Transformations of functions

IB Mathematics AA SL & HL | Section 2.11

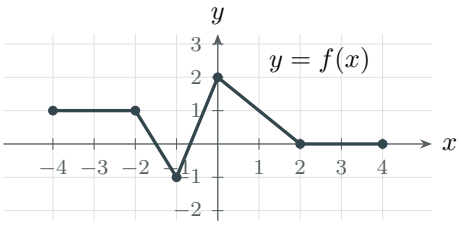
Paper 1 exam-style worksheet • 14 questions • 158 marks

Learning goal	Recognize, describe and combine transformations of functions.
Syllabus link	AA 2.11; related function and inverse skills from 2.5, 2.8 and 2.9.
Structure	Questions 1–10: SL & HL, 102 marks. Questions 11–14: HL, 56 marks.
Technology	Calculator not permitted. Give exact answers.

Instructions. Show your working. State the scale factor of each stretch and the axis of each reflection. Give transformations in the order requested.

1. Maximum mark: 4

The diagram shows the graph of $y = f(x)$ for $-4 \leq x \leq 4$.

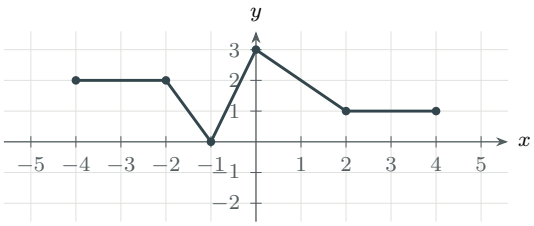


The six graphs A–F below are transformations of this graph.

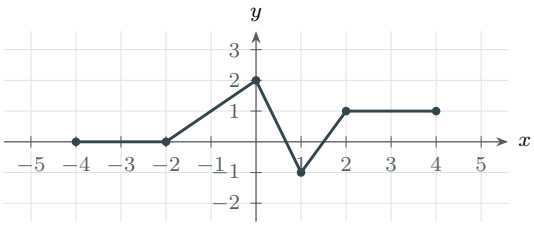
For each expression in the table, write down the letter of the corresponding graph. [4]

Transformed function	Graph
$y = f(x - 1)$	
$y = f(x) + 1$	
$y = f(-x)$	
$y = f(2x)$	

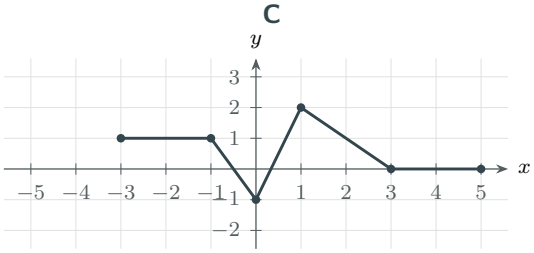
A



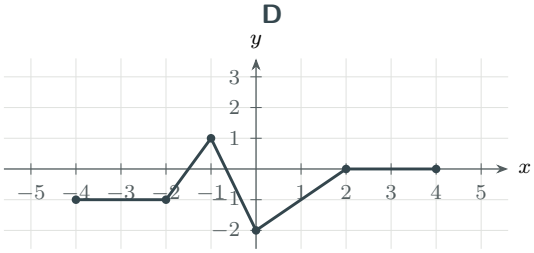
B



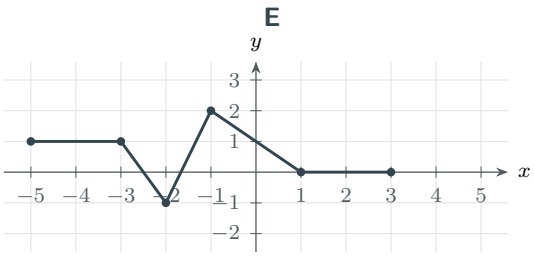
C



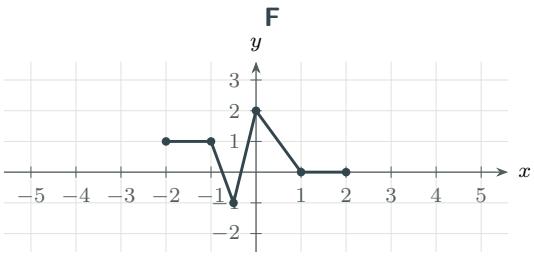
D



E



F



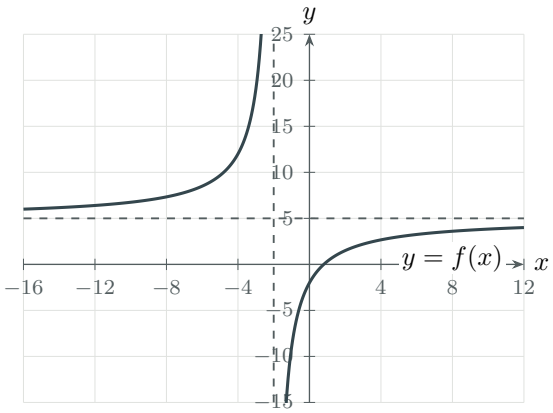
Original questions inspired by the supplied past-paper question types. All marks are suggested practice allocations.

2. Maximum mark: 5

The function f is defined by

$$f(x) = \frac{5x-4}{x+2}, \qquad x \in \mathbb{R}, \quad x \neq -2.$$

The diagram shows part of the graph of f.



(a) Write down $f(0)$. [1]

(b) Write down the equation of the horizontal asymptote. [1]

The function g is defined by $g(x) = -f(x)$ for $x \geq 0$.

