



Transformations of Graphs

IB Mathematics AA SL & HL | Worksheet 2.11

50 shared questions + 10 HL challenges

At a glance

Learning goal	Describe transformations; map points; sketch graphs; track domain, range and asymptotes.
Syllabus link	Shared: SL 2.11. HL extension: AHL 2.16, restricted here to combined inputs $f(ax + b)$.
Structure	1–15 short response; 16–30 extended response; 31–50 IB-style; 51–60 HL exclusive.
Technology	All questions are non-calculator. Use separate paper for working and sketches.

Instructions

Give exact answers and label key features on sketches. State a valid *ordered sequence* when requested; more than one sequence may be correct. Use **stretch** with its scale factor. Unless stated otherwise, use the natural real domain. Questions 1–50 are shared SL/HL practice; Questions 51–60 extend the syllabus for HL.

1. Short response

Identify the change to the input or output, and track the corresponding coordinates.

1. For each equation, name the transformation, state its direction or scale factor, and classify it as horizontal or vertical.

(a) $y = f(x) + 6$.

(b) $y = f(x - 4)$.

(c) $y = \frac{1}{3}f(x)$.

2. Describe each transformation fully.

(a) $y = f(5x)$.

(b) $y = f(x/4)$.

(c) $y = -f(x)$.

(d) $y = f(-x)$.

3. The point $P = (-2, 7)$ lies on $y = f(x)$. Find its corresponding point on each graph.

(a) $y = f(x) - 3$.

(b) $y = f(x + 5)$.

(c) $y = 2f(x)$.

(d) $y = f(2x)$.

4. Write the point mapping for each transformation, using a generic point (x, y) .

(a) $y = -f(x)$.

(b) $y = f(-x)$.

(c) $y = f(x/3)$.

5. The point $A = (8, -3)$ lies on $y = f(x)$. Its corresponding point on $y = f(kx)$ is $A' = (2, -3)$, where $k > 0$. Find k and the horizontal scale factor.

6. The graph of f has roots $x = -3$ and $x = 5$. State the roots of $y = 4f(x)$ and $y = -f(x)$. Explain why these roots stay fixed.

7. The domain of f is $[-5, 2)$ and its range is $(-1, 6]$. State the domain and range of $y = f(x) + 4$.

8. The domain of f is $[-5, 2)$ and its range is $(-1, 6]$. State the domain and range of $y = f(x - 3)$.

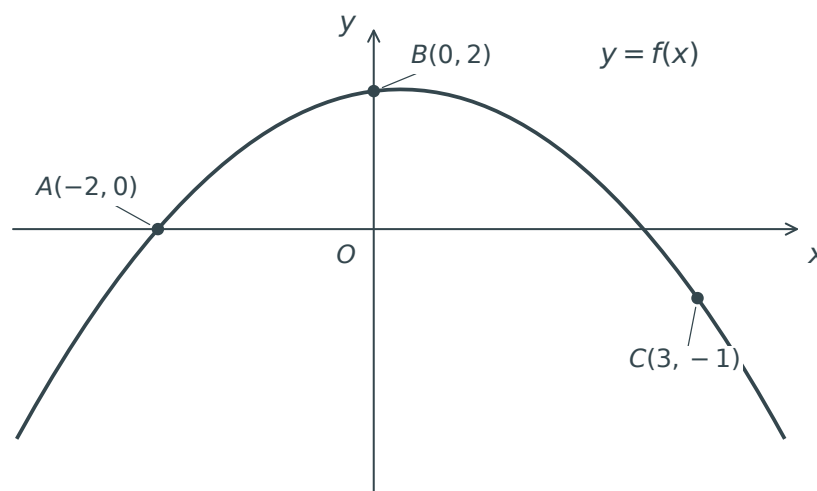
9. The graph of f has range $[-2, 5)$. State the range of $y = -2f(x)$. Explain which endpoint is included.

10. Write an equation for the image of $y = f(x)$ after each sequence.

(a) Translate 2 units left, then 5 units down.

(b) Stretch vertically with SF 3, then reflect across the x -axis.

11. The sketch shows an original function with labelled points A , B and C . State their images under $y = 2f(x)$, and explain why the new curve is not simply a vertical translation.





12. A graph has a vertical asymptote $x = -1$ and a horizontal asymptote $y = 2$. State the corresponding asymptotes after a translation 4 units right and 3 units down.

13. Given $f(3) = 8$, explain why the point $(5, 8)$ lies on $y = f(x - 2)$. Use the input expression in your explanation.

14. Given $f(-6) = 4$, find the corresponding point on $y = f(3x)$. Explain why the horizontal scale factor is not 3.

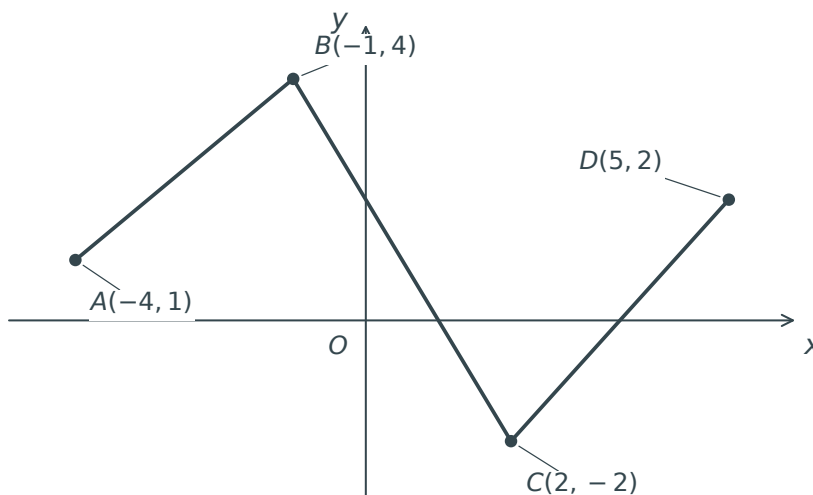
15. A student writes: “For $y = f(x) + 5$, every root of f moves 5 units up and remains a root.” Identify the error and correct the statement.

2. Extended response

Use the original graph to follow key points, domain and range through each transformation.

16. The graph shown consists of straight segments joining $A(-4, 1)$, $B(-1, 4)$, $C(2, -2)$ and $D(5, 2)$, including its endpoints. Let $g(x) = -f(x) + 2$.

- State the images of A, B, C, D .
- Sketch g and state its domain and range.
- Explain why the domain is unchanged.



17. Let $f(x) = x^2 - 6x + 5$.

- Find and simplify $f(x - 2)$.
- State the original roots and vertex, then their images under this transformation.
- State the domain and range of $f(x - 2)$.

18. Starting from $y = \sqrt{x}$, obtain $y = 2\sqrt{x + 3} - 4$.

- List a valid sequence, completing one category before the other and leaving the shift last within each category.
- Make a small sketch after each step. At every stage, state the domain and range, and track $(1, 1)$.

19. Starting from $y = 2^x$, obtain $y = 3 - 2^{x-2}$.

- List a valid sequence and write the equation after every step.
- Sketch each stage, tracking $(0, 1)$. State the domain, range and horizontal asymptote at every stage.



20. Begin with $y = 1/x$. After each instruction below, write the new equation, sketch the graph and state its domain, range and both asymptotes.

- (a) Translate 1 unit left.
 - (b) Stretch the current graph vertically with SF 3.
 - (c) Translate the current graph 2 units down.
 - (d) Express the final equation as a single fraction.
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21. The domain of f is $(-2, 6]$ and its range is $[-3, 5)$. Let $g(x) = -2f(x + 4) + 1$.

- (a) State one valid sequence of transformations.
 - (b) Find the domain and range of g .
 - (c) A point $P = (2, -1)$ lies on f . Find its image.
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22. Compare these vertical sequences applied to $y = f(x)$:

- (a) Stretch with SF 4, then translate 3 units down.
 - (b) Translate 3 units down, then stretch with SF 4.
 - (c) If the translation is to be performed first, how far down should it be to produce the result in part (a)?
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23. Let $g(x) = -3/(x - 2) + 1$.

- (a) List a valid sequence from $y = 1/x$ to $y = g(x)$.
 - (b) State the domain, range and asymptotes.
 - (c) Find the intercepts and sketch the graph.
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24. A function f has a minimum at $(2, -3)$. The graph of $g(x) = af(x) + d$, with $a < 0$, has a maximum at $(2, 10)$ and contains the image $(5, 2)$ of the point $(5, 1)$ on f .

