Straight Lines

IB Mathematics: Analysis and Approaches SL & HL

Topic 2: Functions | Syllabus SL 2.1 | 60 questions

Learning goal	Use gradients, intercepts and line equations to solve coordinate and modelling problems.
Syllabus link	AA SL 2.1 (shared by SL and HL): $y = mx + c$, $ax + by + d = 0$, $y - y_1 = m(x - x_1)$; parallel and perpendicular lines.
Structure	Questions 1–15: short response; 16–30: extended response; 31–50: original IB exam-style; 51–60: HL-exclusive challenge.
Technology	No calculator required. All exam-style questions are designed for Paper 1 practice.

Instructions. Work on separate paper and show your reasoning. Give exact answers unless stated otherwise; label intercepts as coordinate pairs and include units in context. Use the supplied grids for sketches. SL students complete Questions 1–50; HL students also attempt 51–60. The HL section deepens the shared content and introduces no extra AHL syllabus code.

Section 1: Short response

Core fluency

1.

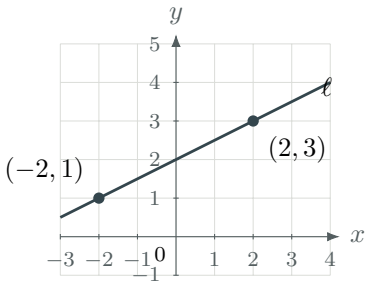
For the line $y = -3x + 7$, write down its gradient and the coordinates of its y-intercept.

2.

Find the gradient of the line through $A(-2, 5)$ and $B(4, -1)$.

3.

The line ℓ passes through the two labelled points. Find its equation in the form $y = mx + c$.



4.

Write $3x + 2y - 12 = 0$ in the form $y = mx + c$. Hence state its gradient.

5.

A line has gradient 4 and passes through $(3, -2)$. Write its equation in point-gradient form and then in the form $y = mx + c$.

6.

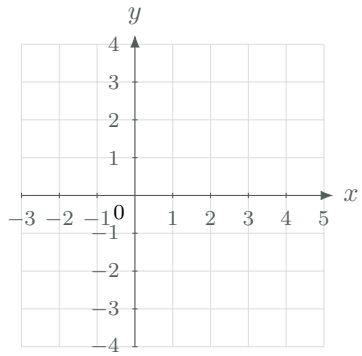
Find the coordinates of both axis intercepts of $2x - 5y + 10 = 0$.

7.
For each pair of points, state the equation and gradient of the line joining them.
(a) $A(-4, 2)$ and $B(3, 2)$.
(b) $C(5, -2)$ and $D(5, 4)$.

8.
Find the equation of the line through $(-2, 3)$ parallel to $y = 2x + 1$.

9.
Find the gradient of a line perpendicular to $y = -\frac{3}{4}x + 2$.

10.
On the supplied grid, sketch both lines $y = x - 2$ and $y = -\frac{1}{2}x + 1$. Label each line and its axis intercepts.



11.
Determine which of $P(1, 4)$ and $Q(3, 1)$ lies on $2x + 3y = 14$. Show your checks.

12.
The gradient of the line joining $A(-1, 2)$ and $B(3, t)$ is $\frac{3}{2}$. Find t.

13.
Classify each pair as *parallel and distinct*, *perpendicular*, or *coincident* (the same line).

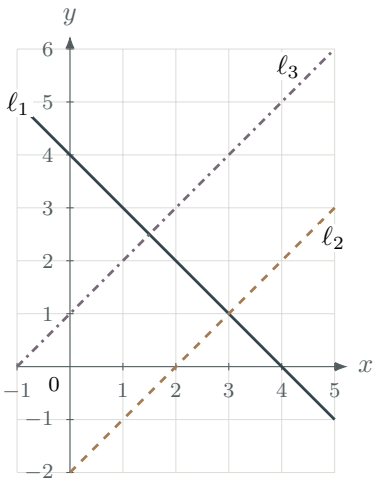
Pair	First line	Second line
(a)	$2x - y + 1 = 0$	$4x - 2y - 3 = 0$
(b)	$x + 2y = 6$	$2x - y = 5$
(c)	$2x + 4y = 8$	$x + 2y = 4$

14.
A taxi fare is modelled by $C = 4 + 1.8d$, where C is the cost in euros and d is the distance travelled in kilometres. Interpret both 4 and 1.8.

15.
A student says, "A line perpendicular to $y = -2x + 5$ has gradient 2." Explain the error and give the correct perpendicular line through $(0, -1)$.

Section 2: Extended response — Mixed skills and applications

16.



The three lines are shown on an equal-scale grid.

(a) Find an equation for each line.

(b) Identify the parallel pair.

(c) Identify every perpendicular pair.

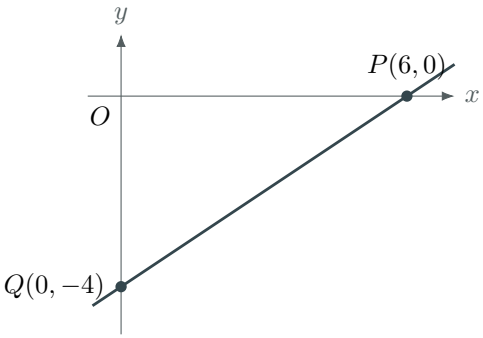
17.
The points $A(-3, 5)$ and $B(5, 1)$ lie on a line ℓ.

(a) Find the equation of ℓ in point-gradient form.

(b) Write the equation as $ax + by + d = 0$, using integer coefficients.

(c) Find the coordinates of both axis intercepts.

18.



Line ℓ meets the axes at $P(6, 0)$ and $Q(0, -4)$.

(a) Find an equation of ℓ.

(b) Find the equation of the line through P perpendicular to ℓ.

(c) Find the y-intercept of this perpendicular line.

19.
The line through $A(2, k)$ and $B(8, 11)$ is parallel to $3x - 2y + 7 = 0$.

(a) Find k.