(c) Find the range of g. [3]

3. Maximum mark: 14

The function f is defined by $f(x) = \log_3(9x)$ for $x > 0$.

(a) Find the value of

(i) $f(9)$; [2]

(ii) $f\left(\frac{1}{9}\right)$. [1]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

For $x > 0$, the graph of $y = f(3x^2)$ can be obtained by stretching and translating the graph of $y = \log_3 x$.

(d) Describe these two transformations, specifying the order. Justify your answer algebraically. [6]

4. Maximum mark: 14

The function f is defined by

$$f(x) = \log_2(16x), \qquad x > 0.$$

(a) Find the value of

(i) $f\left(\frac{1}{2}\right)$; [2]

(ii) $f\left(\frac{1}{16}\right)$. [1]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

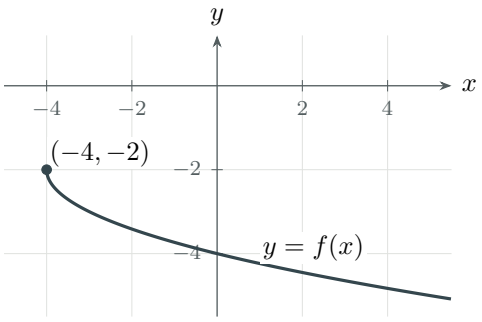
For $x > 0$, the graph of $y = f(2x^3)$ can be obtained by stretching and translating the graph of $y = \log_2 x$.

(d) Describe a sequence of these two transformations, specifying the order. Justify your answer algebraically. [6]

5. Maximum mark: 14

The diagram shows part of the graph of the function

$$f(x) = -2 - \sqrt{x+4}, \qquad x \geq -4.$$



(a) Describe a sequence of transformations that takes the graph of $y = \sqrt{x}$, $x \geq 0$, to the graph of $y = f(x)$. [3]

(b) State the range of f. [1]

(c) Find an expression for $f^{-1}(x)$, stating its domain. [5]

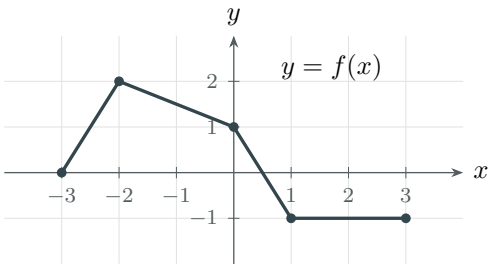
(d) The graphs of $y = f(x)$ and $y = f^{-1}(x)$ meet at a point (x, y).

(i) Show that $(x - y)(x + y + 5) = 0$. [2]

(ii) Hence find the coordinates of all points of intersection of the two graphs. [3]

6. Maximum mark: 4

The diagram shows the graph of $y = f(x)$ for $-3 \leq x \leq 3$.

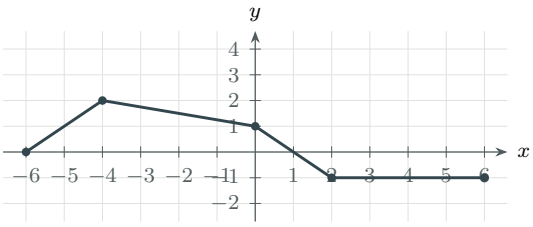


The six graphs A–F below are transformations of this graph.

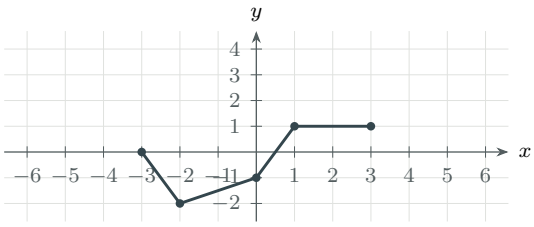
For each expression in the table, write down the letter of the corresponding graph. [4]

Transformed function	Graph
$y = 2f(x)$	
$y = f\left(\frac{x}{2}\right)$	
$y = -f(x)$	
$y = f(x - 2)$	