- (a) Find a and d .
 - (b) Describe the vertical transformations.
 - (c) Explain why the minimum becomes a maximum.
-

25. Let $f(x) = x^3$ and $g(x) = -(x + 2)^3 + 1$.

- (a) List a valid sequence from f to g .
- (b) Map $(-1, -1)$, $(0, 0)$, $(1, 1)$.
- (c) Sketch g , labelling its intercepts, and state its domain and range.



26. For $f(x) = x^2 - 4$, compare $g(x) = 3f(x)$ and $h(x) = f(x) + 3$.

- (a) Find their equations, vertices and roots.
- (b) Sketch both graphs on the same axes.
- (c) Explain which transformation preserves the original roots and why.

27. The graph of f has range $[2, \infty)$ and contains $(1, 4)$. The graph of $g(x) = af(x) + d$, where $a < 0$, has range $(-\infty, 7]$ and contains $(1, 1)$. Find a, d , and give a valid transformation sequence.

28. The graph of f has domain $[-4, 8]$, range $[-2, 3]$ and contains $P = (6, 1)$. For each graph, give its domain, range and the image of P .

- (a) $y = f(2x)$.
- (b) $y = f(x/2)$.
- (c) $y = f(-x)$.

29. A model $H(t)$ gives the height, in metres, of a moving object for $0 \leq t \leq 12$. Its maximum is 9 m at $t = 4$. Consider $K(t) = \frac{1}{2}H(t/2) + 1$.

- (a) Describe the transformations.
- (b) State the domain of K , its maximum height and the time at which this is reached.

30. The following table gives some values of f .

x	-2	0	3	7
$f(x)$	5	-1	4	0

Let $g(x) = -2f(x - 1) + 3$.

- (a) Make a table of the corresponding inputs and outputs of g .
- (b) Evaluate $g(4)$.
- (c) Explain why the table alone does not determine the full domain or range of g .

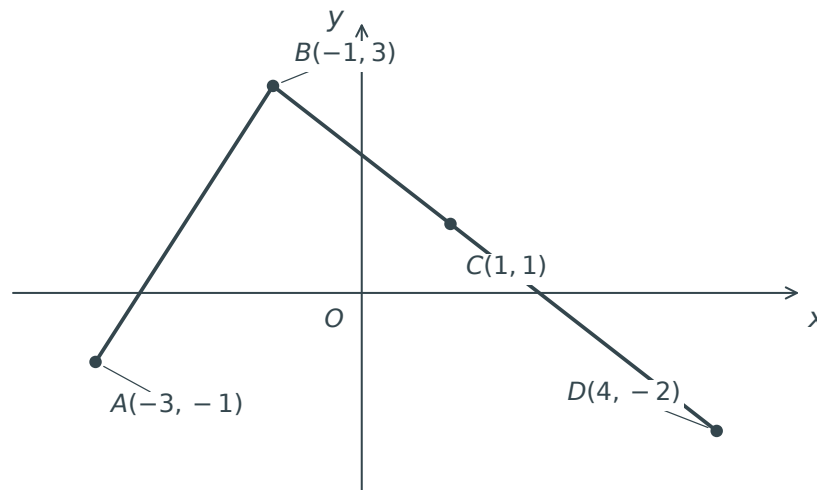
3. IB exam-style questions | Paper 1

These are original practice questions. The maximum mark and marks for each part are shown.

31. [Maximum mark: 8]

The graph of f consists of the straight segments shown, with endpoints included. Let $g(x) = 2f(x + 1) - 1$.

- Describe a valid sequence from f to g . [3]
- Find the images of A, B, C, D . [2]
- Sketch g , and state its domain and range. [3]



32. [Maximum mark: 9]

Let $g(x) = -2(x - 1)^2 + 8$.

- List a valid sequence taking $y = x^2$ to $y = g(x)$. [3]
- Find the vertex and both x -intercepts. [3]
- Sketch g , labelling all intercepts, and state its range. [3]

33. [Maximum mark: 9]

The graph of $g(x) = 5 - \sqrt{x + 4}$ is obtained from $y = \sqrt{x}$.

- State a valid ordered sequence of transformations. [3]
- State the endpoint, domain and range of g . [3]
- Find both intercepts and sketch g . [3]

34. [Maximum mark: 9]

Let $g(x) = a3^x + d$, where $a < 0$. Its horizontal asymptote is $y = 4$, and its graph passes through $(1, -2)$.

- Find a and d . [3]
- Describe a sequence from $y = 3^x$ to $y = g(x)$. [3]
- State the domain, range and y -intercept of g . [3]

35. [Maximum mark: 9]

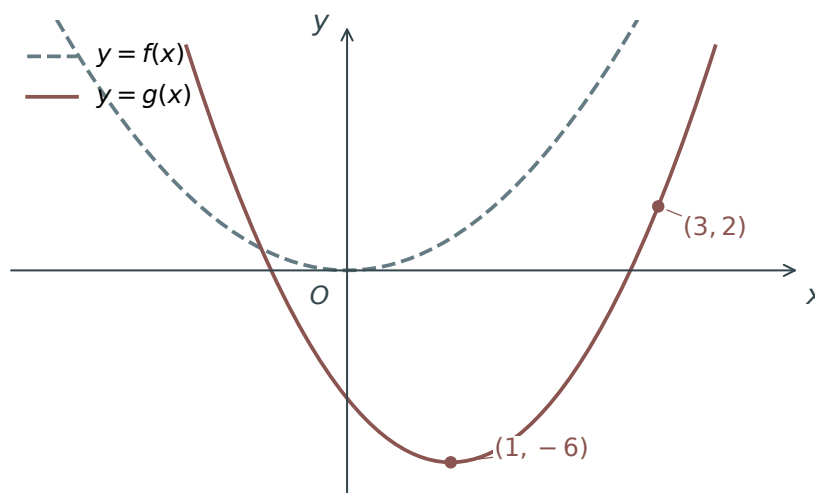
Let $g(x) = a/(x - h) + k$. Its asymptotes are $x = 3$ and $y = -2$, and its graph passes through $(4, 2)$.

- Find h, k, a . [3]
- Describe a sequence from $y = 1/x$ to $y = g(x)$. [3]
- Find the x -intercept and state the domain and range. [3]

36. [Maximum mark: 9]

The sketch shows $y = f(x)$ and $y = g(x)$, where $f(x) = x^2$ and $g(x) = a(x - h)^2 + k$. The point $(3, 2)$ lies on g .

- Write down h and k , and hence find a . [3]
- List a sequence of transformations from f to g . [3]
- State the range of g and find its roots. [3]



37. [Maximum mark: 6]

A graph $y = f(x)$ is stretched vertically with SF 3, then translated 8 units upwards to give $y = g(x)$.

- Express $g(x)$ in terms of $f(x)$. [1]
- A student wants to translate first and then apply the same stretch. Find the required translation. [2]
- If $(2, -1)$ lies on f , track this point through both sequences to verify that they agree. [3]

**38. [Maximum mark: 6]**

Let $f(x) = 2^x$ and $g(x) = 2^{x+3}$.

- (a) Describe a single horizontal transformation taking f to g . [1]
- (b) Show that a single vertical stretch also produces g , and state its SF. [2]
- (c) The point $P = (0, 1)$ lies on f . Find its image under each transformation, and explain why different image points can lie on the same final graph. [3]

39. [Maximum mark: 8]

The domain of f is $[-1, 4)$ and its range is $(2, 9]$. Define $g(x) = -\frac{1}{2}f(x-3) + 6$.

- (a) State a valid transformation sequence. [3]
- (b) Find the domain and range of g , justifying endpoint inclusion. [4]
- (c) Given $f(2) = 6$, find the corresponding point on g . [1]

40. [Maximum mark: 5]

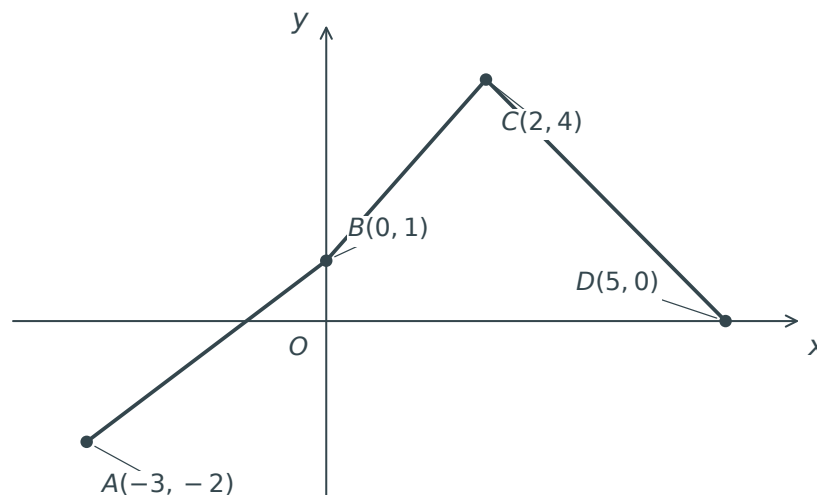
The only solutions of $f(x) = 0$ are $x = -4$ and $x = 2$. Let $g(x) = f(x+3)$.