(b) Find the equation of AB in the form $y = mx + c$.

20.
A tank drains at a constant rate. The table shows its volume V, in litres, at time t, in minutes.

t (min)	0	4	10
V (L)	180	140	80

(a) Find a linear model for V in terms of t.

(b) Interpret the gradient and find when 50 litres remain.

(c) State the time interval for which the draining model is physically meaningful.

21.
Two bicycle-hire firms use the following pricing models, where $h \geq 0$ is the hire duration in hours and C is the cost in euros. Hourly charges are calculated using the exact hire duration.

$$C_A = 12 + 3h, \qquad C_B = 30 + 1.5h.$$

(a) Find the duration at which the costs are equal, and the common cost.

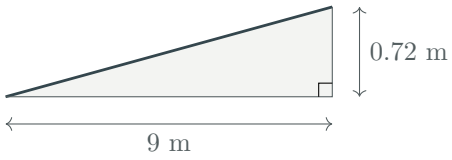
(b) Which firm is cheaper for an 8-hour hire? Find the saving.

(c) State when firm B is cheaper.

22.
A ramp rises 0.72 m over a *horizontal* distance of 9 m. For this design, the maximum permitted gradient is $\frac{1}{12}$.

(a) Find the ramp's gradient. Does it meet the design limit?

(b) Another ramp must rise 0.90 m. Find its minimum horizontal run under the same limit.



Side profile; not to scale.

23.
The lines p and q have equations $(2a-1)x + 3y = 12$ and $y = -x + 5$.

(a) Find a when the lines are parallel and distinct.

(b) Find a when the lines are perpendicular.

24.
Point P has coordinates $(4, -3)$.

(a) Write the horizontal and vertical lines through P.

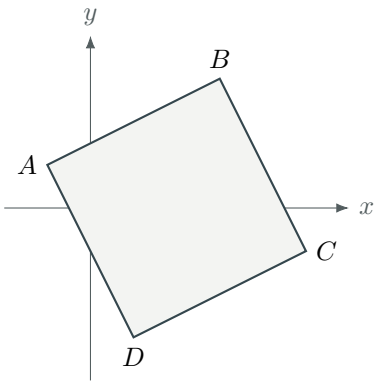
(b) Find where each of these lines meets $2x + y = 6$.

25.

A quadrilateral has vertices $A(-1,1)$, $B(3,3)$, $C(5,-1)$ and $D(1,-3)$.

(a) Use gradients to show that it is a rectangle.

(b) Find the equation of diagonal AC in the form $ax + by + d = 0$.



26.

A path lies on $y = 2x - 1$. A connecting path from $P(0,4)$ meets it at a right angle.

(a) Find the equation of the connecting path.

(b) Find the coordinates of the meeting point.

27.

Consider $A(-2,7)$, $B(1,1)$ and $C(4,-5)$.

(a) Show that the three points are collinear.

(b) Find the equation of their line and its x-intercept.

28.

A family of lines is given by $y = kx + 2k - 3$, where $k \in \mathbb{R}$.

(a) Show that every line in the family passes through $(-2,-3)$.

(b) Find the member of the family through $(4,9)$.

(c) Find k for the member parallel to $3x - y = 8$.

29.

Celsius and Fahrenheit temperatures are related linearly. A temperature of 0°C is 32°F, and 100°C is 212°F.

(a) Find F in terms of C.

(b) Convert 77°F to degrees Celsius.

(c) Find the temperature at which the two numerical readings are equal.

30.

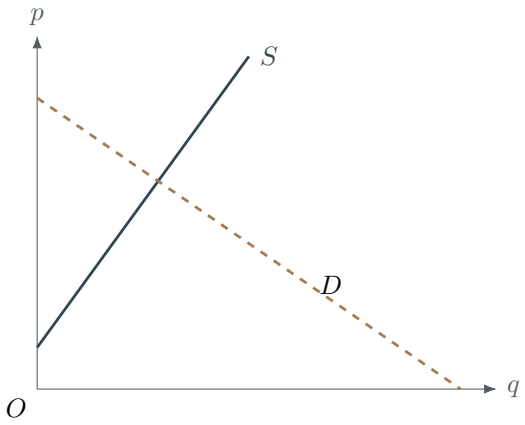
The supply and demand prices for a product are modelled by

$$p_S = 2q + 10, \qquad p_D = -q + 70,$$

where q is the number of batches and p is the price in euros per batch.

(a) Find the equilibrium quantity and price, where $p_S = p_D$.

(b) At a price of 40 euros, find the quantity supplied and the quantity demanded. State the shortfall.



Supply and demand; scales omitted.

Section 3: IB exam-style / Paper 1 — Original questions

These original questions use IB-style command terms and mark allocations. They are not an official IB examination.

31. [Maximum mark: 7]

The line ℓ passes through $A(-2, 7)$ and $B(4, -5)$.

(a) Find the gradient of ℓ. [2]

(b) Find an equation of ℓ in the form $y = mx + c$. [2]

(c) The line meets the x-axis at P and the y-axis at Q. Find the area of triangle OPQ, where O is the origin. [3]

32. [Maximum mark: 7]

The points $A(1, 1)$ and $B(7, 5)$ lie on a line ℓ. The line n passes through $C(6, 0)$ and is perpendicular to ℓ.

(a) Find the gradient of ℓ. [2]

(b) Find an equation of n. [2]

(c) Find the coordinates of the intersection of ℓ and n. [3]

33. [Maximum mark: 6]

The lines ℓ and n have equations $2x + 3y = 12$ and $kx - 2y = 5$, where $k \in \mathbb{R}$.

(a) Find both axis intercepts of ℓ. [2]

(b) Find k when the lines are parallel. [2]

(c) Find k when the lines are perpendicular. [2]

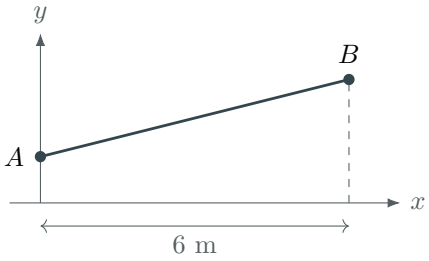
34. [Maximum mark: 6]

A straight ramp starts at $A(0, 0.30)$ and reaches $B(6, 0.80)$. Coordinates are in metres; x measures horizontal distance and y measures height above the ground.