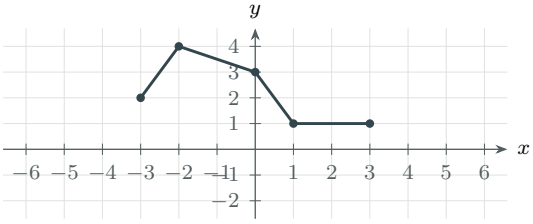
A



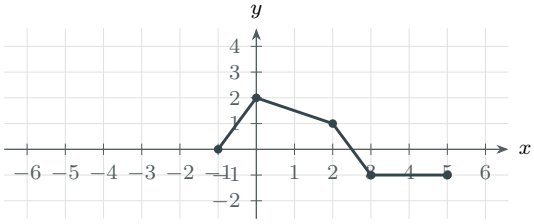
B



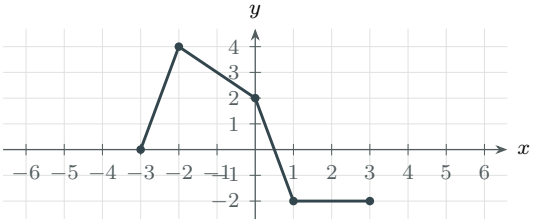
C



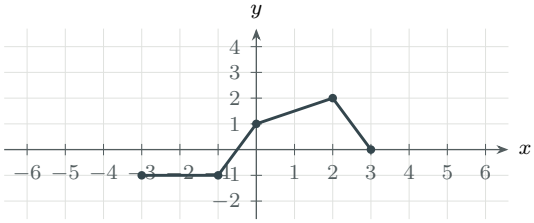
D



E



F

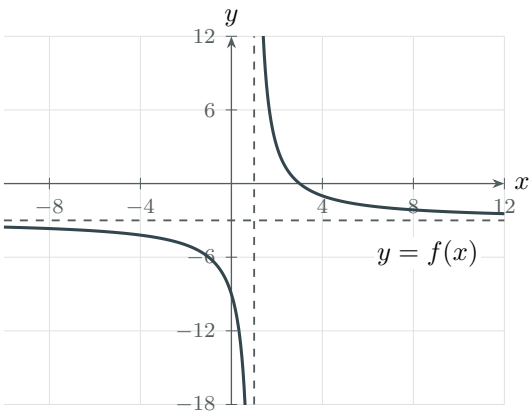


7. Maximum mark: 5

The function f is defined by

$$f(x) = \frac{9 - 3x}{x - 1}, \qquad x \in \mathbb{R}, \quad x \neq 1.$$

The diagram shows part of the graph of f.



(a) Write down $f(0)$. [1]

(b) Write down the equation of the horizontal asymptote. [1]

The function g is defined by $g(x) = -f(x)$ for $x \leq 0$.

(c) Find the range of g. [3]

8. Maximum mark: 14

The function f is defined by $f(x) = \log_5\left(\frac{x}{25}\right)$ for $x > 0$.

(a) Find the value of

(i) $f(125)$; [2]

(ii) $f(25)$. [1]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

For $x > 0$, the graph of $y = f(5x^3)$ can be obtained by stretching and translating the graph of $y = \log_5 x$.

(d) Describe these two transformations, specifying the order. Justify your answer algebraically. [6]

9. Maximum mark: 14

The function f is defined by

$$f(x) = \log_2\left(\frac{x}{8}\right), \qquad x > 0.$$

(a) Find the value of

(i) $f(32)$; [2]

(ii) $f(8)$. [1]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

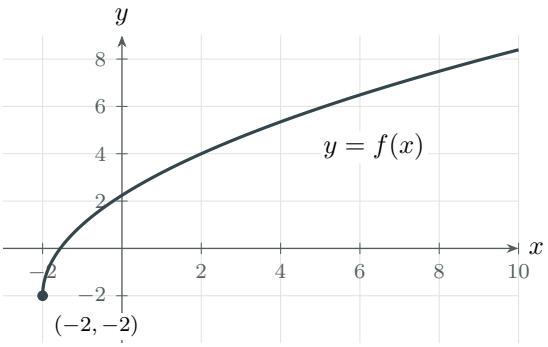
For $x > 0$, the graph of $y = f(2\sqrt{x})$ can be obtained by stretching and translating the graph of $y = \log_2 x$.

(d) Describe these two transformations, specifying the order. Justify your answer algebraically. [6]

10. Maximum mark: 14

The diagram shows part of the graph of the function

$$f(x) = 3\sqrt{x+2} - 2, \qquad x \geq -2.$$



(a) Describe a sequence of transformations that takes the graph of $y = \sqrt{x}$, $x \geq 0$, to the graph of $y = f(x)$. [3]

(b) State the range of f. [1]

(c) Find an expression for $f^{-1}(x)$, stating its domain. [5]

(d) The graphs of $y = f(x)$ and $y = f^{-1}(x)$ meet at a point (x, y).

(i) Show that $(x - y)(x + y + 13) = 0$. [2]

(ii) Hence find the coordinates of all points of intersection of the two graphs. [3]

HL-exclusive challenge Questions 11–14 | 56 marks

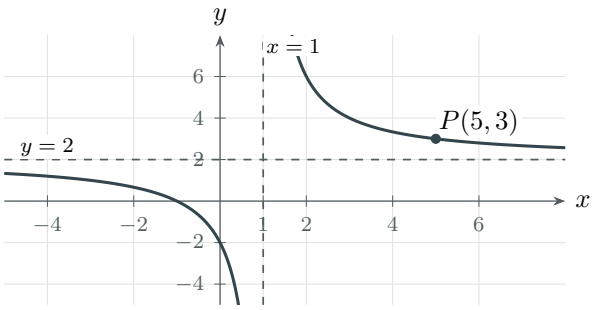
These questions use the same transformation ideas with more demanding algebra and reasoning.

11. Maximum mark: 14

The function f is defined by

$$f(x) = \frac{4}{x-1} + 2, \qquad x \in \mathbb{R}, \quad x \neq 1.$$

The diagram shows part of the graph of f, including the point $P(5, 3)$.



A function g is defined by $g(x) = a\,f(bx) + c$, where $a, b, c \in \mathbb{R}$ and $a, b \neq 0$. Its graph has asymptotes $x = -2$ and $y = -1$, and passes through $(-4, -5)$.

