



Section 1.3: Geometric Sequences & Series

IB Mathematics: Analysis and Approaches SL & HL

60-question worksheet: 50 core questions plus 10 AA HL-only challenges on finite geometric sequences and series.

Learning goal	Recognise, model and solve problems involving geometric sequences and finite geometric series, including term and sum thresholds and applications.
Syllabus link	IB Mathematics AA SL/HL: SL 1.3 . Geometric sequences and series; the n th term; the sum of the first n terms; sigma notation; applications and modelling.
Structure	Section 1: 15 short-response questions. Section 2: 15 extended-response questions. Section 3: 20 original IB-style questions. Section 4: 10 additional AA HL challenge questions.
Calculator	Questions marked GDC are designed for technology. Section 3 is Paper 1-style unless a question is explicitly marked for GDC use.

Instructions

Show clear mathematical working. Use exact values when practical. Unless another accuracy is stated, give non-exact numerical answers to **3 significant figures**. When a question asks for the least or greatest integer term number, justify the integer selected.

1. Short response — core fluency (15 questions)

1. State whether 6, 18, 54, 162, ... is geometric. If it is, state the common ratio r .

2. Find the common ratio of 160, 80, 40, 20, ...

3. The first term of a geometric sequence is 7 and $r = 3$. Find u_6 .



4. For a geometric sequence, $u_1 = 96$ and $r = \frac{1}{2}$. Find u_8 .

5. Write an expression for the n th term of the sequence $5, 15, 45, 135, \dots$

6. A geometric sequence has $u_4 = 54$ and $r = 3$. Find u_1 .

7. A geometric sequence has $u_3 = 20$ and $u_6 = 160$. Find the positive common ratio.

8. The terms $12, x, 75$ are three consecutive terms of a geometric sequence, where $x > 0$. Find x .

9. Find the sum of the first 7 terms of $3, 6, 12, 24, \dots$

10. For a geometric sequence, $u_1 = 80$ and $r = 0.6$. Find S_5 .

11. Evaluate $\sum_{k=1}^6 4(2)^{k-1}$.

12. The partial sums of a sequence are given by $S_n = 5(2^n - 1)$. Find u_7 using $u_n = S_n - S_{n-1}$.

13. A quantity increases by 8% each year. State the corresponding geometric ratio.

14. A quantity decreases by 17% each year. State the corresponding geometric ratio.

15. A geometric sequence has $r = -1$ and $u_1 = 9$. Find S_{18} .

**2. Extended response — mixed skills and applications (15 questions)**

16. A geometric sequence satisfies $u_2 = 18$ and $u_5 = 486$. Find u_1 and r .

17. A geometric sequence satisfies $u_3 = 50$ and $u_7 = 4050$. Find all possible real values of r .

18. The three consecutive terms $2x - 1$, $x + 5$ and $5x + 1$ form a geometric sequence. Find all possible values of x .

19. The first term of a geometric sequence is 12 and its common ratio is 1.5.

(a) Find u_9 .

(b) Find S_9 .

(c) Find $u_4 + u_5 + u_6 + u_7 + u_8 + u_9$ without adding the terms individually.

20. A finite geometric sequence has first term 5, common ratio 2 and final term 2560. Find the number of terms and the sum of the sequence.

21.

GDC

Given $u_n = 3.14(2.7)^{n-1}$, find the least value of n such that $u_n > 10^6$.

22.

GDC

Given $u_n = 31420(0.7)^{n-1}$, find the largest value of n such that $u_n > 100$.

23.

GDC

A sequence has $u_1 = 18$ and $r = 1.2$. Find the smallest n for which $S_n > 500$.



24.

GDC

A bacteria culture contains 2400 cells at the start of an observation. The population increases by 14% every hour. Model the population by a geometric sequence and estimate the population after 12 hours.

25.

GDC

A machine is worth \$68 000 when new. Its value decreases by 11% per year. Find its value after 8 complete years.

26.

GDC

A ball is dropped from a height of 3.2 m. Each rebound reaches 72% of the previous maximum height.

- (a) Find the height of the fifth rebound.
 - (b) Find the total vertical distance travelled from release until the ball hits the ground immediately after the fifth rebound.
-

27. The sum of the first n terms of a sequence is $S_n = 7(3^n - 1)$.

- (a) Find u_n in terms of n .
 - (b) Hence state u_1 and r .
-

28. A geometric sequence has $S_4 = 120$ and $r = 2$. Find u_1 and u_6 .

29. A geometric sequence has first term $a > 0$, common ratio $r > 0$, $u_4 = 40$ and $S_4 = 75$. Find a and r .

30.