(a) Find the gradient of the ramp. [2]

(b) Find the ramp's equation. [2]

(c) The ramp is extended in a straight line to $x = 9$. Find its height there. [2]



Side profile; not to scale.

35. [Maximum mark: 7]

The lines $s : 3x - y = 4$ and $t : x + 2y = 6$ intersect at P.

(a) Find P. [3]

(b) Find the equation of the line r through P parallel to $4x + 3y = 12$. [2]

(c) Find the equation of the line through the origin perpendicular to r. [2]

36. [Maximum mark: 7]

Two taxi companies quote costs in euros for a journey of $d \geq 0$ kilometres:

Company	Fixed charge	Charge per kilometre
A	5	1.50
B	2	2.00

(a) Calculate each company's cost for an 8 km journey. [2]

(b) Find the distance at which the costs are equal and the common cost. [3]

(c) State, with a reason, the distances for which company A is cheaper. [2]

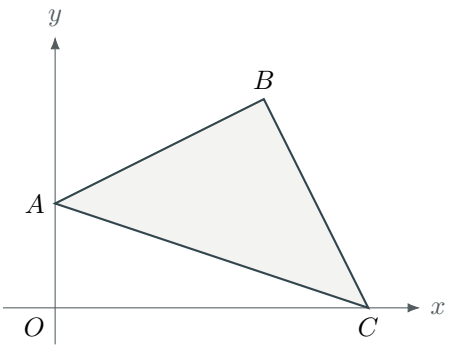
37. [Maximum mark: 7]

Triangle ABC has vertices $A(0, 2)$, $B(4, 4)$ and $C(6, 0)$.

(a) Show that AB is perpendicular to BC. [3]

(b) Find an equation of AC. [2]

(c) Find the equation of the line through B perpendicular to AC. [2]



38. [Maximum mark: 6]

A line with gradient $-\frac{3}{2}$ meets the positive axes at $A(a, 0)$ and $B(0, 6)$, where $a > 0$.

(a) Find a. [2]

(b) Write the line's equation in general form, using integer coefficients. [2]

(c) The point (t, t) lies on the line. Find t. [2]

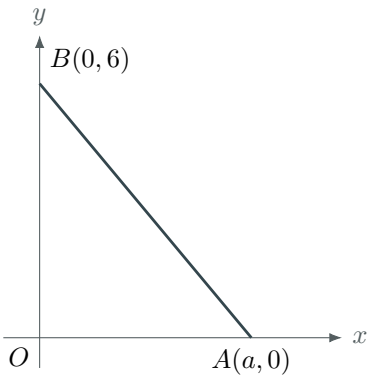


Diagram not to scale.

39. [Maximum mark: 6]

Points $A(2, 3)$, $B(6, k)$ and $C(-2, 7)$ are given. The line AB is parallel to $x - 2y + 1 = 0$.

(a) Find k. [2]

(b) Find the equation of AB. [2]

(c) Determine whether AC is perpendicular to AB. Justify your answer. [2]

40. [Maximum mark: 7]

A family of lines is given by $(k - 1)x + 2y = 4k + 2$, where $k \in \mathbb{R}$.

(a) Find k for which the line is horizontal, and write its equation. [2]

(b) Show that every member of the family passes through the same point, and find that point. [3]

(c) Find k for which the line is perpendicular to $y = 2x + 1$. [2]

41. [Maximum mark: 9]

A cooling liquid has temperature $T = 90 - 4t$, where T is in degrees Celsius and t is in minutes, for $0 \le t \le 15$. A second liquid warms linearly: it is at 10°C when $t = 0$ and 70°C when $t = 10$.

(a) Interpret the values 90 and -4 in the cooling model. [2]

(b) Find when the cooling liquid reaches 54°C. [2]

(c) Find a linear model for the second liquid's temperature S. [2]

(d) Find when the temperatures are equal, and their common temperature. [3]

42. [Maximum mark: 6]

For real k, the line ℓ_k has equation $kx + (k - 2)y = 6$.

(a) Find k when ℓ_k is horizontal and write its equation. [2]

(b) Find k when ℓ_k is vertical and write its equation. [2]

(c) Find the point where the lines in parts (a) and (b) intersect. [1]

(d) Write the equation of the line through $(-1, 4)$ perpendicular to the line in part (a). [1]

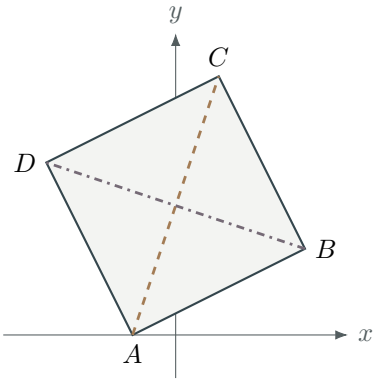
43. [Maximum mark: 8]

The vertices of square $ABCD$ are $A(-1,0)$, $B(3,2)$, $C(1,6)$ and $D(-3,4)$.

(a) Use gradients to verify that $AB \perp BC$. [3]

(b) Find an equation of AD. [2]

(c) Find the intersection of diagonals AC and BD by solving their equations. [3]



44. [Maximum mark: 7]

An electric bicycle's remaining battery charge B, as a percentage of full capacity, decreases linearly during a journery. After 1 hour the charge is 85%; after 4 hours it is 40%.

(a) Find the gradient of the charge-time graph and interpret it. [2]

(b) Find B in terms of time t hours, measured from the start. [2]

(c) Assuming the same rate continues, find when the battery is empty. Give an exact value in hours and also in hours and minutes. [2]

(d) State a physically meaningful time interval for the model. [1]

45. [Maximum mark: 8]

The lines $\ell : y = px + 3$ and $n : y = (p+2)x - 1$ intersect at P, where $p \in \mathbb{R}$.