(a) Find the values of a, b and c. [6]

(b) Describe a sequence of transformations that takes the graph of f to the graph of g. State the order. [4]

(c) State the domain and range of g. [2]

(d) Find the coordinates of the image of $P(5, 3)$ under these transformations. [2]

HL-exclusive challenge continued

12. Maximum mark: 14

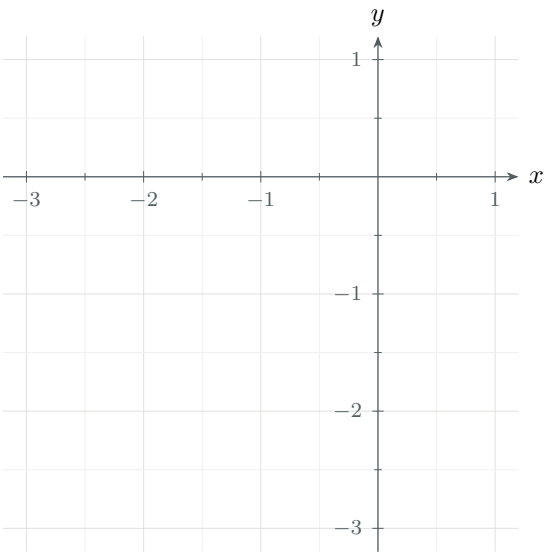
The function h is defined by

$$h(x) = -1 - \sqrt{x+2}, \qquad x \geq -2.$$

(a) Find an expression for $h^{-1}(x)$, stating its domain. [3]

(b) Find the exact coordinates of all points where the graphs of h and h^{-1} intersect. [6]

(c) On the axes below, sketch both graphs. Label their endpoints and all their points of intersection. Also draw the line $y = x$. [3]



(d) A student solves $h(x) = x$ and concludes that the two graphs have only one point of intersection. Explain why this conclusion is incomplete. [2]

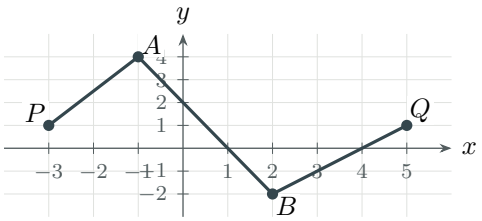
HL-exclusive challenge continued

13. Maximum mark: 14

The graph of f consists of straight-line segments joining the points

$$P(-3,1), \qquad A(-1,4), \qquad B(2,-2), \qquad Q(5,1),$$

in that order. Both endpoints are included.

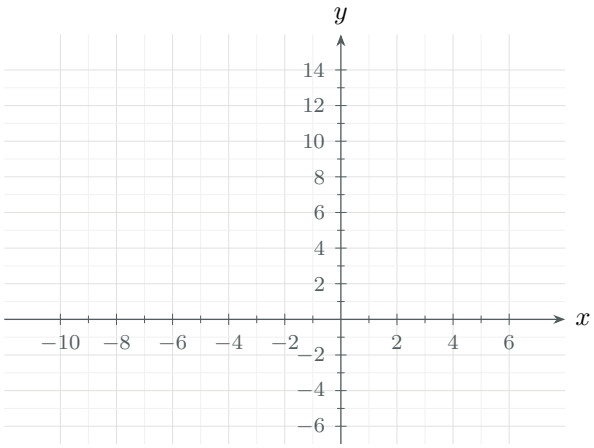


A function g is defined by $g(x) = a\,f(bx) + c$, where $a < 0$, $b < 0$ and $c \in \mathbb{R}$. Its domain is $[-10, 6]$ and its range is $[-5, 13]$.

(a) Find the values of a, b and c. [5]

(b) Describe a sequence of stretches, reflections and translations that takes the graph of f to the graph of g. Specify the order. [5]

(c) Sketch g on the axes below. Label the image of each vertex with its coordinates. [4]



HL-exclusive challenge continued

14. Maximum mark: 14

The function f is defined by $f(x) = \log_2 x$ for $x > 0$.

A function g is defined by $g(x) = a\,f(x) + b$, where $a, b \in \mathbb{R}$ and $a \neq 0$. Its graph passes through $(1, 6)$ and $(8, -3)$.

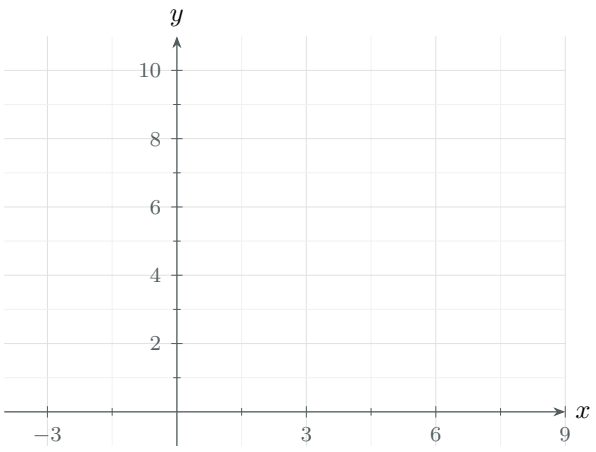
(a) Find the values of a and b. [2]

(b) Describe a sequence of transformations that takes the graph of f to the graph of g. Specify the order. [3]

(c) Find an expression for $g^{-1}(x)$, stating its domain and range. [4]

(d) Using only stretches and reflections, describe a sequence of transformations that takes the graph of $y = 2^x$ to the graph of $y = g^{-1}(x)$. [3]

(e) Sketch the graph of g^{-1} on the axes below. Label its intercept with the y-axis and indicate its horizontal asymptote. [2]



End of student questions. Worked solutions begin on the next page.

Answer key and worked solutions

Suggested marks are shown for guidance. Equivalent correct methods and valid transformation orders should receive credit.

Questions 1–3 | Shared SL & HL questions

1. Graph matching *4 marks*

Function	Graph	Transformation
$f(x-1)$	C	Translation 1 unit to the right.
$f(x)+1$	A	Translation 1 unit upwards.
$f(-x)$	B	Reflection in the y-axis.
$f(2x)$	F	Horizontal stretch with scale factor $\frac{1}{2}$.