GDC

The number of subscribers to a channel is modelled by $u_n = 1250(1.09)^{n-1}$, where $n = 1$ corresponds to the first month.

- (a) Find the predicted number of subscribers in month 18.



- (b) Find the first month in which the model predicts more than 5000 subscribers.
- (c) State one reason why the model may become less reliable far into the future.

3. IB exam-style / Paper 1 — original sister questions (20 questions)

31. The first three terms of a geometric sequence are 8, 24 and 72.

- (a) State r . [1 marks]
- (b) Find u_9 . [2 marks]
- (c) Find S_{10} . [2 marks]
-

32. A geometric sequence satisfies $u_4 = 48$ and $u_7 = 384$.

- (a) Find the common ratio. [2 marks]
- (b) Find the first term. [2 marks]
- (c) Find the sum of the first 9 terms. [2 marks]
-

33. The numbers $x - 2$, $2x + 1$ and $5x - 1$ are consecutive terms of a geometric sequence. Find the possible values of x . [5 marks]

34. A geometric sequence has $u_1 = 160$ and $r = \frac{3}{4}$.

- (a) Find u_7 . [2 marks]
- (b) Find S_7 . [2 marks]
- (c) Hence find $u_3 + u_4 + \cdots + u_7$. [2 marks]
-

35. The partial sum of a sequence is $S_n = 2^{n+3} - 8$.

- (a) Find u_1 . [1 marks]
- (b) Show that u_n is geometric. [3 marks]
- (c) State the common ratio. [1 marks]
-

36. A geometric sequence has $u_2 = 6$ and $u_5 = -48$.

- (a) Find r . [2 marks]
- (b) Find u_1 . [2 marks]
- (c) Find S_8 . [2 marks]



37. A finite geometric series has first term 3 and common ratio 4. Its sum is 4095. Find the number of terms. [4 marks]

38. A geometric sequence has positive terms. The third term is 18 and the sixth term is 486.

(a) Find r . [2 marks]

(b) Find S_6 . [3 marks]

39.

GDC

A student saves \$250 in the first month. Each month the amount saved is 6% greater than in the previous month.

(a) Find the amount saved in month 12. [2 marks]

(b) Find the total saved during the first 12 months. [3 marks]

40. A geometric sequence has first term 10 and common ratio r . Given $S_5 = 310$, find r . [4 marks]

41. The terms 3, p and 12 are consecutive terms of a geometric sequence.

(a) Find the possible values of p . [2 marks]

(b) For each value of p , state the corresponding common ratio. [2 marks]

42. A geometric sequence has $u_5 = 162$ and $r = -3$. Find u_1 and S_6 . [5 marks]

43.

GDC

The sales of a product are modelled by a geometric sequence. In year 2 there are 5400 sales and in year 5 there are 9331.2 sales.

(a) Find the common ratio. [2 marks]

(b) Find the predicted number of sales in year 8. [2 marks]

(c) Find the predicted total sales in years 1 to 8. [3 marks]



44.

GDC

A ball is dropped from 2.5 m. Each rebound reaches 80% of the previous maximum height.

- (a) Find the maximum height after the fourth bounce. [2 marks]
 - (b) Find the total vertical distance travelled up to the instant just before the fifth impact with the ground. [4 marks]
-

45. The sum of the first n terms of a sequence is $S_n = 6(5^n - 1)$.

- (a) Find u_n . [3 marks]
 - (b) Find u_6 . [1 marks]
-

46. A geometric sequence has $u_1 = 4$ and $r > 1$. Given $u_n = 26244$, find n and S_n . [6 marks]

47.

GDC

A town has population 85 000. A model predicts annual growth of 2.4%.

- (a) Write a geometric model for the population n years after the initial count. [2 marks]
 - (b) Estimate the population after 15 years. [2 marks]
 - (c) State a limitation of this geometric model. [1 marks]
-

48. A finite geometric sequence has 12 terms. The first term is 5 and the last term is 10240.

- (a) Find r . [3 marks]
 - (b) Find the sum of the 12 terms. [2 marks]
-

49. A geometric sequence has first term a and common ratio r , where $a \neq 0$ and $r \neq 0$. Given $u_3 = 24$ and $u_5 = 216$:

- (a) find all possible real values of r ; [2 marks]
- (b) for each value of r , find a ; [2 marks]
- (c) determine which sequence has $u_6 < 0$. [1 marks]



50.

GDC

The first term of a geometric sequence is 20 and $r = 1.08$.

- (a) Find the first term that exceeds 50. [4 marks]
- (b) Find the least n such that $S_n > 500$. [4 marks]

**AA HL extension**

The questions below are deliberately more algebraically demanding and are intended for AA HL students. They remain focused on the SL 1.3 geometric-sequences and finite-series skills developed in this worksheet.