(a) Show that the x-coordinate of P is independent of p, and find it. [3]

(b) Given that P also lies on $y = 3x + 1$, find p. [2]

(c) In a different case, find p for which ℓ and n are perpendicular. [3]

46. [Maximum mark: 7]

The line $\ell : ax + by = 12$ passes through $(2,2)$ and $(6,0)$.

(a) Find a and b. [3]

(b) Write down the gradient of ℓ. [1]

(c) Find the equation of the perpendicular to ℓ through $(1,-1)$. [2]

(d) Find the x-intercept of the perpendicular line. [1]

47. [Maximum mark: 8]

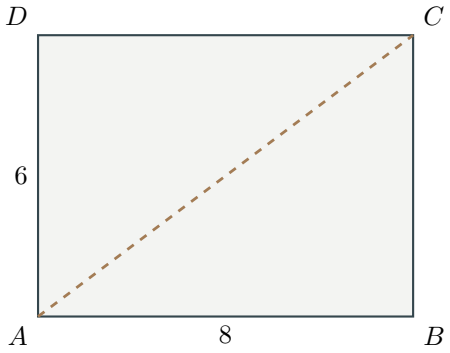
A rectangular field has vertices $A(0,0)$, $B(8,0)$, $C(8,6)$ and $D(0,6)$. A straight path from B is perpendicular to diagonal AC.

(a) Find the equation of AC. [2]

(b) Find the equation of the path. [2]

(c) Find the point where the path meets side CD. [3]

(d) Explain why the path does not meet side AD within the field. [1]



48. [Maximum mark: 8]

A student claims that $\ell : 2x - 5y = 7$ and $n : 5x - 2y = 3$ are perpendicular because "the coefficients have been swapped".

(a) Find both gradients and show that the claim is false. [3]

(b) Change only the coefficient of y in the equation of n to make it perpendicular to ℓ. Write the corrected equation. [2]

(c) Find the intersection of ℓ and the corrected line. [3]

49. [Maximum mark: 8]

The line ℓ passes through $A(-4, 5)$ and $B(2, 2)$. Line n passes through the origin and is perpendicular to ℓ.

(a) Find an equation of ℓ. [2]

(b) Find an equation of n. [2]

(c) Find their intersection P. [3]

(d) A line r passes through P and has the same gradient as ℓ. Explain why r must be the same line as ℓ. [1]

50. [Maximum mark: 8]

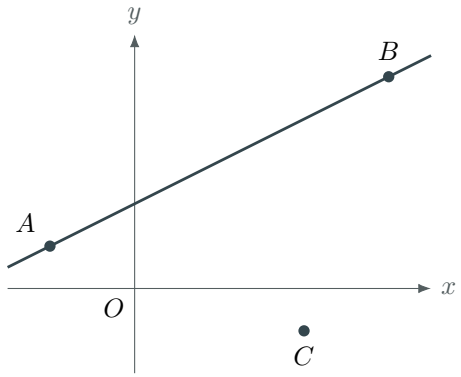
A straight bridge lies on the line through $A(-2, 1)$ and $B(6, 5)$. A footpath from $C(4, -1)$ joins the bridge at a right angle at D.

(a) Find an equation for the bridge. [3]

(b) Find an equation for the footpath. [2]

(c) Find D. [2]

(d) A road lies on $y = \frac{1}{2}x - 1$. Explain why it is parallel to, and distinct from, the bridge. [1]



Section 4: AA HL exclusive challenge

HL extension

HL students: Questions 51–60 extend the reasoning and algebra of SL 2.1. No additional AHL topic is assumed. SL students may stop after Question 50.

51.

The lines ℓ_k and r have equations $(k-1)x + (k+1)y = 6$ and $2x - y = 1$, respectively, where $k \in \mathbb{R}$. Find the value of k for which ℓ_k is

(a) parallel to r,

(b) perpendicular to r,

(c) horizontal, and

(d) vertical. In part (a), justify that the lines are distinct.

52.

For $k \in \mathbb{R}$, consider $\ell_k : (k+2)x + (2k-1)y = 3k+4$.

(a) Show that every member of this family passes through the same point P, and find P.

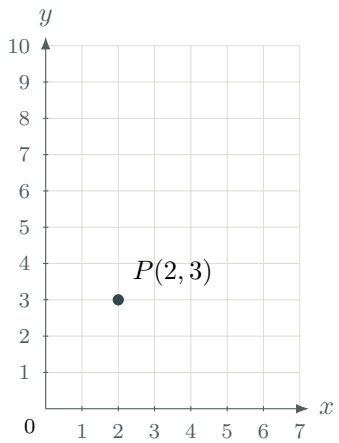
(b) The line r through P is parallel to $x+2y=0$. Find the equation of r and show that r is not a member of the family for any finite real value of k.

53.

The points $A(t,0)$, $B(1,t)$ and $C(-1,6)$ depend on a real parameter t. Find all values of t for which the three points are collinear. For each value, find the equation of their common line.

54.

A straight line passes through $P(2,3)$ and meets the positive coordinate axes at $A(a,0)$ and $B(0,b)$, where $a,b>0$. The area of triangle OAB is $\frac{27}{2}$ square units. Find the equations of both possible lines and sketch them on the supplied axes.



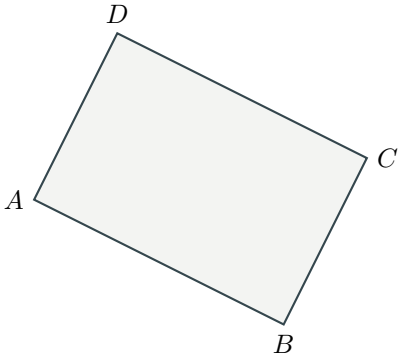
55.