- (a) Find the roots of g . [2]
- (b) Given $f(-1) = 5$, find the corresponding point on g . [1]
- (c) A second graph is $h(x) = f(x) + 3$. Explain why the roots of h cannot be found from the given information alone. [2]

41. [Maximum mark: 7]

The graph shown consists of straight segments, including both endpoints. Let $g(x) = f(-x)$.

- Describe the transformation and find the images of the four labelled points. [3]
- Sketch g and state its domain and range. [3]
- The original graph has a root at $x = -1$. State the corresponding root of g . [1]



42. [Maximum mark: 9]

The graph of $g(x) = a\sqrt{x-h} + k$, with $a > 0$, has endpoint $(2, -3)$ and passes through $(6, 5)$.

- Find h, k, a . [3]
- Describe a sequence from $y = \sqrt{x}$ to $y = g(x)$. [3]
- Find the root of g and state its domain and range. [3]

43. [Maximum mark: 11]

Let $g(x) = (2x + 5)/(x + 1)$.

- Show that $g(x) = 2 + 3/(x + 1)$. [2]
- Hence describe a sequence of transformations from $y = 1/x$. [3]
- State the domain, range and asymptotes. [3]
- Find the intercepts and sketch g . [3]

**44. [Maximum mark: 8]**

A temperature model $T(t)$ is defined for $0 \leq t \leq 8$ hours and has range $[10, 22]$ degrees Celsius. Its maximum occurs at $t = 3$. A second model is $S(t) = \frac{1}{2}T(t/3) + 4$.

- (a) Describe the three transformations. [3]
- (b) State the domain and range of S , with units. [3]
- (c) State when S reaches its maximum, and explain how the input expression gives your answer. [2]

45. [Maximum mark: 8]

Let $g(x) = 2f(x - 4) - 3$. It is known that $g(1) = 7$ and $g(6) = -5$.

- (a) Find $f(-3)$ and $f(2)$. [4]
- (b) The range of g is $[-7, 9]$. Find the range of f . [2]
- (c) The domain of g is $[0, 10]$. Find the domain of f . [2]

46. [Maximum mark: 8]

A function f has domain $[-2, 7]$ and range $[-4, 1]$. Its graph has a maximum at $(3, 1)$. Let $g(x) = -3f(-x) + 2$.

- (a) List a valid transformation sequence. [3]
- (b) State the domain and range of g . [3]
- (c) Find the corresponding turning point and explain whether it is a maximum or minimum. [2]

47. [Maximum mark: 6]

Let $f(x) = x^2 - 2x - 3$. The graph is reflected across both coordinate axes.

- (a) Find the final equation when the x -axis reflection is performed first. [2]
- (b) Find the final equation in the reverse order. [2]
- (c) Explain, using (x, y) , why these two reflections commute for any function. [2]

48. [Maximum mark: 8]

The parabola $f(x) = x^2 - 6x + 5$ is translated vertically by d to produce g .

- (a) Write g in vertex form. [2]
- (b) Find d if the translated graph touches the x -axis. [2]
- (c) State the values of d for which g has no real roots. [1]
- (d) For $d = 4$, state the vertex and range, and sketch g . [3]

**49. [Maximum mark: 6]**

A graph is stretched vertically with SF $p > 0$ and translated upwards by $d > 0$.

- (a) Write the final equation for each of the two possible orders. [2]
- (b) Find the condition on p for these two graphs to be identical for every function f . [2]
- (c) Explain why a reflection across the x -axis and a vertical stretch can always be interchanged. [2]

50. [Maximum mark: 10]

Let $g(x) = 6 - 3 \cdot 2^{x+1}$.

- (a) List a valid sequence from $y = 2^x$ to $y = g(x)$. [3]
- (b) State the domain, range and horizontal asymptote after every transformation in your sequence. [4]
- (c) Find both intercepts and sketch g . [3]

4. AA HL exclusive challenge

SL students: the shared questions end at Question 50. This extension uses combined inputs $f(ax + b)$ from **AHL 2.16**. It is also useful preparation for mathematical modelling and the IA.

51. [Maximum mark: 7]

Let $g(x) = f(3x - 6)$.

- Factor the input and list a valid sequence from f to g , leaving the horizontal translation last. [3]
- Find the image of $P = (9, -2)$. [2]
- Give a second sequence with the translation first. [2]

52. [Maximum mark: 7]

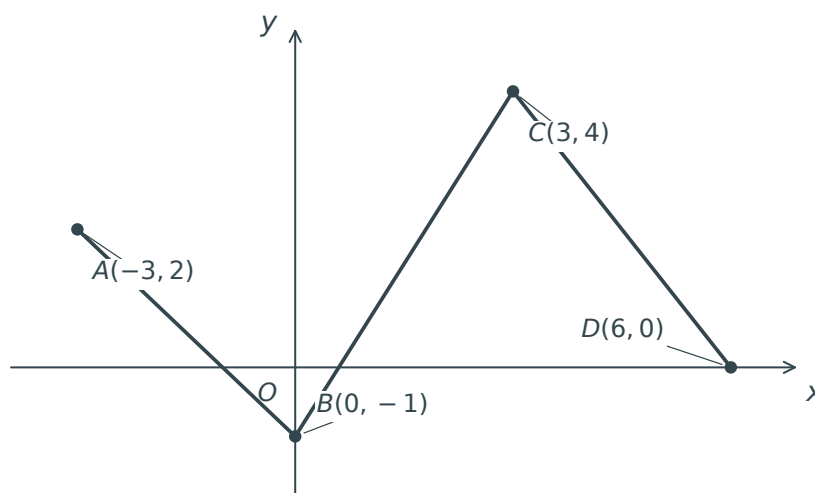
The domain of f is $[-3, 5)$ and its range is $(-2, 6]$. Let $g(x) = f(4 - 2x)$.

- List a valid sequence, with the horizontal shift last. [3]
- Find the domain and range of g . [3]
- Find the image of $(1, 4)$. [1]

53. [Maximum mark: 10]

The supplied piecewise linear graph has endpoints included. Let $g(x) = -2f(3x - 6) + 1$.

- Find the images of all four labelled points. [4]
- Sketch g , and state its domain and range. [3]
- List one valid ordered sequence from f to g . [3]



54. [Maximum mark: 10]

Under the transformation $g(x) = af(bx + c) + d$, the points $P = (1, 2)$ and $Q = (5, -1)$ map to $P' = (3, -3)$ and $Q' = (5, 6)$.

- (a) Find b, c, a, d . [6]
- (b) Factor the input and state a valid ordered sequence. [4]

55. [Maximum mark: 10]

Let $g(x) = 4 - 2\sqrt{6 - 3x}$.

- (a) Starting from $y = \sqrt{x}$, list a valid sequence with all horizontal transformations first and each category's translation last. [4]
- (b) State the domain, range and endpoint of g . [3]
- (c) Find both intercepts and sketch g . [3]

56. [Maximum mark: 11]

Let $f(x) = 1/x$ and $g(x) = -6f(2x + 4) + 1$.

- (a) Describe a valid sequence corresponding directly to the stated input and output operations. [4]
- (b) Simplify $g(x)$ and give an alternative sequence using no horizontal stretch. [3]
- (c) State its domain, range, asymptotes and intercepts. [4]

57. [Maximum mark: 7]

Let S_p denote a horizontal stretch with SF $p > 0$, and T_h a translation h units to the right, where $h \neq 0$.

- (a) Starting from (x, y) , find the final point for S_p followed by T_h , and for the reverse order. [3]
- (b) Determine when these two point mappings are identical. [2]
- (c) A student must apply the translation first. Find the translation that, followed by S_p , gives the same mapping as S_p followed by T_h . [2]

58. [Maximum mark: 10]

A function f has domain $[-4, -1) \cup (2, 8]$ and range $[-2, 5)$. Define $g(x) = -3f(2 - 2x) + 4$.

