty} - S_n| < 0.01$.

117. An arithmetic sequence and a geometric sequence both begin $10, 15, \dots$. Find the least n for which the geometric term exceeds twice the arithmetic term.

118. A geometric sequence has $u_3 = 18$ and $u_7 = \frac{2}{9}$. Find all possible real common ratios and the corresponding sums to infinity.



119. A self-similar construction starts with one shaded region of area A . Each region produces two new shaded regions, each with area q times its parent. Find the condition on q for the total shaded area to be finite, and find that total area in terms of A and q .

120. A convergent geometric series satisfies $S_{\infty} = 3S_2$.

- (a) Find all possible real values of r .
- (b) If $S_{\infty} = 12$, find the corresponding possible first terms.

5. Answer key and solution outlines

Section 1 answers — Questions 1–30

1. $d = 5$.
2. $u_{20} = 61$.
3. $u_1 = 3$.
4. $d = 4$.
5. $n = 18$.
6. $S_{20} = 820$.
7. $S_{15} = -30$.
8. 670.
9. $x = 6$.
10. $u_n = 6n - 1$.
11. $r = 3$.
12. $u_7 = 3645$.
13. $u_7 = 2$.
14. $r = 2, u_1 = 6$.
15. $x = 12$.
16. $S_8 = 1785$.
17. $S_5 = 121$.
18. $n = 8$.
19. $n = 10$.
20. $63/8$.
21. Yes: $r = 0.8$, so $|r| < 1$.
22. No: $r = -1.1$, so $|r| > 1$.
23. $S_\infty = 24$.
24. $S_\infty = 16$.
25. $u_1 = 20$.
26. $r = 0.4$.
27. $-1 < k < 5$.
28. 2.
29. $3/11$.
30. 12 m.

Section 2 answers — Questions 31–60

31. $d = 4, u_1 = 5, S_{25} = 1325$.
32. $n = 39$, sum = 4251.
33. $u_n = 4n + 1; u_1 = 5, d = 4, u_{10} = 41$.
34. $x = 3$; terms 4, 7, 10.
35. $u_{15} = \$73\,200$, total = \$909 000.
36. $u_{18} = -2$ is the first negative term; maximum $S_{17} = 884$.
37. 1384.
38. $n = 14$.
39. $d \approx 2.65, u_{10} \approx 76.2$.
40. The sum is $n(5n + 1)/2; n = 13$.
41. $r = 2, u_1 = 5, S_8 = 1275$.
42. $r = 1/2, u_1 = 48, S_7 = 381/4$.
43. $x = 48, r = 1/4, S_6 = 4095/64 \approx 64.0$.
44. $n = 10$.
45. $n = 14$.
46. $u_1 = 2, r = 3, u_7 = 1458$.
47. $u_n = 3 \cdot 2^{n-1}$, so $u_1 = 3, r = 2$.
48. $u_{12} \approx 2280; S_{12} \approx 20\,200$.
49. $V_8 \approx \$28\,800$; below \$30 000 after 8 years.
50. 22.7 m.

51. $u_1 = 128$, $S_\infty = 256$.
 52. $-1 < k < 2$.
 53. $u_1 = 24$, $u_3 = 8/3$.
 54. $S_\infty = 250/7 \approx 35.7$; $S_5 = 36.08 \approx 36.1$.
 55. $u_1 = 48$, $S_\infty = 96$.
 56. $184/999$.
 57. 200 cm.
 58. 16 .
 59. $r = 0.4$, $u_1 = 12.6$, $S_\infty = 21$.
 60. $n = 11$.