The sides of quadrilateral $ABCD$ lie on the lines AB : $x+2y=0$, BC : $2x-y=15$, CD : $x+2y=10$ and DA : $2x-y=0$.

(a) Show that $ABCD$ is a rectangle.

(b) Find the coordinates of its four vertices.

(c) Find the equations of both diagonals and the coordinates of their point of intersection.



Sides lie on the four given lines.

56.

(a) The non-zero numbers a,b,c,d define the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{c}+\frac{y}{d}=1$. Show that the lines are perpendicular if and only if $ac+bd=0$.

(b) Hence find all real values of k for which $\frac{x}{k}+\frac{y}{k+1}=1$ and $\frac{x}{k-1}+\frac{y}{2k}=1$ are perpendicular. State all values excluded by the given equations.

57.

A tank drains at a constant rate. Its volume $V(t)$ litres, t minutes after timing begins, satisfies $V(2) = 740$. Safety checks require $500 \leq V(5) \leq 560$ and $V(10) \geq 200$.

(a) Determine all possible models $V(t) = mt + c$, stating the allowable range of m and its relation to c.

(b) Assume the same rate continues until the tank empties. Find the exact range of possible emptying times T, measured from when timing began.

58.

For real k, let $\ell_1 : (k+1)x + 2y = 4$ and $\ell_2 : 2x + (k+1)y = 4$.

(a) Classify all values of k for which the lines are coincident, parallel and distinct, perpendicular, or intersecting but not perpendicular.

(b) Whenever there is a unique intersection, find its coordinates in terms of k. Your reasoning must include cases involving a vertical line.

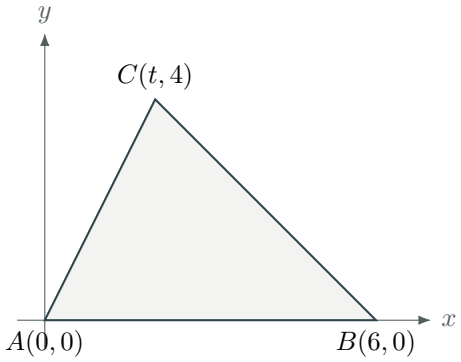
59.

Triangle ABC has vertices $A(0,0)$, $B(6,0)$ and $C(t,4)$, where $t \in \mathbb{R}$. An altitude is a line through a vertex perpendicular to the opposite side.

(a) Find the equations of the altitudes from A and B, including the cases $t = 0$ and $t = 6$.

(b) Show that all three altitudes pass through one point H, and find H in terms of t.

(c) Determine the values of t for which H lies strictly inside the triangle.



One possible configuration.

60.

The points $A(-2,0)$ and $B(6,0)$ are fixed. Point P lies on the line $y = 3$, and AP is perpendicular to BP.

(a) Find the exact coordinates of both possible positions of P.

(b) For the position with the larger x-coordinate, find the equations of AP and BP in point-gradient form, with rationalized gradients.

Answer key and solution outlines — Questions 1–60

Equivalent line equations are accepted. For perpendicular lines, the product $m_1 m_2 = -1$ applies when both gradients exist; a horizontal and a vertical line are also perpendicular.

Section 1: Short response | Questions 1–15

1. Gradient -3; y-intercept $(0, 7)$.

2. $m = \dfrac{-1-5}{4-(-2)} = -1$.

3. $m = \dfrac{3-1}{2-(-2)} = \frac{1}{2}$ and $3 = \frac{1}{2}(2) + c$, so $y = \frac{1}{2}x + 2$.

4. $2y = -3x + 12$, so $y = -\frac{3}{2}x + 6$ and $m = -\frac{3}{2}$.

5. $y + 2 = 4(x - 3)$; hence $y = 4x - 14$.

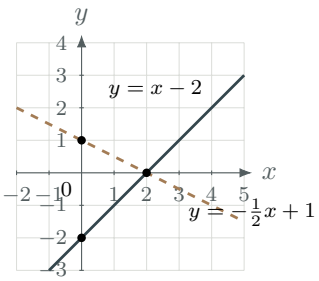
6. Set $y = 0$: $x = -5$. Set $x = 0$: $y = 2$. Intercepts $(-5, 0)$ and $(0, 2)$.

7. (a) $y = 2$; gradient 0.
(b) $x = 5$; gradient undefined, since the horizontal change is zero.

8. The required gradient is 2. $y - 3 = 2(x + 2)$, so $y = 2x + 7$.

9. $(-\frac{3}{4})m = -1$, so $m = \frac{4}{3}$.

10. $y = x - 2$ passes through $(0, -2)$ and $(2, 0)$. $y = -\frac{1}{2}x + 1$ passes through $(0, 1)$ and $(2, 0)$. Both are straight lines; see the answer sketch below.



11. For P, $2(1) + 3(4) = 14$, so P lies on the line. For Q, $2(3) + 3(1) = 9 \neq 14$, so Q does not.

12. $\dfrac{t-2}{3-(-1)} = \frac{3}{2}$, so $t - 2 = 6$ and $t = 8$.

13. (a) Parallel and distinct: both gradients are 2, with intercepts 1 and $-\frac{3}{2}$.
(b) Perpendicular: $(-\frac{1}{2})(2) = -1$.
(c) Coincident: dividing the first equation by 2 gives the second.

14. The fixed charge is 4 euros. The fare increases by 1.80 euros for each kilometre travelled; its gradient is 1.8 euros per kilometre.

15. Changing only the sign is insufficient: the product must be -1. The gradient is $\frac{1}{2}$, and the line is $y = \frac{1}{2}x - 1$.

Section 2: Extended response | Questions 16–30

16. (a) $\ell_1 : y = -x + 4$; $\ell_2 : y = x - 2$; $\ell_3 : y = x + 1$.
(b) $\ell_2 \parallel \ell_3$, since both have gradient 1.
(c) $\ell_1 \perp \ell_2$ and $\ell_1 \perp \ell_3$, since $(-1)(1) = -1$.