Award one mark for each correct letter.

2. Rational function and range *5 marks*

(a) $f(0) = \dfrac{-4}{2} = \boxed{-2}$. [1]

(b) The horizontal asymptote is $\boxed{y = 5}$. [1]

(c) Rewrite the function before restricting its domain:

$$f(x) = 5 - \frac{14}{x+2}, \qquad g(x) = -5 + \frac{14}{x+2}.$$

For $x \geq 0$, $x+2 \geq 2$, so $0 < \dfrac{14}{x+2} \leq 7$. Therefore

$$\boxed{-5 < g(x) \leq 2}, \qquad \text{or} \qquad \boxed{(-5, 2]}.$$

The value 2 is attained at $x = 0$; -5 is approached but never attained. [3]

3. Base-three logarithm *14 marks*

(a(i)) $f(9) = \log_3(81) = \boxed{4}$. [2]

(a(ii)) $f(\frac{1}{9}) = \log_3(1) = \boxed{0}$. [1]

(b) Let $y = \log_3(9x)$. Then $3^y = 9x$, so $x = 3^y/9$. Interchanging x and y gives

$$\boxed{f^{-1}(x) = \frac{3^x}{9} = 3^{x-2}} \qquad (x \in \mathbb{R}).$$

Suggested marks: exponential form [1], rearrangement [1], interchange [1], correct inverse [1].

(c) $f^{-1}(0) = 3^{-2} = \boxed{\frac{1}{9}}$. [1]

(d) For $x > 0$,

$$f(3x^2) = \log_3(27x^2) = 3 + 2\log_3 x.$$

Start with $y = \log_3 x$. Apply a **vertical stretch of scale factor** 2, then a **translation by** $\binom{0}{3}$. The ordered functions are

$$\log_3 x \longrightarrow 2\log_3 x \longrightarrow 2\log_3 x + 3.$$

Suggested marks: composition [1], logarithm laws and simplification [2], stretch [1], translation [1], correct order [1]. Translating up $\frac{3}{2}$ before stretching vertically by 2 is also valid.

Answer key and worked solutions continued

Questions 4–5 | Shared SL & HL questions

4. Base-two logarithm *14 marks*

(a(i)) $f(\frac{1}{2}) = \log_2(8) = \boxed{3}$. [2]

(a(ii)) $f(\frac{1}{16}) = \log_2(1) = \boxed{0}$. [1]

(b) $y = \log_2(16x) \Rightarrow 2^y = 16x \Rightarrow x = 2^y/16$. Interchange the variables:

$$\boxed{f^{-1}(x) = \frac{2^x}{16} = 2^{x-4}} \qquad (x \in \mathbb{R}).$$

Suggested marks: exponential form [1], rearrangement [1], interchange [1], correct inverse [1].

(c) $f^{-1}(0) = 2^{-4} = \boxed{\frac{1}{16}}$. [1]

(d) For $x > 0$,

$$f(2x^3) = \log_2(32x^3) = 5 + 3\log_2 x.$$

Apply a **vertical stretch of scale factor** 3, then a **translation by** $\binom{0}{5}$. The ordered functions are

$$\log_2 x \longrightarrow 3\log_2 x \longrightarrow 3\log_2 x + 5.$$

Suggested marks: composition [1], logarithm laws and simplification [2], stretch [1], translation [1], correct order [1]. Translating up $\frac{5}{3}$ before stretching vertically by 3 is also valid.

5. Reflected square-root function *14 marks*

(a) Reflect in the x-axis, translate 4 units left, then 2 units down:

$$\sqrt{x} \longrightarrow -\sqrt{x} \longrightarrow -\sqrt{x+4} \longrightarrow -\sqrt{x+4} - 2.$$

Equivalently, reflect in the x-axis and then translate by $\binom{-4}{-2}$. [3]

(b) Since $\sqrt{x+4} \geq 0$, the range is $\boxed{(-\infty, -2]}$. [1]

(c) Start with $y = -2 - \sqrt{x+4}$. Then

$$\sqrt{x+4} = -y - 2, \qquad x + 4 = (y+2)^2, \qquad x = (y+2)^2 - 4.$$

Thus $\boxed{f^{-1}(x) = (x+2)^2 - 4, \quad x \leq -2}$. The inverse domain is the range of f.

Suggested marks: isolate root [1], square and rearrange [2], interchange [1], domain [1].

(d(i)) At an intersection, both graph equations hold. Since $x, y \leq -2$,

$$x = y^2 + 4y, \qquad y = x^2 + 4x.$$

Subtracting and rearranging gives $\boxed{(x-y)(x+y+5) = 0}$. [2]

(d(ii)) If $x = y$, then $x = x^2 + 4x$, so $x(x+3) = 0$. The root $x = 0$ is outside the domain of f^{-1}; $x = -3$ gives $\boxed{(-3, -3)}$.

If $x + y = -5$, then $x, y \leq -2$ force $-3 \leq x \leq -2$. Substitution gives

$$x^2 + 5x + 5 = 0 \iff (x+2)(x+3) = 1.$$

On this interval the left side is non-positive, so this case is impossible. Therefore $\boxed{(-3, -3)}$ is the **only** intersection. [3]

Answer key and worked solutions continued

Questions 6–8 | Shared SL & HL questions

6. Graph matching *4 marks*

Function	Graph	Transformation
$2f(x)$	E	Vertical stretch with scale factor 2.
$f(x/2)$	A	Horizontal stretch with scale factor 2.
$-f(x)$	B	Reflection in the x-axis.
$f(x-2)$	D	Translation 2 units to the right.