- (a) Find the domain and range of g , retaining the correct open and closed endpoints. [5]
- (b) Find the images of $A = (-4, -2)$ and $B = (8, 1)$. [2]
- (c) State a valid ordered transformation sequence. [3]

**59. [Maximum mark: 9]**

Let $g(x) = 2f(3x - 6) - 5$.

- (a) Express $f(x)$ in terms of g . [3]
- (b) Hence describe a sequence that takes the graph of g back to the graph of f . [3]
- (c) A point $P = (3, 4)$ lies on f . Find its image on g and verify that your reverse transformation recovers P . [3]

60. [Maximum mark: 11]

The function f has domain $[-2, 4]$, range $[-1, 3]$, and contains $P = (2, 1)$. Define

$$g(x) = -2f(2x - 6) + 5, \quad h(x) = 3g(-x) + 1.$$

- (a) Express $h(x)$ directly in terms of f . [2]
- (b) Find a direct point mapping from f to h by solving the input equation and transforming the output. [3]
- (c) State the domain and range of h , and find the final image of P . [3]
- (d) Give one valid sequence from f directly to h , finishing each category before beginning the other. [3]

Answer key and solution outlines

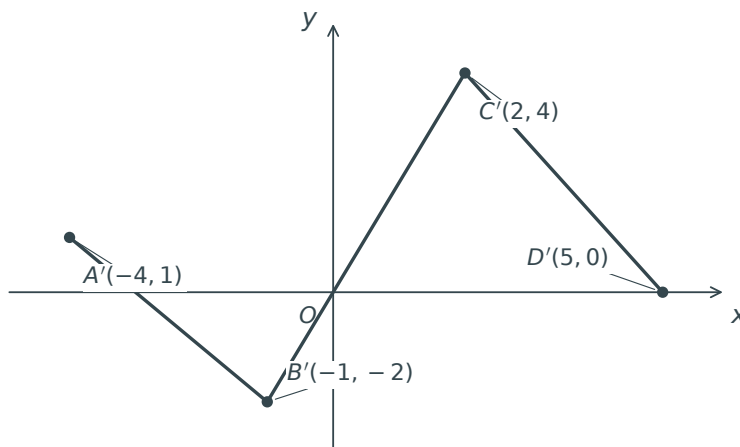
One valid sequence is given when several are possible. Equivalent sequences with correctly adjusted shifts are accepted. Sketches are indicative; labelled coordinates and asymptotes determine their positions.

1. Short response

1. (a) Vertical translation 6 units up. (b) Horizontal translation 4 units right. (c) Vertical stretch, SF $\frac{1}{3}$.
2. (a) Horizontal stretch, SF $\frac{1}{5}$. (b) Horizontal stretch, SF 4. (c) Reflection across the x -axis. (d) Reflection across the y -axis.
3. (a) $(-2, 4)$; (b) $(-7, 7)$; (c) $(-2, 14)$; (d) $(-1, 7)$. Horizontal changes alter x ; vertical changes alter y .
4. (a) $(x, y) \mapsto (x, -y)$. (b) $(x, y) \mapsto (-x, y)$. (c) $(x, y) \mapsto (3x, y)$.
5. Recover input 8: $k(2) = 8$, so $k = 4$. The horizontal SF is $1/k = \frac{1}{4}$.
6. Both graphs have roots -3 and 5 . Their zero outputs stay zero when multiplied by 4 or by -1 .
7. Domain $[-5, 2)$ is unchanged. Range $(-1 + 4, 6 + 4] = (3, 10]$.
8. Add 3 to each input: domain $[-2, 5)$. Range $(-1, 6]$ is unchanged.
9. The original upper boundary 5 maps to -10 and is excluded; -2 maps to 4 and is included. Range $(-10, 4]$.
10. (a) $y = f(x + 2) - 5$. (b) $y = -3f(x)$.
11. $A' = (-2, 0)$, $B' = (0, 4)$, $C' = (3, -2)$. Heights are doubled: A is fixed, B moves up and C moves down.
12. Vertical asymptote $x = 3$; horizontal asymptote $y = -1$.
13. At $x = 5$, the input to f is $5 - 2 = 3$. Hence $f(5 - 2) = f(3) = 8$: the original input and output are recovered. The point has moved 2 units right.
14. Solve $3x = -6$, giving $x = -2$. The point is $(-2, 4)$. Dividing original inputs by 3 gives horizontal SF $\frac{1}{3}$.
15. A root $(r, 0)$ maps to $(r, 5)$, which is not on the x -axis. The new roots occur where $f(x) = -5$, if such inputs exist.

2. Extended response

- 16.** (a) Use $(x, y) \mapsto (x, 2 - y)$: $A'(-4, 1)$, $B'(-1, -2)$, $C'(2, 4)$, $D'(5, 0)$. (b) Join in the same order. Domain $[-4, 5]$, range $[-2, 4]$. (c) Both changes are vertical, so the x -coordinates stay the same.



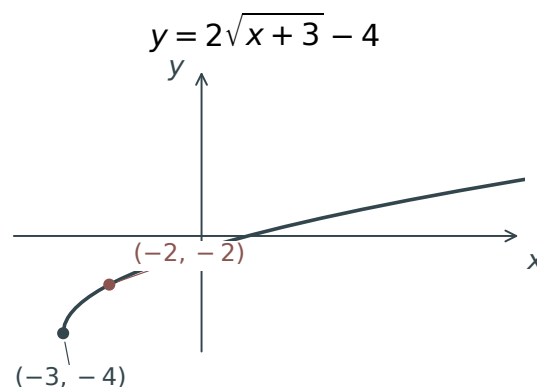
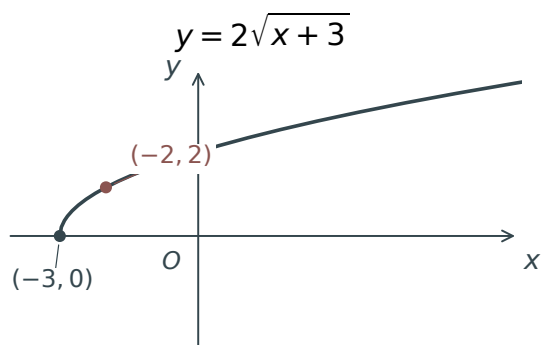
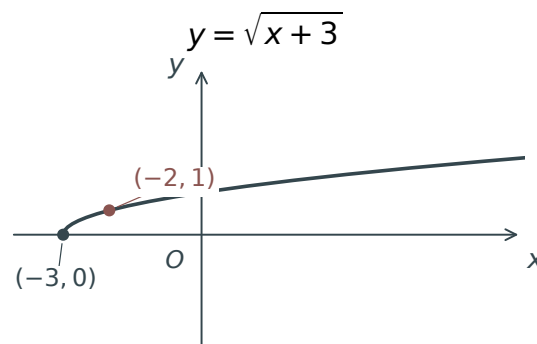
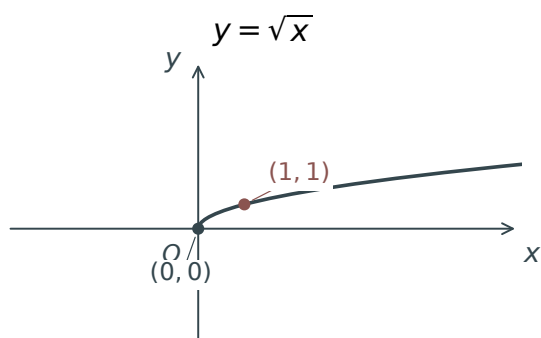
- 17.** (a) $(x - 2)^2 - 6(x - 2) + 5 = x^2 - 10x + 21$. (b) Since $f = (x - 1)(x - 5) = (x - 3)^2 - 4$, roots $(1, 0)$, $(5, 0)$ move to $(3, 0)$, $(7, 0)$; vertex $(3, -4)$ moves to $(5, -4)$. (c) Domain $\mathbb{R}$; range $[-4, \infty)$.