17. (a) $m = -\frac{1}{2}$; $y - 5 = -\frac{1}{2}(x + 3)$.
(b) $x + 2y - 7 = 0$.
(c) $(7, 0)$ and $(0, \frac{7}{2})$.

18. (a) $m = \frac{4}{6} = \frac{2}{3}$, so $y = \frac{2}{3}x - 4$.
(b) Perpendicular gradient $-\frac{3}{2}$; $y = -\frac{3}{2}(x - 6) = -\frac{3}{2}x + 9$.
(c) $(0, 9)$.

19. (a) $\dfrac{11 - k}{6} = \frac{3}{2}$, so $k = 2$.
(b) $y - 2 = \frac{3}{2}(x - 2)$, hence $y = \frac{3}{2}x - 1$.

20. (a) $m = (140 - 180)/4 = -10$; $V = 180 - 10t$.
(b) The volume decreases by 10 L/min. $50 = 180 - 10t$ gives $t = 13$ min.
(c) $0 \le t \le 18$ min, when the volume falls from 180 L to 0 L.

21. (a) $12 + 3h = 30 + 1.5h$ gives $h = 12$ h and $C = 48$ euros.
(b) At $h = 8$, $C_A = 36$ and $C_B = 42$ euros; A saves 6 euros.
(c) B is cheaper when $h > 12$.

22. (a) $m = 0.72/9 = \frac{2}{25}$. Since $\frac{2}{25} < \frac{1}{12}$ (because $24 < 25$), the ramp meets the limit.
(b) $0.90/r \le \frac{1}{12}$ gives $r \ge 10.8$ m; minimum horizontal run 10.8 m.

23. (a) $m_p = -(2a - 1)/3 = -1$ gives $a = 2$. Then $p : y = -x + 4$, distinct from q.
(b) $m_p = 1$ gives $-(2a - 1) = 3$, so $a = -1$.

24. (a) Horizontal: $y = -3$. Vertical: $x = 4$.
(b) At $y = -3$, $2x = 9$, so $(\frac{9}{2}, -3)$. At $x = 4$, $y = -2$, so $(4, -2)$.

25. (a) $m_{AB} = m_{CD} = \frac{1}{2}$ and $m_{BC} = m_{DA} = -2$. Opposite sides are parallel and adjacent slopes multiply to -1, so the quadrilateral is a rectangle.
(b) $m_{AC} = (-1 - 1)/(5 + 1) = -\frac{1}{3}$. Thus $y - 1 = -\frac{1}{3}(x + 1)$, or $x + 3y - 2 = 0$.

26. (a) Perpendicular gradient $-\frac{1}{2}$; $y = -\frac{1}{2}x + 4$.
(b) $2x - 1 = -\frac{1}{2}x + 4$, so $x = 2$ and $y = 3$. Meeting point $(2, 3)$.

27. (a) $m_{AB} = (1 - 7)/3 = -2$ and $m_{BC} = (-5 - 1)/3 = -2$. The segments share B, so all three points are collinear.
(b) $y = -2x + 3$, or $2x + y - 3 = 0$; x-intercept $(\frac{3}{2}, 0)$.

28. (a) Substituting $x = -2$ gives $y = -2k + 2k - 3 = -3$ for every k.
(b) $9 = 4k + 2k - 3$, so $k = 2$ and $y = 2x + 1$.
(c) The given line has gradient 3, so $k = 3$.

29. (a) $m = (212 - 32)/100 = \frac{9}{5}$, so $F = \frac{9}{5}C + 32$.
(b) $77 - 32 = \frac{9}{5}C$, so $C = 25°$C.
(c) $C = \frac{9}{5}C + 32$ gives $C = F = -40$.

30. (a) $2q + 10 = -q + 70$ gives $q = 20$ batches and $p = 50$ euros per batch.
(b) Supply: $40 = 2q + 10$, so $q_S = 15$. Demand: $40 = -q + 70$, so $q_D = 30$. Shortfall $30 - 15 = 15$ batches.

Section 3: IB exam-style / Paper 1 | Questions 31–50

31. (a) $\dfrac{-5 - 7}{4 - (-2)} = -2$.
(b) $y - 7 = -2(x + 2)$, so $y = -2x + 3$.
(c) $P = (\frac{3}{2}, 0)$ and $Q = (0, 3)$. Area $= \frac{1}{2}(\frac{3}{2})(3) = \frac{9}{4}$ square units.

32. (a) $m_\ell = (5 - 1)/(7 - 1) = \frac{2}{3}$.
(b) $m_n = -\frac{3}{2}$, so $n : y = -\frac{3}{2}(x - 6) = -\frac{3}{2}x + 9$.
(c) $\ell : y = \frac{2}{3}x + \frac{1}{3}$. Equating gives $\frac{13}{6}x = \frac{26}{3}$, so the intersection is $(4, 3)$.

33. (a) $(6,0)$ and $(0,4)$.
(b) $m_\ell = -\frac{2}{3}$ and $m_n = \frac{k}{2}$. Equality gives $k = -\frac{4}{3}$.
(c) $(-\frac{2}{3})(\frac{k}{2}) = -1$, so $k = 3$.

34. (a) $m = (0.80 - 0.30)/6 = \frac{1}{12}$.
(b) $y = \frac{1}{12}x + \frac{3}{10}$.
(c) At $x = 9$, $y = \frac{3}{4} + \frac{3}{10} = \frac{21}{20} = 1.05$ m.

35. (a) $y = 3x - 4$ gives $x + 2(3x - 4) = 6$, so $P = (2,2)$.
(b) $m_r = -\frac{4}{3}$; $y - 2 = -\frac{4}{3}(x - 2)$, or $4x + 3y = 14$.
(c) Perpendicular gradient $\frac{3}{4}$; $y = \frac{3}{4}x$.

36. (a) A: $5 + 1.5(8) = 17$ euros; B: $2 + 2(8) = 18$ euros.
(b) $5 + 1.5d = 2 + 2d$ gives $d = 6$ km and cost 14 euros.
(c) $5 + 1.5d < 2 + 2d$ gives $d > 6$ km. A has the lower rate after the costs cross.