4. AA HL exclusive challenge (10 additional questions)

51. A geometric sequence v_n satisfies $v_2 = 6$ and $v_6 = 486$. Find all possible real values of v_1 and r .
-
52. Let u_n be an arithmetic sequence with $u_1 = 4$ and $u_6 = 24$. A geometric sequence v_n satisfies $v_2 = u_1$ and $v_4 = u_6$.
- (a) Find the possible values of the common ratio of v_n .
- (b) Given $v_{99} < 0$, find v_5 .
-
53. The three expressions $x + 1$, $2x - 3$ and $5x - 11$ are consecutive non-zero terms of a geometric sequence. Find all real values of x and reject any value that makes a term zero.
-
54. A geometric sequence has $u_1 = a$, ratio r and $S_4 = 30$. Also, $u_2 + u_4 = 18$ and $r > 0$. Find a and r .
-
55. A geometric sequence has positive terms and satisfies $u_2 u_5 = 288$ and $u_3 + u_4 = 36$. Find all possible pairs (u_1, r) .
-
56. For a geometric sequence with $r \neq 1$, $S_8 = 5S_4$. Find all possible real values of r .
-
57. A geometric sequence has first term $a > 0$ and common ratio $r > 1$. The sum of terms 5 to 8 is 15 times the sum of terms 1 to 4. Find r .
-
58. The partial sums of a sequence satisfy $S_n = A(q^n - 1)$, where $A \neq 0$ and $q \neq 0, 1$.
- (a) Show that the corresponding terms u_n form a geometric sequence.
- (b) Find its common ratio.



59. A geometric sequence has $u_1 = 2$ and $r > 1$. The sum of the first $2n$ terms is 21 times the sum of the first n terms. Find r^n and hence find u_{n+1} .

60. A finite geometric series has $2m$ terms, first term $a \neq 0$, and common ratio $r = -2$. The sum of the final m terms is -32 times the sum of the first m terms. Find m .



Answer key and solution outlines

Section 1 — Short response

1. Yes, $r = 3$. 2. $r = \frac{1}{2}$. 3. $u_6 = 1701$. 4. $u_8 = \frac{3}{4}$. 5. $u_n = 5(3)^{n-1}$.
6. $u_1 = 2$. 7. $r = 2$. 8. $x = 30$. 9. $S_7 = 381$. 10. $S_5 = 184.448 \approx 184$.
11. 252. 12. $u_7 = S_7 - S_6 = 320$. 13. 1.08. 14. 0.83. 15. 0.

Section 2 — Extended response

16. $r = 3$, $u_1 = 6$.
17. $r^4 = 81 \Rightarrow r = \pm 3$.
18. $(x + 5)^2 = (2x - 1)(5x + 1) \Rightarrow 9x^2 - 13x - 26 = 0$, so $x = \frac{13 \pm \sqrt{1105}}{18}$.
19. (a) $u_9 = 307.547 \approx 308$; (b) $S_9 = 898.641 \approx 899$; (c) $S_9 - S_3 = 841.641 \approx 842$.
20. $5(2^{n-1}) = 2560 \Rightarrow n = 10$; $S_{10} = 5115$.
21. $n = 14$.
22. Crossing occurs at about 17.1, so the largest integer with $u_n > 100$ is $n = 17$.
23. $18(1.2^n - 1)/(0.2) > 500 \Rightarrow n = 11$.
24. $u_n = 2400(1.14)^n$ if n counts elapsed hours; after 12 h: $2400(1.14)^{12} \approx 11500$ cells.
25. $68000(0.89)^8 \approx 26700$ dollars.
26. (a) $h_5 = 3.2(0.72)^5 \approx 0.619$ m. (b) $3.2 + 2 \sum_{k=1}^5 3.2(0.72)^k \approx 16.5$ m.
27. $u_n = S_n - S_{n-1} = 14(3^{n-1})$; $u_1 = 14$, $r = 3$.
28. $120 = u_1(1 - 2^4)/(1 - 2) = 15u_1 \Rightarrow u_1 = 8$; $u_6 = 256$.
29. $ar^3 = 40$ and $a(1 + r + r^2 + r^3) = 75$. Solving gives $r = 2$, $a = 5$.
30. (a) $1250(1.09)^{17} \approx 5410$; (b) $n = 18$; (c) e.g. constant percentage growth is unlikely to persist indefinitely.