Award one mark for each correct letter.

7. Rational function and range *5 marks*

(a) $f(0) = \dfrac{9}{-1} = \boxed{-9}$. [1]

(b) The horizontal asymptote is $\boxed{y = -3}$. [1]

(c) Rewrite the functions as

$$f(x) = -3 + \frac{6}{x-1}, \qquad g(x) = 3 + \frac{6}{1-x}.$$

For $x \le 0$, $1 - x \ge 1$, so $0 < \dfrac{6}{1-x} \le 6$. Therefore

$$\boxed{3 < g(x) \le 9}, \qquad \text{or} \qquad \boxed{(3, 9]}.$$

The value 9 is attained at $x = 0$; 3 is approached as $x \to -\infty$ but never attained. [3]

8. Base-five logarithm *14 marks*

(a(i)) $f(125) = \log_5(5) = \boxed{1}$. [2]

(a(ii)) $f(25) = \log_5(1) = \boxed{0}$. [1]

(b) Let $y = \log_5(x/25)$. Then $5^y = x/25$, so $x = 25 \cdot 5^y$. Interchanging the variables gives

$$\boxed{f^{-1}(x) = 25 \cdot 5^x = 5^{x+2}} \qquad (x \in \mathbb{R}).$$

Suggested marks: exponential form [1], rearrangement [1], interchange [1], correct inverse [1].

(c) $f^{-1}(0) = 5^2 = \boxed{25}$. [1]

(d) For $x > 0$,

$$f(5x^3) = \log_5\left(\frac{5x^3}{25}\right) = \log_5\left(\frac{x^3}{5}\right) = 3\log_5 x - 1.$$

Apply a **vertical stretch of scale factor** 3, then a **translation by** $\binom{0}{-1}$. The ordered functions are

$$\log_5 x \;\longrightarrow\; 3\log_5 x \;\longrightarrow\; 3\log_5 x - 1.$$

Suggested marks: composition [1], logarithm laws and simplification [2], stretch [1], translation [1], valid order [1]. Translating down $\frac{1}{3}$ before stretching vertically by 3 is also valid.

Answer key and worked solutions continued

Questions 9–10 | Shared SL & HL questions

9. A stretch with scale factor $\frac{1}{2}$ *14 marks*

(a(i)) $f(32) = \log_2(4) = \boxed{2}$. [2]

(a(ii)) $f(8) = \log_2(1) = \boxed{0}$. [1]

(b) $y = \log_2(x/8) \Rightarrow 2^y = x/8 \Rightarrow x = 8 \cdot 2^y$. Interchanging the variables gives

$$\boxed{f^{-1}(x) = 8 \cdot 2^x = 2^{x+3}} \qquad (x \in \mathbb{R}).$$

Suggested marks: exponential form [1], rearrangement [1], interchange [1], correct inverse [1].

(c) $f^{-1}(0) = 2^3 = \boxed{8}$. [1]

(d) For $x > 0$,

$$f(2\sqrt{x}) = \log_2\left(\frac{2\sqrt{x}}{8}\right) = \log_2\left(\frac{\sqrt{x}}{4}\right) = \frac{1}{2}\log_2 x - 2.$$

Apply a **vertical stretch of scale factor** $\frac{1}{2}$, then a **translation by** $\binom{0}{-2}$:

$$\log_2 x \longrightarrow \frac{1}{2}\log_2 x \longrightarrow \frac{1}{2}\log_2 x - 2.$$

Suggested marks: composition [1], logarithm laws and simplification [2], stretch [1], translation [1], valid order [1]. Translating down 4 before stretching vertically by $\frac{1}{2}$ is also valid.

10. Increasing square-root function *14 marks*

(a) Stretch vertically by scale factor 3, then translate 2 units left and 2 units down:

$$\sqrt{x} \longrightarrow 3\sqrt{x} \longrightarrow 3\sqrt{x+2} \longrightarrow 3\sqrt{x+2} - 2.$$

Equivalently, stretch vertically by 3 and then translate by $\binom{-2}{-2}$. [3]

(b) Since $\sqrt{x+2} \geq 0$, the range is $\boxed{[-2, \infty)}$. [1]

(c) Starting with $y = 3\sqrt{x+2} - 2$,

$$\frac{y+2}{3} = \sqrt{x+2} \quad \Rightarrow \quad x = \frac{(y+2)^2}{9} - 2.$$

Thus $\boxed{f^{-1}(x) = \dfrac{(x+2)^2}{9} - 2, \quad x \geq -2}$. The inverse domain is the range of f.

Suggested marks: isolate root [1], square and rearrange [2], interchange [1], domain [1].