37. (a) $m_{AB} = \frac{2}{4} = \frac{1}{2}$, $m_{BC} = \frac{-4}{2} = -2$. Their product is -1, so $AB \perp BC$.
(b) $m_{AC} = -\frac{1}{3}$ and $A = (0,2)$, so $y = -\frac{1}{3}x + 2$.
(c) Gradient 3; $y - 4 = 3(x - 4)$, hence $y = 3x - 8$.

38. (a) $-6/a = -\frac{3}{2}$, so $a = 4$.
(b) $y = -\frac{3}{2}x + 6$, or $3x + 2y - 12 = 0$.
(c) $3t + 2t = 12$, so $t = \frac{12}{5}$.

39. (a) $(k - 3)/4 = \frac{1}{2}$, so $k = 5$.
(b) $y - 3 = \frac{1}{2}(x - 2)$, hence $y = \frac{1}{2}x + 2$.
(c) $m_{AC} = (7 - 3)/(-2 - 2) = -1$. The product is $-\frac{1}{2} \neq -1$, so the lines are not perpendicular.

40. (a) $m = -(k - 1)/2 = 0$ gives $k = 1$ and $y = 3$.
(b) Write $k(x - 4) + (-x + 2y - 2) = 0$. Both brackets vanish at $x = 4$, $y = 3$, independently of k; the fixed point is $(4,3)$.
(c) The required gradient is $-\frac{1}{2}$; $-(k - 1)/2 = -\frac{1}{2}$ gives $k = 2$.

41. (a) Initial temperature 90°C; temperature decreases by 4°C per minute.
(b) $54 = 90 - 4t$ gives $t = 9$ min.
(c) Gradient $(70 - 10)/10 = 6$, so $S = 6t + 10$.
(d) $90 - 4t = 6t + 10$ gives $t = 8$ min and $T = S = 58$°C.

42. (a) $k = 0$, so $-2y = 6$ and $y = -3$.
(b) $k = 2$, so $2x = 6$ and $x = 3$.
(c) $(3,-3)$.
(d) The perpendicular is vertical: $x = -1$.

43. (a) $m_{AB} = \frac{1}{2}$ and $m_{BC} = -2$; product -1.
(b) $m_{AD} = 4/(-2) = -2$, so $y = -2(x + 1) = -2x - 2$.
(c) $AC : y = 3x + 3$ and $BD : y = -\frac{1}{3}x + 3$. Equating gives $x = 0$, $y = 3$, so the diagonals meet at $(0,3)$.

44. (a) $m = (40 - 85)/(4 - 1) = -15$: the charge decreases by 15 percentage points per hour.
(b) $85 = -15(1) + c$ gives $c = 100$; $B = 100 - 15t$.
(c) $B = 0$ gives $t = \frac{20}{3}$ h, or 6 h 40 min.
(d) $0 \le t \le \frac{20}{3}$ h.

45. (a) Equating gives $px + 3 = (p + 2)x - 1$, hence $2x = 4$ and $x = 2$ for every p. Thus $P = (2, 2p + 3)$.
(b) On $y = 3x + 1$, $y_P = 7$, so $2p + 3 = 7$ and $p = 2$.
(c) $p(p + 2) = -1$, so $(p + 1)^2 = 0$ and $p = -1$.

46. (a) From $(6,0)$, $6a = 12$, so $a = 2$. Then $2(2) + 2b = 12$, giving $b = 4$.
(b) $2x + 4y = 12$ gives gradient $-\frac{1}{2}$.
(c) Gradient 2; $y + 1 = 2(x - 1)$, so $y = 2x - 3$.
(d) $(\frac{3}{2}, 0)$.

47. (a) $m_{AC} = 6/8 = \frac{3}{4}$, so $y = \frac{3}{4}x$.
(b) Gradient $-\frac{4}{3}$; $y = -\frac{4}{3}(x - 8)$.
(c) On CD, $y = 6$; $18 = -4x + 32$ gives $x = \frac{7}{2}$. The point is $(\frac{7}{2}, 6)$.
(d) At $x = 0$, $y = \frac{32}{3} > 6$, beyond side AD.

48. (a) $m_\ell = \frac{2}{5}$ and $m_n = \frac{5}{2}$; their product is 1, not -1.
(b) The corrected line is $5x + 2y = 3$, with gradient $-\frac{5}{2}$.
(c) Solve $2x - 5y = 7$ and $5x + 2y = 3$. Multiplying by 2 and 5 respectively gives $29x = 29$, so $x = 1$ and $y = -1$. Intersection $(1, -1)$.

49. (a) $m_\ell = (2-5)/(2+4) = -\frac{1}{2}$; $y = -\frac{1}{2}x + 3$.
(b) $y = 2x$.
(c) $2x = -\frac{1}{2}x + 3$ gives $x = \frac{6}{5}$, $y = \frac{12}{5}$.
(d) A point and a finite gradient determine a unique line. Since P lies on ℓ, both equations are $y - \frac{12}{5} = -\frac{1}{2}(x - \frac{6}{5})$.

50. (a) $m = (5-1)/(6+2) = \frac{1}{2}$. With A, $1 = -1 + c$ gives $c = 2$, so $y = \frac{1}{2}x + 2$.
(b) Gradient -2; $y + 1 = -2(x - 4)$, so $y = -2x + 7$.
(c) $\frac{1}{2}x + 2 = -2x + 7$ gives $D = (2, 3)$.
(d) Both gradients are $\frac{1}{2}$, but their y-intercepts differ (-1 and 2).