Section 3 solution outlines — Questions 61–100

61. (a) $d = -6$, so $u_{13} = -36$. (b) $S_n = \frac{n}{2}(72 - 6(n-1)) = 3n(13 - n)$, hence $n = 13$.
 62. $u_1 + 6d = 6$ and $2u_1 + 16d = 24$. Hence $d = 3$, $u_1 = -12$.
 63. (a) $2(-2k-4) = (k+5) + (3k+4)$ gives $k = -17/8$. (b) $(-2k-4)^2 = (k+5)(3k+4)$ gives $(k+1)(k-4) = 0$, so $k = -1, 4$.
 64. $50r^3 = 0.4$, so $r = 0.2$. Then $S_\infty = 62.5$. Since $S_4 = 62.4$, the least n with $S_n > 62.4$ is 5.
 65. (a) $8(0.6)^4 = 1.04$ m (3 s.f.). (b) $8 + 2 \sum_{k=1}^4 8(0.6)^k \approx 28.9$ m. (c) $8 + 2 \cdot \frac{8(0.6)}{1-0.6} = 32$ m.
 66. $H_n = 50000 + 2000(n-1)$, $J_n = 50000(1.04)^{n-1}$. Jane first exceeds Helen in year 3. Her first-10-year total is about \$600 000 (3 s.f.).
 67. $0.4181818... = 0.4 + 0.0181818... = \frac{2}{5} + \frac{18/1000}{1-1/100} = \frac{23}{55}$.
 68. Convergence requires $|c| < 1$. For $c = 0.75$, $S_\infty = 24$. Since $|S_\infty - S_n| = 24(0.75)^n$, the least n is 22.
 69. $u_n = S_n - S_{n-1} = 6n + 2$, which is arithmetic with $d = 6$. Thus $u_{20} = 122$ and $S_{20} = 1300$.
 70. $u_1 = 4$, $r = 1/2$, $S_\infty = 8$. The tail is $8(1/2)^n$, giving least $n = 10$.
 71. Arithmetic: $d = 6$, so $u_{10} = 60$, $S_8 = 216$. Geometric: $r = 2$, so $u_{10} = 3072$, $S_8 = 1530$.
 72. Area ratio $= (1.2)^2 = 1.44$. Sixth area ≈ 149 cm²; first-six total ≈ 432 cm².
 73. The row counts have constant difference 3. $S_{20} = 670$. Solve $\frac{n}{2}(3n+7) > 2000$ to obtain least $n = 36$.
 74. Year 10: $2400(1.08)^9 \approx 4800$. First above 5000: year 11. First-10 total ≈ 34800 .
 75. After year 1: \$36 960. After 8 years: $36960(0.91)^7 \approx \$19\,100$. It first falls below \$21 000 after 7 years.
 76. $S_6 = 1200 \frac{1 - (1/2)^6}{1 - 1/2} = 2362.5$. $S_\infty = 2400$, so the tail after year 6 is 37.5.
 77. $0.2474747... = \frac{1}{5} + \frac{47/1000}{1-1/100} = \frac{49}{198}$.
 78. $r = (a-2)/(2a)$. For $a > 0$, $|r| < 1$ gives $a > 2/3$. Also $S_\infty = \frac{2a^2}{a+2} = 12$, so $a = 3 + \sqrt{21}$.
 79. For height H , total distance is $H + 2 \frac{(2/3)H}{1-2/3} = 5H$. We need $5[8 - 0.5(n-1)] < 20$, giving first $n = 10$.
 80. $S_5 = 10 \frac{1 - 0.7^5}{0.3} = 27.731 \approx 27.7$ m. $S_\infty = 10/0.3 = 100/3$ m.
 81. $A_n = 400 + 25(n-1)$ and $B_n = 400(1.05)^{n-1}$. The first week with $B_n > A_n$ is week 11.
 82. $S_\infty - S_4 = 2.5 = 40r^4$, so $r = \pm 1/2$. Then $u_1 = 40(1-r)$, giving $(r, u_1) = (1/2, 20)$ or $(-1/2, 60)$.
 83. Even-indexed sum $= r$ times odd-indexed sum, so $r = 6/15 = 0.4$. Then $u_1 = 15(1-r^2) = 12.6$ and $S_\infty = 21$.
 84. $S_\infty = 256$ and the tail is $256(0.75)^n$. The least n is 20.
 85. Arithmetic $S_{10} = 215$. Geometric $S_{10} \approx 162$. Solving $8(1.15)^{n-1} > 8 + 3(n-1)$ gives first $n = 14$.
 86. $u_1 = 54(2/3) = 36$, $u_3 = 4$. The tail from u_5 is $u_5/(1-r) = (36/81)/(2/3) = 2/3$.
 87. $(x+3)^2 = 16x$ gives $x = 1$ or 9 . For $x = 1$, $r = 2$ so no infinite sum. For $x = 9$, $r = 2/3$ and $S_\infty = 18/(1-2/3) = 54$.
 88. $r^4 = (3/4)/12 = 1/16$, hence $r = \pm 1/2$. Since $u_1 = 48$, the sums are 96 and 32.
 89. $u_n = 80 - 3(n-1)$. The first negative term is $u_{28} = -1$. The maximum is $S_{27} = 1107$.
 90. $u_{12} = 4250$, so $S_{12} = \frac{12}{2}(1500 + 4250) = \$34\,500$.
 91. A has differences close to 4, so arithmetic; next ≈ 100 . B has ratios close to 1.205, so geometric; next ≈ 126 .



(reasonable models may vary slightly).

92. Solve $5(1.12)^{n-1} > 5 + 1.5(n-1)$ using technology: first $n = 17$.

93. $r = 0.6$, so the total is $12/(1-0.6) = 30$ cm.

94. Area ratio $= 0.7^2 = 0.49$, so total area $= 100/(1-0.49) \approx 196$ cm².

95. $0.1252525\ldots = 0.1 + 0.0252525\ldots = \frac{1}{10} + \frac{25/1000}{1-1/100} = \frac{62}{495}$.