2. Extended response (continued)

18. Translate left 3, stretch vertically with SF 2, then translate down 4.

Equation $y =$	Tracked point	Domain	Range
$\sqrt{x}$	(1, 1)	$[0, \infty)$	$[0, \infty)$
$\sqrt{x+3}$	(-2, 1)	$[-3, \infty)$	$[0, \infty)$
$2\sqrt{x+3}$	(-2, 2)	$[-3, \infty)$	$[0, \infty)$
$2\sqrt{x+3} - 4$	(-2, -2)	$[-3, \infty)$	$[-4, \infty)$

The endpoints are $(0, 0)$, $(-3, 0)$, $(-3, 0)$, $(-3, -4)$, all included.

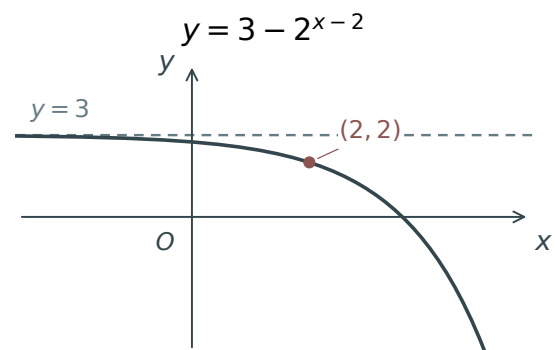
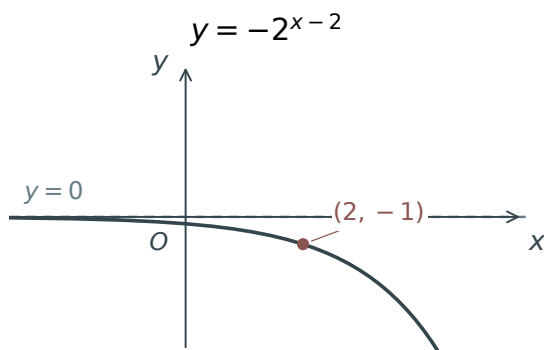
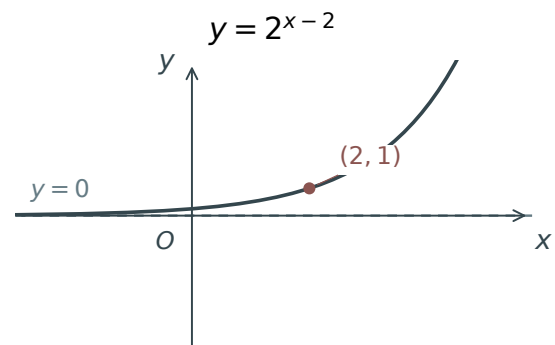
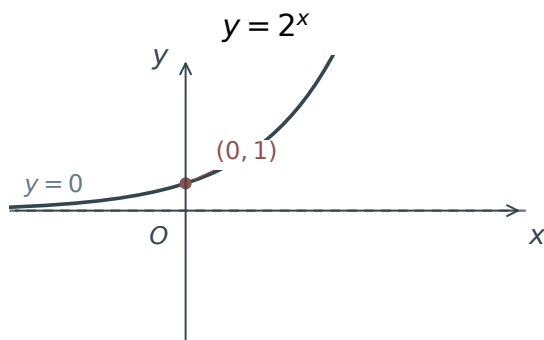


2. Extended response (continued)

19. Translate right 2, reflect across the x -axis, then translate up 3.

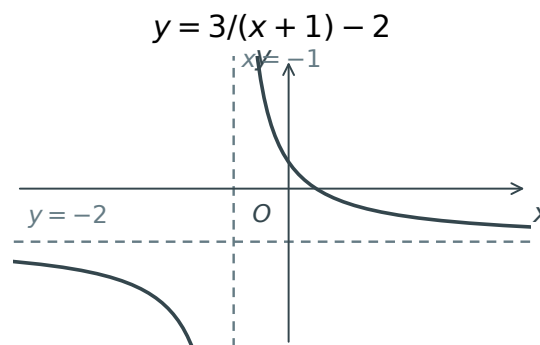
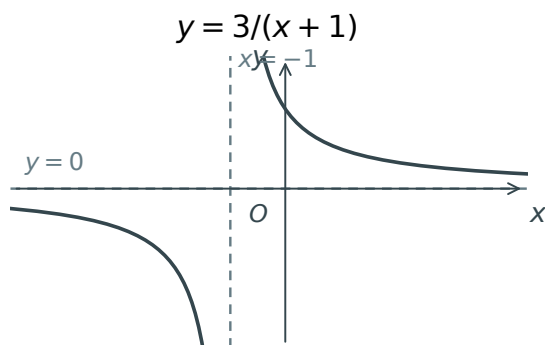
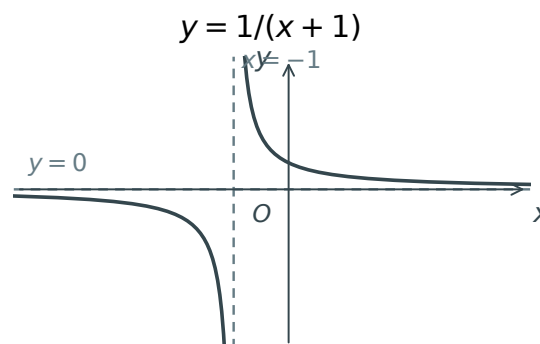
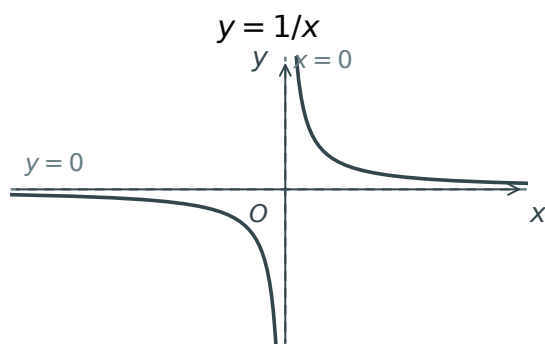
Equation $y =$	Tracked point	Domain	Range
2^x	$(0, 1)$	$\mathbb{R}$	$(0, \infty)$
2^{x-2}	$(2, 1)$	$\mathbb{R}$	$(0, \infty)$
-2^{x-2}	$(2, -1)$	$\mathbb{R}$	$(-\infty, 0)$
$3 - 2^{x-2}$	$(2, 2)$	$\mathbb{R}$	$(-\infty, 3)$

The horizontal asymptotes are $y = 0, 0, 0, 3$, respectively. No range boundary is attained.

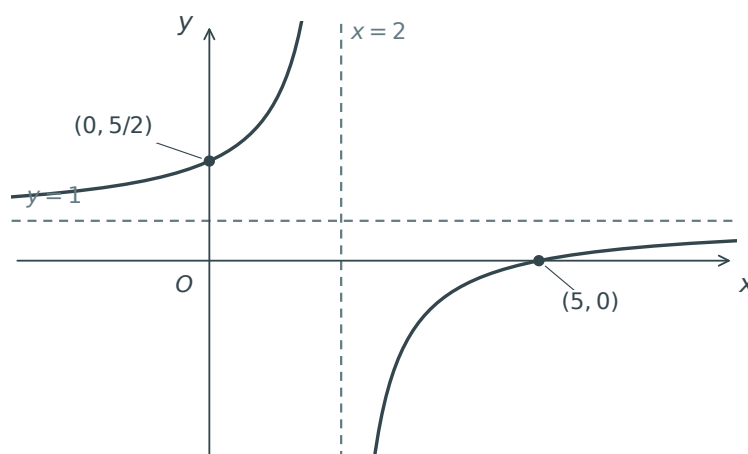


2. Extended response (continued)

- 20.** (a) $y = 1/(x + 1)$: domain $\mathbb{R} \setminus \{-1\}$, range $\mathbb{R} \setminus \{0\}$; asymptotes $x = -1, y = 0$.
 (b) $y = 3/(x + 1)$: the same domain, range and asymptotes.
 (c) $y = 3/(x + 1) - 2$: domain unchanged; range $\mathbb{R} \setminus \{-2\}$; asymptotes $x = -1, y = -2$.
 (d) $[3 - 2(x + 1)]/(x + 1) = (1 - 2x)/(x + 1), x \neq -1$.