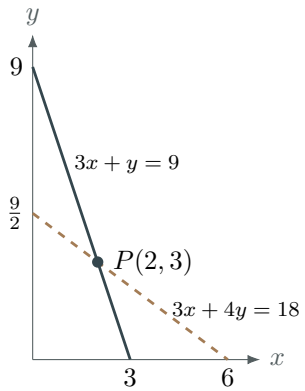
Section 4: AA HL exclusive challenge | Questions 51–60

51. For $k \neq -1$, $m_\ell = -(k-1)/(k+1)$ and $m_r = 2$. (a) $-(k-1) = 2(k+1)$ gives $k = -\frac{1}{3}$. Then ℓ_k is $2x - y = -9$, distinct from $2x - y = 1$.
(b) $-(k-1)/(k+1) = -\frac{1}{2}$ gives $k = 3$.
(c) $k = 1$, giving $y = 3$.
(d) $k = -1$, giving $x = -3$.

52. (a) Rearranging gives $k(x + 2y - 3) + (2x - y - 4) = 0$. Solve $x + 2y = 3$ and $2x - y = 4$ to obtain $P = (\frac{11}{5}, \frac{2}{5})$; both brackets vanish there for every k.
(b) $r : x + 2y = 3$. For $k \neq \frac{1}{2}$, equality of gradients would require $-(k+2)/(2k-1) = -\frac{1}{2}$, hence $2k + 4 = 2k - 1$, which is impossible. At $k = \frac{1}{2}$, ℓ_k is vertical and also cannot equal r.

53. When $t = 1$, AB is vertical but C is not on it, so this case is excluded. Otherwise AB has equation $(1-t)y = t(x-t)$. Substituting C gives $6(1-t) = t(-1-t)$, so $t^2 - 5t + 6 = 0$. Thus $t = 2$ or $t = 3$. For $t = 2$, $y = -2x + 4$; for $t = 3$, $y = -\frac{3}{2}x + \frac{9}{2}$.

54. The area gives $ab = 27$. From $x/a + y/b = 1$ and $P(2,3)$, $2b + 3a = 27$. Using $b = 27/a$ gives $a^2 - 9a + 18 = 0$, so $(a-3)(a-6) = 0$. Thus $(a, b) = (3, 9)$ or $(6, \frac{9}{2})$. The lines are $3x + y = 9$ and $3x + 4y = 18$. Both pass through P and have positive intercepts.



55. (a) AB and CD have gradient $-\frac{1}{2}$; BC and DA have gradient 2. Opposite sides are parallel and adjacent gradient products are -1, so the quadrilateral is a rectangle.
(b) Solving adjacent pairs gives $A = (0, 0)$, $B = (6, -3)$, $C = (8, 1)$, $D = (2, 4)$.
(c) $AC : y = \frac{1}{8}x$ and $BD : 7x + 4y = 30$. Their intersection is $(4, \frac{1}{2})$.

56. (a) The gradients are $-b/a$ and $-d/c$, so perpendicularity is equivalent to $bd/(ac) = -1$, or $ac + bd = 0$. All denominators are non-zero by assumption.
(b) The equations exclude $k = -1, 0, 1$. The criterion gives $k(k-1) + 2k(k+1) = k(3k+1) = 0$. The root $k = 0$ is excluded; the only answer is $k = -\frac{1}{3}$.

57. (a) $V(t) = 740 + m(t-2)$, so $c = 740 - 2m$. The check at $t = 5$ gives $-80 \le m \le -60$; the check at $t = 10$ gives $m \ge -\frac{135}{2}$. Therefore $-\frac{135}{2} \le m \le -60$, with $c = 740 - 2m$ (hence $860 \le c \le 875$, not independently).

(b) Put $r = -m$, so $60 \le r \le \frac{135}{2}$. Since $T = 2 + 740/r$, $\frac{350}{27} \le T \le \frac{43}{3}$ minutes.

58. For $k \neq -1$, the gradients are $-(k+1)/2$ and $-2/(k+1)$. Equality requires $(k+1)^2 = 4$: $k = 1$ gives the same line $x + y = 2$, whereas $k = -3$ gives the distinct parallel lines $y = x + 2$ and $y = x - 2$. At $k = -1$, the equations are $y = 2$ and $x = 2$, so the lines are perpendicular. For all $k \notin \{-3, -1, 1\}$ the lines intersect but are not perpendicular (their gradient product is 1). For a unique intersection, $k \notin \{-3, 1\}$: subtracting the equations gives $(k-1)(x-y) = 0$, so $x = y = 4/(k+3)$.

59. (a) The altitudes are $4y = (6-t)x$ from A and $4y = -t(x-6)$ from B. These formulas also give the correct horizontal altitudes when $t = 6$ and $t = 0$, respectively.

(b) Solving gives $H = (t, \frac{t(6-t)}{4})$. The altitude from C is $x = t$, so it also passes through H.

(c) For $0 < t < 6$, $0 < t(6-t) = 9 - (t-3)^2 \le 9$, so $0 < y_H \le \frac{9}{4} < 4$. Thus H lies inside the vertical segment from $(t, 0)$ to C, which lies inside the triangle except at its endpoints. If $t \le 0$ or $t \ge 6$, then $y_H \le 0$, so H cannot lie strictly inside. Hence $0 < t < 6$.

60. Write $P = (p, 3)$. If $p = -2$ or $p = 6$, one of AP, BP would be vertical and the other would not be horizontal, so these cases are not perpendicular. Otherwise $\frac{3}{p+2}\frac{3}{p-6} = -1$. Thus $(p+2)(p-6) = -9$, or $p^2 - 4p - 3 = 0$, giving $P = (2 \pm \sqrt{7}, 3)$. For $p = 2 + \sqrt{7}$, the gradients are $m_{AP} = 3/(4+\sqrt{7}) = (4-\sqrt{7})/3$ and $m_{BP} = 3/(-4+\sqrt{7}) = -(4+\sqrt{7})/3$. Hence $AP : y = \frac{4-\sqrt{7}}{3}(x+2)$ and $BP : y = -\frac{4+\sqrt{7}}{3}(x-6)$.

Syllabus scope checked against the supplied IB Mathematics: Analysis and Approaches guide, SL 2.1, p. 37. Original practice questions and illustrations by Radford Mathematics.