96. $|(3k-2)/4| < 1$ gives $-2/3 < k < 2$. Also $r = 1 - u_1/S_\infty = 3/4$, so $k = 5/3$.

97. $R_n = u_{n+1} + u_{n+2} + \cdots = \frac{u_1 r^n}{1-r} = r^n S_\infty$. Here $S_\infty = 27$; $27(2/3)^n < 0.1$ gives least $n = 14$.

98. Arithmetic: $d = 6$, so $u_5 = 28$, $S_5 = 80$. Geometric: $r = 2.5$, so $u_5 = 156.25$, $S_5 = 257.75$.

99. $u_6 = 150(0.7)^5 \approx 25.2$. $S_{10} = 150 \frac{1-0.7^{10}}{0.3} \approx 486$. $S_\infty = 500$.

100. First eight: $12 \frac{1-0.75^8}{0.25} \approx 43.2$ m. Infinite total $= 48$ m. Tail $= 48(0.75)^n < 0.1$ gives least $n = 22$. The arithmetic robot travels 82 m in the first eight legs.

AA HL exclusive answers — Questions 101–120

101. Geometric condition gives $(k+1)(k-4) = 0$. For $k = -1$: terms 4, -2 , 1, $r = -1/2$, convergent, $S_\infty = 8/3$. For $k = 4$: terms 9, -12 , 16, $r = -4/3$, divergent.

102. $|\log_2 c| < 1 \iff 1/2 < c < 2$. For $c = 1.5$, $r \approx 0.584963$, $S_\infty \approx 12.047$ and the least n is 9.

103. $0.4171717\ldots = 0.4 + 0.0171717\ldots = \frac{2}{5} + \frac{17/1000}{1-1/100} = \frac{413}{990}$.

104. $T_n = \frac{a}{1-r} \left[n - \frac{r(1-r^n)}{1-r} \right]$. For $a = 3$, $r = 1/2$, $n = 10$, $T_{10} = 54.005859375 \approx 54.0$.

105. Even sum $= 32 - 20 = 12 = r(20)$, so $r = 0.6$. Then $u_1 = 20(1-r^2) = 12.8$.

106. Convergence requires $\left| \frac{k-1}{k+2} \right| < 1$, giving $k > -1/2$. Also $S_\infty = \frac{(2k+1)(k+2)}{3}$.

107. $18 - 13.5 = 18r^2$, so $r = \pm 1/2$. Then $u_1 = 18(1-r)$, giving $u_1 = 9$ or 27 .

108. Convergence of A needs $|r| < 1$; convergence of B needs $|1/r| < 1$, i.e. $|r| > 1$. These cannot both hold.

109. Geometric condition: $(p \ln x)^2 = (\ln x)(\frac{1}{4} \ln x)$, so $p = \pm 1/2$. The sums are $2 \ln x$ and $\frac{2}{3} \ln x$, respectively.

110. $u_n^2 = 64(1/9)^{n-1}$, so the sum is $64/(1-1/9) = 72$.

111. $u_n u_{n+1} = 18(1/4)^{n-1}$, so the sum is $18/(1-1/4) = 24$.

112. $R_n = u_{n+1}/(1-r) = 4u_{n+1}$, so $1/(1-r) = 4$ and $r = 3/4$.

113. $\left| \frac{2k+3}{k-4} \right| < 1$ gives $-7 < k < 1/3$.

114. If arithmetic, $u_{n+1} = u_n + d$. If geometric, $u_{n+1} = ru_n$. Hence $(r-1)u_n = d$ for every n . Unless $r = 1$, this forces all u_n equal anyway; therefore $d = 0$, $r = 1$, and the sequence is constant.

115. $0.2686868\ldots = 0.2 + 0.0686868\ldots = \frac{1}{5} + \frac{68/1000}{1-1/100} = \frac{133}{495}$.

116. $S_\infty = 50/(1+0.6) = 31.25$. The absolute tail is $31.25(0.6)^n$; least $n = 16$.

117. Arithmetic term $= 10+5(n-1)$; geometric term $= 10(1.5)^{n-1}$. The first n with $10(1.5)^{n-1} > 2[10+5(n-1)]$ is $n = 6$.

118. $r^4 = (2/9)/18 = 1/81$, so $r = \pm 1/3$. Then $u_1 = 162$. The sums are 243 and $243/2$.

119. At generation n , total new shaded area is $A(2q)^n$. The total is finite iff $|2q| < 1$; for area scaling $q \geq 0$, this is $0 \leq q < 1/2$. Total area $= A/(1-2q)$.

120. $\frac{u_1}{1-r} = 3u_1(1+r)$ gives $1 = 3(1-r^2)$, so $r = \pm \sqrt{2/3}$. If $S_\infty = 12$, then $u_1 = 12(1-r)$, giving $12(1 \mp \sqrt{2/3})$.