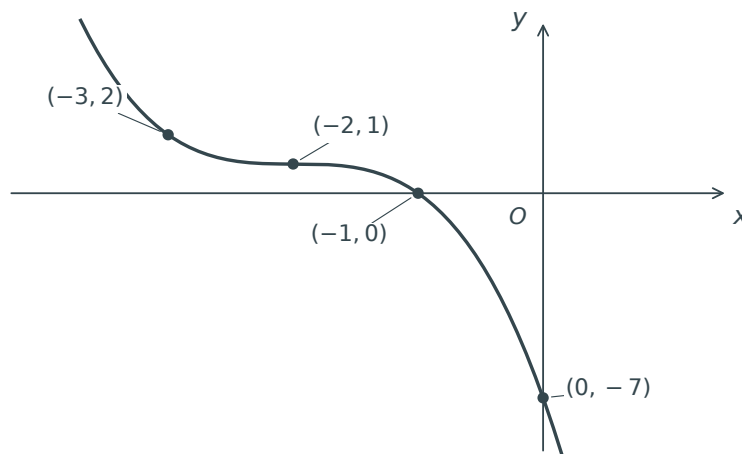
- 21.** (a) Translate left 4; stretch vertically, SF 2; reflect across the x -axis; translate up 1. (b) Domain $(-6, 2]$. Outputs become $-2y + 1$: $5 \mapsto -9$ excluded and $-3 \mapsto 7$ included, so range $(-9, 7]$. (c) $(2, -1) \mapsto (-2, 3)$.
22. (a) $4f(x) - 3$. (b) $4(f(x) - 3) = 4f(x) - 12$. (c) Use a downward shift of $3/4$, since $4(f(x) - 3/4) = 4f(x) - 3$.
23. (a) Right 2; vertical stretch SF 3; reflect across the x -axis; up 1. (b) Domain $\mathbb{R} \setminus \{2\}$, range $\mathbb{R} \setminus \{1\}$; asymptotes $x = 2, y = 1$. (c) $g(0) = 5/2$. For $g(x) = 0$, $3/(x - 2) = 1$, so $x = 5$. Intercepts $(0, 5/2), (5, 0)$.



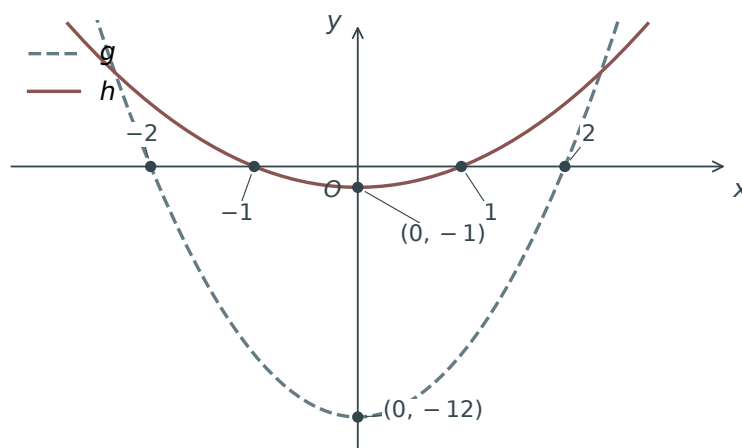
2. Extended response (continued)

24. (a) $-3a + d = 10$ and $a + d = 2$, so $-4a = 8$, $a = -2$, $d = 4$. (b) Vertical stretch SF 2, reflection across the x -axis, then up 4. (c) Multiplication by a negative number reverses the order of the outputs.

25. (a) Left 2, reflect across the x -axis, up 1. (b) $(x, y) \mapsto (x - 2, 1 - y)$ gives $(-3, 2)$, $(-2, 1)$, $(-1, 0)$. (c) $g(0) = -7$; the x -intercept is $(-1, 0)$. Domain and range are $\mathbb{R}$.



26. (a) $g = 3x^2 - 12$: vertex $(0, -12)$, roots ± 2 . $h = x^2 - 1$: vertex $(0, -1)$, roots ± 1 . (c) The stretch preserves the original roots ± 2 because zero outputs stay zero; the upward translation does not.



27. The included lower boundary 2 becomes the maximum 7: $2a + d = 7$. Also $4a + d = 1$. Hence $a = -3$, $d = 13$. Stretch vertically by SF 3, reflect across the x -axis, then translate up 13.

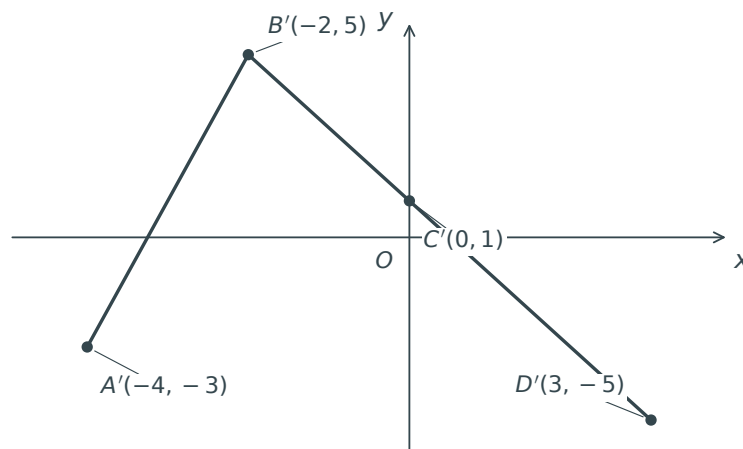
28. (a) Domain $[-2, 4]$, range $[-2, 3]$, image $(3, 1)$. (b) Domain $[-8, 16]$, same range, image $(12, 1)$. (c) Domain $[-8, 4]$, same range, image $(-6, 1)$.

29. (a) Horizontal stretch SF 2, vertical stretch SF $\frac{1}{2}$, then up 1. (b) Domain $[0, 24]$; maximum $\frac{1}{2}(9) + 1 = 11/2$ m, reached at $t = 8$.

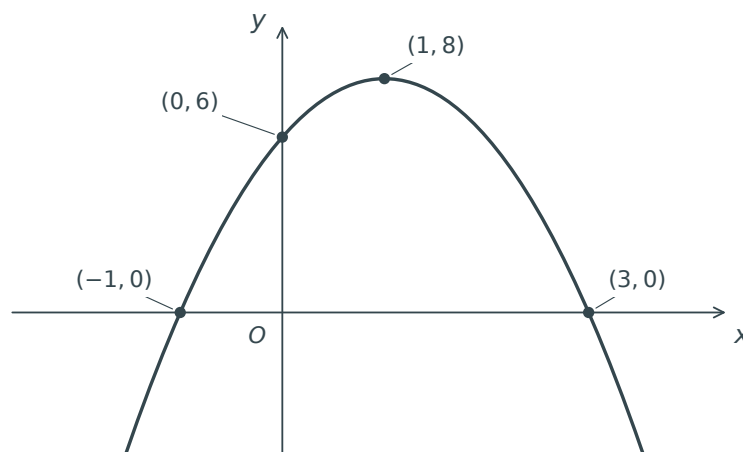
30. (a) Corresponding pairs: $(-1, -7)$, $(1, 5)$, $(4, -5)$, $(8, 3)$. (b) $g(4) = -2f(3) + 3 = -8 + 3 = -5$. (c) Only four values of f are supplied; other inputs and outputs may exist.

3. IB exam-style questions

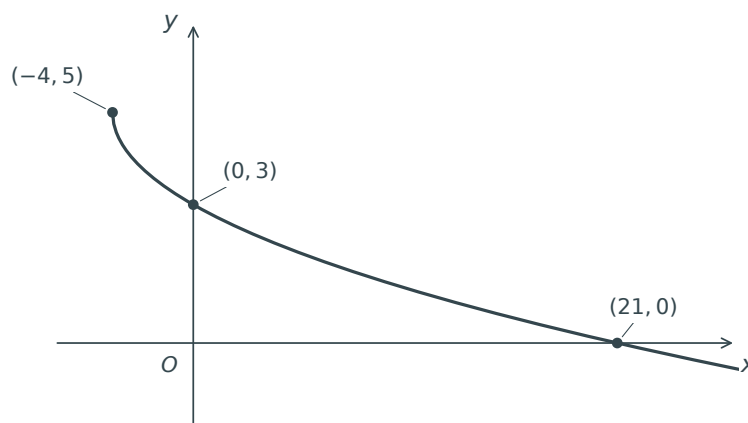
31. (a) Left 1, vertical stretch SF 2, then down 1. (b) $(x, y) \mapsto (x - 1, 2y - 1)$ gives $A'(-4, -3), B'(-2, 5), C'(0, 1), D'(3, -5)$. (c) Join in order; domain $[-4, 3]$, range $[-5, 5]$.