(d(i)) Both graph equations hold at a common point:

$$(y+2)^2 = 9(x+2), \qquad (x+2)^2 = 9(y+2).$$

Subtracting gives $(x-y)(x+y+4) = -9(x-y)$, so $\boxed{(x-y)(x+y+13) = 0}$. [2]

(d(ii)) Both coordinates satisfy $x, y \geq -2$, so $x + y + 13 \geq 9 > 0$. Hence $x = y$. Substitution gives

$$(x+2)^2 = 9(x+2) \quad \Rightarrow \quad (x+2)(x-7) = 0.$$

Both roots are valid. The intersections are $\boxed{(-2,-2)}$ and $\boxed{(7,7)}$. [3]

HL-exclusive challenge | Worked solutions

Question 11 | Recovering transformation parameters

(a) The vertical asymptote of f occurs when its input is 1. For g, it therefore occurs where $bx = 1$. Since its equation is $x = -2$,

$$-2b = 1 \quad \Rightarrow \quad \boxed{b = -\frac{1}{2}}. \qquad [2]$$

The horizontal asymptote of f is $y = 2$, so that of g is $y = 2a + c$. Hence

$$2a + c = -1. \qquad [1]$$

At the given point $(-4, -5)$, the input to f is $b(-4) = 2$. Since $f(2) = 6$,

$$g(-4) = a\,f(2) + c = 6a + c = -5. \qquad [1]$$

Subtracting $2a + c = -1$ gives $4a = -4$, hence

$$\boxed{a = -1, \qquad b = -\frac{1}{2}, \qquad c = 1}. \qquad [2]$$

The transformed function is

$$g(x) = -f\left(-\frac{x}{2}\right) + 1 = -\left(\frac{4}{-x/2 - 1} + 2\right) + 1 = \boxed{\frac{8}{x + 2} - 1}.$$

(b) One valid ordered sequence is:

1. Horizontal stretch with scale factor 2.
2. Reflection in the y-axis.
3. Reflection in the x-axis.
4. Translation by $\binom{0}{1}$.

In function notation,

$$f(x) \to f\left(\frac{x}{2}\right) \to f\left(-\frac{x}{2}\right) \to -f\left(-\frac{x}{2}\right) \to -f\left(-\frac{x}{2}\right) + 1.$$

Award one mark per correctly described transformation in a valid order. [4]

(c) Domain: $\boxed{\mathbb{R} \setminus \{-2\}}$; range: $\boxed{\mathbb{R} \setminus \{-1\}}$. [2]

(d) For an original point (u, v), solve $bx = u$ and use $y = av + c$:

$$(u, v) \longmapsto \left(\frac{u}{b}, av + c\right) = (-2u, 1 - v).$$

Therefore $P(5, 3) \longmapsto \boxed{(-10, -2)}$. [2]

Point-mapping check. Horizontal changes affect the input coordinate: solve the new input equal to the old x-coordinate. Vertical changes act directly on the old y-coordinate. Here $g(-10) = 8/(-8) - 1 = -2$, as required.

HL-exclusive challenge | Worked solutions continued

Question 12 | A function and its inverse can meet off $y = x$

(a) $y = -1 - \sqrt{x+2}$ gives $(y+1)^2 = x+2$. Therefore

$$\boxed{h^{-1}(x) = (x+1)^2 - 2, \qquad x \le -1}.$$

Award one mark for rearrangement, one for the inverse, and one for the domain. [3]

(b) At any common point, $-2 \le x, y \le -1$, and

$$x = (y+1)^2 - 2 = y^2 + 2y - 1, \qquad y = x^2 + 2x - 1.$$

Subtracting gives $(x-y)(x+y+3) = 0$. We must consider **both** factors.

Case 1: $x = y$**.** Then $x = x^2 + 2x - 1$, giving $x^2 + x - 1 = 0$ and

$$x = \frac{-1 \pm \sqrt{5}}{2}.$$

Only the negative root is in $[-2, -1]$, so one intersection is

$$\boxed{\left(-\frac{1+\sqrt{5}}{2}, -\frac{1+\sqrt{5}}{2}\right)}.$$

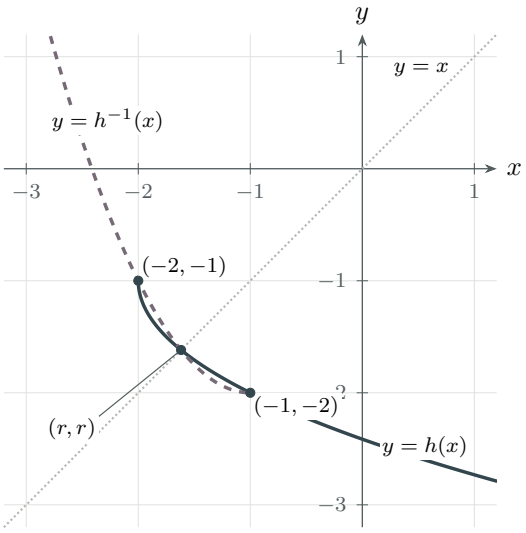
Case 2: $x + y = -3$**.** Substitute $y = -3 - x$:

$$-3 - x = x^2 + 2x - 1 \quad \Rightarrow \quad x^2 + 3x + 2 = 0 \quad \Rightarrow \quad (x+1)(x+2) = 0.$$

Both roots are valid and give $\boxed{(-1, -2)}$ and $\boxed{(-2, -1)}$.

Suggested marks: two equations and factorization [2], diagonal solution with domain check [2], off-diagonal pair [2].

(c) The solid curve is h; the dashed curve is h^{-1}. Each endpoint is also an intersection. Put $r = -\dfrac{1+\sqrt{5}}{2}$.