Section 3 — IB exam-style / Paper 1

31. (a) 3; (b) 52488; (c) $S_{10} = 236192$.
32. $r^3 = 8 \Rightarrow r = 2$; $u_1 = 6$; $S_9 = 3066$.
33. $(2x + 1)^2 = (x - 2)(5x - 1) \Rightarrow x^2 - 13x - 1 = 0$, so $x = \frac{13 \pm \sqrt{173}}{2}$.
34. (a) $u_7 = 160(3/4)^6 \approx 28.5$; (b) $S_7 \approx 555$; (c) $S_7 - S_2 \approx 275$.
35. $u_1 = 8$; $u_n = S_n - S_{n-1} = 2^{n+2}$, hence geometric with $r = 2$.
36. $r^3 = -8 \Rightarrow r = -2$; $u_1 = -3$; $S_8 = 255$.
37. $3(4^n - 1)/3 = 4095 \Rightarrow 4^n = 4096 = 4^6$, so $n = 6$.
38. $r^3 = 27 \Rightarrow r = 3$; $u_1 = 2$; $S_6 = 728$.
39. (a) $250(1.06)^{11} \approx 475$ dollars; (b) $250(1.06^{12} - 1)/0.06 \approx 4210$ dollars.
40. $10(1 - r^5)/(1 - r) = 310 \Rightarrow 1 + r + r^2 + r^3 + r^4 = 31$; hence $r = 2$.
41. $p^2 = 36 \Rightarrow p = \pm 6$; ratios 2 and -2 .
42. $u_1 = 162/81 = 2$; $S_6 = 2(1 - (-3)^6)/(1 + 3) = -364$.
43. $r^3 = 9331.2/5400 = 1.728 \Rightarrow r = 1.2$; $u_1 = 4500$; $u_8 \approx 16100$; $S_8 \approx 74200$.
44. (a) $2.5(0.8)^4 = 1.024 \approx 1.02$ m; (b) $2.5 + 2 \sum_{k=1}^4 2.5(0.8)^k \approx 14.3$ m.
45. $u_n = 24(5^{n-1})$; $u_6 = 75000$.
46. $4r^{n-1} = 26244$. Here $26244/4 = 6561 = 3^8$, so $r = 3$ and $n = 9$; $S_9 = 39364$.
47. (a) $P_n = 85000(1.024)^n$; (b) ≈ 121000 ; (c) e.g. growth rate may change.
48. $5r^{11} = 10240 \Rightarrow r^{11} = 2048 = 2^{11}$, so $r = 2$; $S_{12} = 20475$.
49. $r^2 = 9 \Rightarrow r = \pm 3$. If $r = 3$, $a = 8/3$; if $r = -3$, $a = 8/3$. The sequence with $r = -3$ has $u_6 < 0$.
50. (a) $20(1.08)^{n-1} > 50 \Rightarrow n = 13$; (b) $20(1.08^n - 1)/0.08 > 500 \Rightarrow n = 15$.



Section 4 — AA HL extension

51. $r^4 = 81 \Rightarrow r = \pm 3$. If $r = 3$, $v_1 = 2$; if $r = -3$, $v_1 = -2$.
52. Arithmetic: $d = 4$, so $u_1 = 4$, $u_6 = 24$. Thus $v_2 = 4$, $v_4 = 24$, so $r^2 = 6$, $r = \pm\sqrt{6}$. Since $v_{99} < 0$, choose $r < 0$. Then $v_5 = v_4 r = -24\sqrt{6}$.
53. $(2x - 3)^2 = (x + 1)(5x - 11) \Rightarrow x^2 + 6x - 20 = 0$, so $x = -3 \pm \sqrt{29}$. Neither value makes a term zero.
54. $a(1 + r + r^2 + r^3) = 30$ and $ar + ar^3 = 18$. Solving with $r > 0$ gives $r = \frac{3}{2}$ and $a = \frac{48}{13}$.
55. Let $u_1 = a$. Then $a^2 r^5 = 288$ and $ar^2(1 + r) = 36$. The positive solutions are $(a, r) = (3, 2)$ and $(96, \frac{1}{2})$.
56. $S_8 = S_4(1 + r^4)$, so $1 + r^4 = 5 \Rightarrow r^4 = 4$, hence $r = \pm\sqrt{2}$.
57. Terms 5–8 sum to $r^4 S_4$, so $r^4 = 15$, hence $r = \sqrt[4]{15}$.
58. $u_n = S_n - S_{n-1} = Aq^{n-1}(q - 1)$, hence geometric with ratio q .
59. $S_{2n} = S_n(1 + r^n) = 21S_n$, so $r^n = 20$. Hence $u_{n+1} = 2r^n = 40$.
60. Sum of final m terms is r^m times the sum of first m terms. Thus $(-2)^m = -32 = (-2)^5$, so $m = 5$.