32. (a) Right 1, vertical stretch SF 2, reflect across the x -axis, up 8. (b) Vertex $(1, 8)$. Set $(x - 1)^2 = 4$: roots $-1, 3$. (c) $g(0) = 6$, so the y -intercept is $(0, 6)$. Range $(-\infty, 8]$.



33. (a) Left 4, reflect across the x -axis, up 5. (b) Endpoint $(-4, 5)$, domain $[-4, \infty)$, range $(-\infty, 5]$. (c) $g(0) = 3$; $\sqrt{x + 4} = 5$ gives $x = 21$. Intercepts $(0, 3), (21, 0)$.



34. (a) $d = 4$ and $3a + 4 = -2$, so $a = -2$. (b) Vertical stretch SF 2, reflect across the x -axis, up 4. (c) Domain $\mathbb{R}$, range $(-\infty, 4)$, y -intercept $(0, 2)$.

3. IB exam-style questions (continued)

35. (a) $h = 3, k = -2$ and $a/(4 - 3) - 2 = 2$, so $a = 4$. (b) Right 3, vertical stretch SF 4, down 2. (c) $4/(x - 3) = 2$ gives $x = 5$. Domain $\mathbb{R} \setminus \{3\}$; range $\mathbb{R} \setminus \{-2\}$.

36. (a) The vertex is $(1, -6)$, so $h = 1, k = -6$. Then $2 = 4a - 6$, giving $a = 2$. (b) Right 1, vertical stretch SF 2, down 6. (c) Range $[-6, \infty)$; $2(x - 1)^2 = 6$ gives roots $1 \pm \sqrt{3}$.

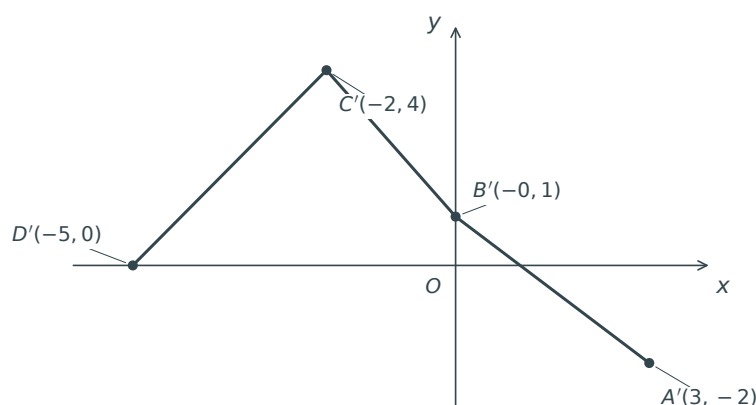
37. (a) $g = 3f + 8$. (b) Translate up $8/3$, since $3(f + 8/3) = 3f + 8$. (c) First route: $(2, -1) \rightarrow (2, -3) \rightarrow (2, 5)$. Second: $(2, -1) \rightarrow (2, 5/3) \rightarrow (2, 5)$.

38. (a) Translate left 3. (b) $2^{x+3} = 8 \cdot 2^x$, so vertical SF 8. (c) The images are $(-3, 1)$ and $(0, 8)$. Both lie on g , but the two transformations pair original points with different points on that same curve.

39. (a) Right 3, vertical stretch SF $\frac{1}{2}$, reflect across the x -axis, up 6. (b) Domain $[2, 7)$. The output 9 maps to $3/2$ and is included; boundary 2 maps to 5 and is excluded. Range $[3/2, 5)$. (c) $(5, 3)$.

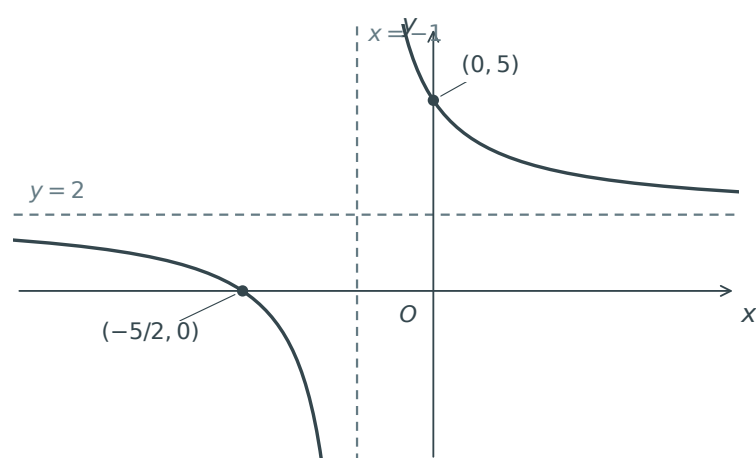
40. (a) $x + 3 = -4$ or 2 , so $x = -7$ or -1 . (b) $(-4, 5)$. (c) Roots of h require $f(x) = -3$, and these inputs are not given.

41. (a) Reflect across the y -axis: $A'(3, -2), B'(0, 1), C'(-2, 4), D'(-5, 0)$. (b) Join in the order A', B', C', D' (or the reverse); domain $[-5, 3]$, range $[-2, 4]$. (c) The root is $x = 1$.



42. (a) $h = 2, k = -3$; $5 = 2a - 3$ gives $a = 4$. (b) Right 2, vertical stretch SF 4, down 3. (c) $4\sqrt{x - 2} = 3$ gives $x = 2 + 9/16 = 41/16$. Domain $[2, \infty)$, range $[-3, \infty)$.

43. (a) $2x + 5 = 2(x + 1) + 3$. (b) Left 1, vertical stretch SF 3, up 2. (c) Domain $\mathbb{R} \setminus \{-1\}$, range $\mathbb{R} \setminus \{2\}$; asymptotes $x = -1, y = 2$. (d) $g(0) = 5$ and $2x + 5 = 0$ gives $x = -5/2$.



44. (a) Horizontal stretch SF 3, vertical stretch SF $\frac{1}{2}$, up 4. (b) Domain $[0, 24]$ hours, range $[9, 15]$ degrees Celsius. (c) Recover the original maximum input: $t/3 = 3$, so $t = 9$ hours.

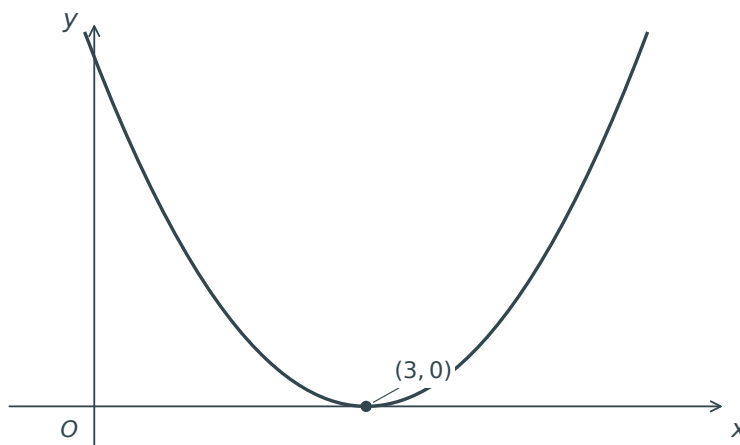
3. IB exam-style questions (continued)

45. (a) $7 = 2f(-3) - 3$ gives $f(-3) = 5$; $-5 = 2f(2) - 3$ gives $f(2) = -1$. (b) Undo $Y = 2y - 3$: $y = (Y + 3)/2$, giving range $[-2, 6]$. (c) Undo the right shift: domain $[-4, 6]$.

46. (a) Reflect across the y -axis; vertical stretch SF 3; reflect across the x -axis; up 2. (b) Domain $[-7, 2]$, range $[-1, 14]$. (c) $(3, 1) \mapsto (-3, -1)$, a minimum because the output order is reversed.

47. (a),(b) Both give $-f(-x) = -(x^2 + 2x - 3) = -x^2 - 2x + 3$. (c) Either $(x, y) \rightarrow (x, -y) \rightarrow (-x, -y)$ or $(x, y) \rightarrow (-x, y) \rightarrow (-x, -y)$; the two coordinates are changed independently.

48. (a) $g = (x - 3)^2 - 4 + d$. (b) Touching requires the minimum to be zero: $d = 4$. (c) No roots for $d > 4$. (d) Vertex $(3, 0)$, range $[0, \infty)$; parabola opening upwards.