Award one mark for each correct curve with its endpoint, and one for $y = x$ and all three labelled intersections. [3]

(d) Solving $h(x) = x$ finds only the common points on $y = x$. Here $h(-2) = -1$ and $h(-1) = -2$, so $(-2, -1)$ and $(-1, -2)$ also lie on both graphs. They are reflections of each other in $y = x$. [2]

HL-exclusive challenge | Worked solutions continued

Question 13 | Domain, range and point mapping

(a) From the original graph, the domain of f is $[-3, 5]$ and its range is $[-2, 4]$. For g to be defined, $-3 \le bx \le 5$. Since $b < 0$,

$$\frac{5}{b} \le x \le -\frac{3}{b}.$$

Matching the endpoints of $[-10, 6]$ gives $5/b = -10$, so $\boxed{b = -\frac{1}{2}}$. [2]

Since $a < 0$, the maximum of f gives the minimum of g, and the minimum of f gives the maximum of g. Hence

$$4a + c = -5, \qquad -2a + c = 13.$$

Subtracting gives $6a = -18$, so $a = -3$ and then $c = 7$. Therefore

$$\boxed{a = -3, \qquad b = -\frac{1}{2}, \qquad c = 7}.$$

Suggested remaining marks: correct range equations [1], $a = -3$ [1], $c = 7$ [1].

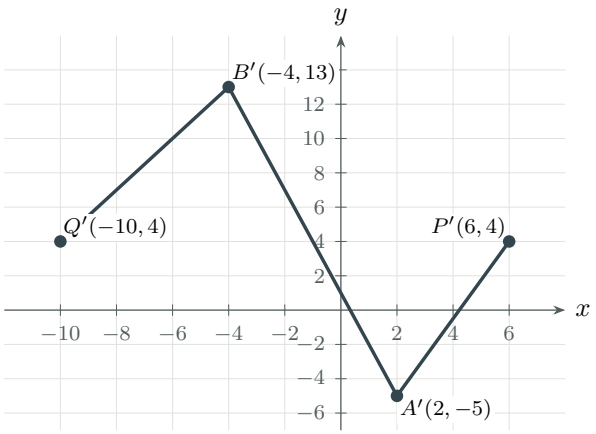
(b) One valid ordered sequence is:

1. Horizontal stretch with scale factor 2.
2. Reflection in the y-axis.
3. Vertical stretch with scale factor 3.
4. Reflection in the x-axis.
5. Translation by $\binom{0}{7}$.

This produces $g(x) = -3f(-x/2) + 7$. Award one mark per correct transformation in a valid order. [5]

(c) The point mapping is $(u, v) \longmapsto (-2u,\ 7 - 3v)$.

Original point	Image
$P(-3, 1)$	$P'(6, 4)$
$A(-1, 4)$	$A'(2, -5)$
$B(2, -2)$	$B'(-4, 13)$
$Q(5, 1)$	$Q'(-10, 4)$



Join the four images by straight segments, with filled endpoints. Award one mark per correctly plotted and labelled image. [4]

HL-exclusive challenge | Worked solutions continued

Question 14 | A logarithm and its transformed inverse

(a) The point $(1, 6)$ gives $a \log_2 1 + b = 6$, so $\boxed{b = 6}$. The point $(8, -3)$ then gives

$$3a + 6 = -3 \quad \Longrightarrow \quad \boxed{a = -3}.$$

Thus $g(x) = -3 \log_2 x + 6$. [2]

(b) Apply a vertical stretch with scale factor 3, a reflection in the x-axis, then a translation by $\binom{0}{6}$:

$$\log_2 x \longrightarrow 3 \log_2 x \longrightarrow -3 \log_2 x \longrightarrow -3 \log_2 x + 6.$$

The stretch and reflection may be interchanged. [3]

(c) Start with $y = -3 \log_2 x + 6$. Then

$$\log_2 x = \frac{6 - y}{3} \quad \Longrightarrow \quad x = 2^{(6-y)/3}.$$

Interchanging x and y gives

$$\boxed{g^{-1}(x) = 2^{(6-x)/3} = 4 \cdot 2^{-x/3}}.$$

Its domain is $\boxed{\mathbb{R}}$ and its range is $\boxed{(0, \infty)}$.

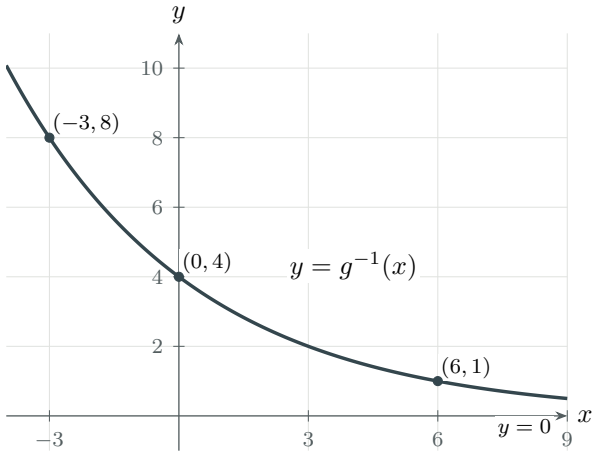
Suggested marks: exponential rearrangement [1], inverse [1], domain [1], range [1].

(d) Starting with $y = 2^x$, apply a horizontal stretch with scale factor 3, a reflection in the y-axis, then a vertical stretch with scale factor 4:

$$2^x \longrightarrow 2^{x/3} \longrightarrow 2^{-x/3} \longrightarrow 4 \cdot 2^{-x/3}.$$

Award one mark per correctly described transformation. These three transformations may be performed in any order. [3]

(e) The graph is a decreasing exponential curve. Its y-intercept is $\boxed{(0, 4)}$, and its horizontal asymptote is $\boxed{y = 0}$. It stays above the x-axis.



Suggested marks: correct decreasing curve with labelled intercept [1]; horizontal asymptote and approach from above [1].