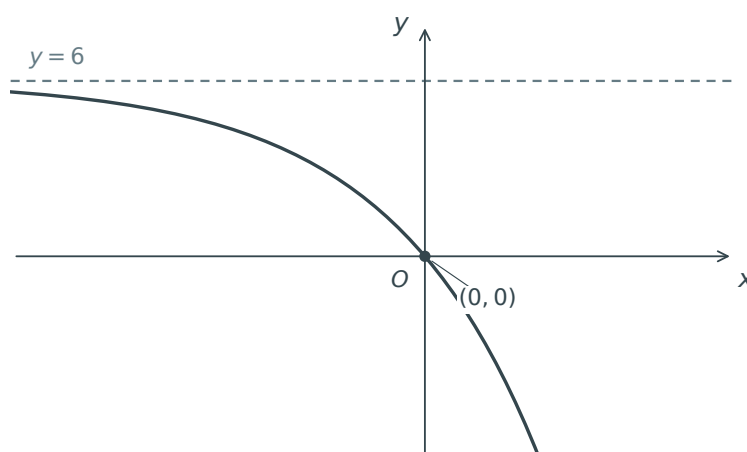


49. (a) Stretch then shift: $pf(x) + d$. Shift then stretch: $p(f(x) + d) = pf(x) + pd$. (b) Equality requires $d = pd$. Since $d > 0$, $p = 1$. (c) Both orders send y to $-py$.

50. (a) Left 1, vertical stretch SF 3, reflect across the x -axis, up 6.

(b) The domain is always $\mathbb{R}$. For equations 2^{x+1} , $3 \cdot 2^{x+1}$, $-3 \cdot 2^{x+1}$, $6 - 3 \cdot 2^{x+1}$, the ranges are $(0, \infty)$, $(0, \infty)$, $(-\infty, 0)$, $(-\infty, 6)$ and horizontal asymptotes are $y = 0, 0, 0, 6$.

(c) $g(0) = 0$; $2^{x+1} = 2$ gives $x = 0$. Both intercepts coincide at $(0, 0)$.

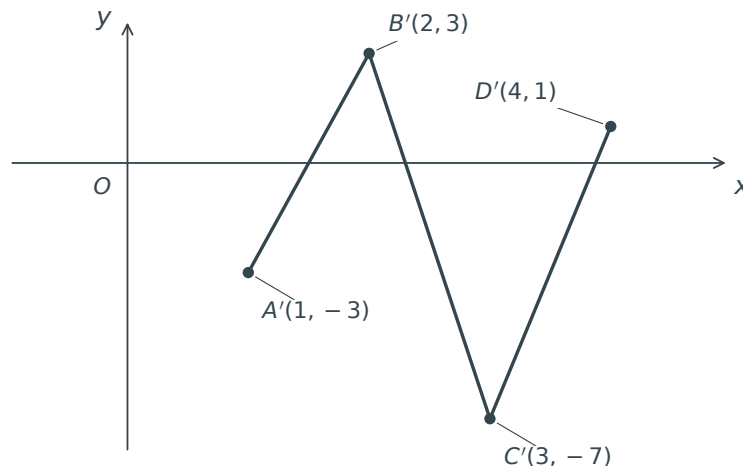


4. AA HL exclusive challenge

51. (a) $3x - 6 = 3(x - 2)$: horizontal stretch SF $\frac{1}{3}$, then right 2. (b) $3x - 6 = 9$ gives $x = 5$, so $(5, -2)$. (c) Right 6, then horizontal stretch SF $\frac{1}{3}$; both send x to $x/3 + 2$.

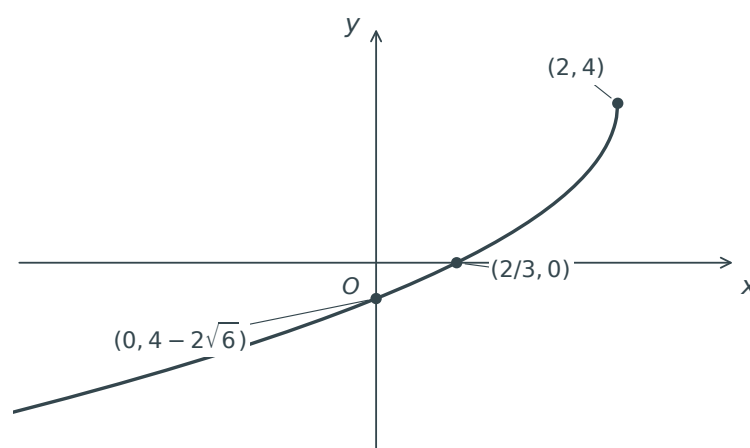
52. (a) $4 - 2x = -2(x - 2)$: horizontal stretch SF $\frac{1}{2}$, reflect across the y -axis, then right 2. (b) New inputs satisfy $-3 \leq 4 - 2x < 5$, giving $-1/2 < x \leq 7/2$. Domain $(-1/2, 7/2]$, range $(-2, 6]$. (c) Solve $4 - 2x = 1$: image $(3/2, 4)$.

53. (a) Recover original input: $3x' - 6 = x$, so $x' = x/3 + 2$; also $y' = -2y + 1$. Thus $A'(1, -3)$, $B'(2, 3)$, $C'(3, -7)$, $D'(4, 1)$. (b) Domain $[1, 4]$, range $[-7, 3]$. (c) Horizontal stretch SF $\frac{1}{3}$, right 2, vertical stretch SF 2, reflection across the x -axis, up 1.



54. (a) $3b + c = 1$, $5b + c = 5$, so $b = 2$, $c = -5$. Also $2a + d = -3$, $-a + d = 6$, so $a = -3$, $d = 3$. (b) $2x - 5 = 2(x - 5/2)$: horizontal stretch SF $\frac{1}{2}$, right $5/2$, vertical stretch SF 3, reflection across the x -axis, up 3.

55. (a) $6 - 3x = -3(x - 2)$: horizontal stretch SF $\frac{1}{3}$, reflection across the y -axis, right 2; vertical stretch SF 2, reflection across the x -axis, up 4. (b) $6 - 3x \geq 0$ gives domain $(-\infty, 2](-\infty, 4]$; endpoint $(2, 4)$. (c) $\sqrt{6} - 3x = 2$ gives root $x = 2/3$; y -intercept $(0, 4 - 2\sqrt{6})$.



56. (a) Horizontal stretch SF $\frac{1}{2}$, left 2, vertical stretch SF 6, reflection across the x -axis, up 1. (b) $g = -3/(x + 2) + 1$. Alternatively left 2, vertical stretch SF 3, reflection across the x -axis, up 1. (c) Domain $\mathbb{R} \setminus \{-2\}$, range $\mathbb{R} \setminus \{1\}$; asymptotes $x = -2$, $y = 1$; intercepts $(1, 0)$, $(0, -1/2)$.

57. (a) Stretch then translate: $(px + h, y)$. Translate then stretch: $(p(x + h), y) = (px + ph, y)$. (b) Equality requires $h = ph$; since $h \neq 0$, $p = 1$. (c) Translate right h/p first: $p(x + h/p) = px + h$.

**4. AA HL exclusive challenge (continued)**

58. (a) The input mapping is $x' = 1 - x/2$. The intervals map to $(3/2, 3]$ and $[-3, 0)$, so the domain is $[-3, 0) \cup (3/2, 3]$. Outputs $y' = -3y + 4$ give range $(-11, 10]$. (b) $A' = (3, 10)$, $B' = (-3, 1)$. (c) Horizontal stretch SF $\frac{1}{2}$, reflection across the y -axis, right 1; vertical stretch SF 3, reflection across the x -axis, up 4.

59. (a) Set $t = x/3 + 2$ so $3t - 6 = x$. Then $f(x) = \frac{1}{2}g(x/3 + 2) + \frac{5}{2}$. (b) Since $x/3 + 2 = (x + 6)/3$, stretch horizontally with SF 3, translate left 6, stretch vertically with SF $\frac{1}{2}$, then translate up $5/2$. (c) $P \mapsto (3, 3)$. Reverse: $(3, 3) \rightarrow (9, 3) \rightarrow (3, 3) \rightarrow (3, 3/2) \rightarrow (3, 4)$.

60. (a) $h(x) = -6f(-2x - 6) + 16$. (b) Solve $-2x' - 6 = x$: $x' = -x/2 - 3$; $y' = -6y + 16$. (c) Domain $[-5, -2]$, range $[-2, 22]$, image $P' = (-4, 10)$. (d) Since $-2x - 6 = -2(x + 3)$: horizontal stretch SF $\frac{1}{2}$, reflection across the y -axis, left 3; vertical stretch SF 6, reflection across the x -axis, up 16.