



Functions, Domain, Range and Inverses

IB Mathematics AA SL & HL · Section 2.2
50 shared questions + 10 HL challenges · Worksheet

Learning goal	Use functions, domain and range, models and inverse representations.
Syllabus link	Core: AA SL 2.2, also studied at HL. Graphing, inverse-rule and parent-function connections follow the handout; HL extensions link to AHL 2.14.
Structure	Q1–15 short response; Q16–30 extended; Q31–50 IB exam-style; Q51–60 HL-exclusive challenge; answer key and solution outlines.
Calculator use	GDC required for Q27–30, Q48–50 and Q60, each clearly labelled. All other questions are non-calculator.

Work on separate paper unless axes are supplied. Show your reasoning. Give exact values in non-calculator questions and numerical GDC answers to **three significant figures**, unless exact values are clear. Use interval or set notation correctly; include units in models. Each inverse is understood as a function on the original range.

1. Short response

Shared SL / HL

1. Each relation has complete domain $\{1, 2, 3\}$. Decide whether it defines a function. Give a reason.

(a) $\{(1, 4), (2, 4), (3, 8)\}$

(b) $\{(1, 4), (1, 6), (2, 5), (3, 8)\}$.

2. Let $f(x) = 2x^2 - x + 1$. Find (a) $f(-3)$ and (b) $f(a - 1)$ in expanded form.

3. Let $g(x) = (x + 2)^2 - 5$, for $x \in \mathbb{R}$. Find all inputs for which $g(x) = 4$.

4. The table gives every input and output of h .

x	-3	0	2	5
$h(x)$	4	-1	4	7

State (a) $h(0)$, (b) all solutions of $h(x) = 4$, and (c) the range of h .

5. Write in interval notation: (a) $-4 \leq x < 2$; (b) $x > 3$. Then describe $(-\infty, 1] \cup (5, \infty)$ using inequalities.

6. State the largest real domain of $p(x) = \frac{2x + 1}{x^2 - 9}$.

7. State the largest real domain and the range of $q(x) = \sqrt{12 - 3x}$.

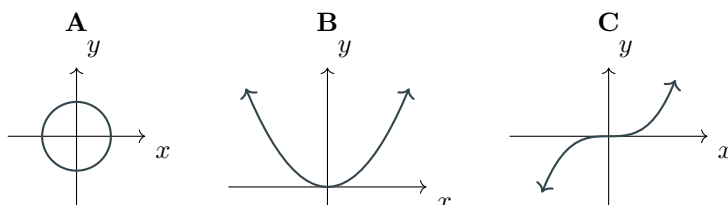
8. State the largest real domain of $r(x) = \frac{1}{\sqrt{x - 6}}$. Explain why $x = 6$ is excluded.



9. The function $s(x) = x^2 - 2$ has domain $\{-2, -1, 1, 3\}$. State its range. Does s have an inverse function on its range? Explain.

10. Both f and g use the rule x^2 . The domain of f is $\mathbb{R}$ and the domain of g is $(2, 5]$. State each range.

11. For each complete graph below, decide (i) whether it represents y as a function of x , and (ii) if so, whether that function is one-to-one.



12. A one-to-one function f has domain $[-3, 4)$, range $(1, 9]$ and $f(2) = 7$. State $f^{-1}(7)$, the domain and range of f^{-1} , and the corresponding point on its graph.

13. For $f(x) = 5x - 2$, find $f^{-1}(8)$ by undoing the rule. Also find $\frac{1}{f(8)}$.

14. The complete domain of g is $\{-1, 2, 4\}$, with $g(-1) = 6$, $g(2) = 0$ and $g(4) = 3$. State $g^{-1}(0)$ and the domain of g^{-1} .

15. A workshop charges a booking fee of €12 and €7 per participant. A booking may contain 0 to 18 participants. Write a cost function $C(n)$ and state its domain.

2. Extended response

Shared SL / HL

16. Let $f(x) = \frac{x^2 - 16}{x - 4}$, using its largest real domain.

- (a) State the domain and simplify the expression for allowed inputs.
- (b) Find the range. Explain why simplification does not make $f(4)$ defined.
- (c) Is $(4, 8)$ on the graph of f ?

17. Let $g(x) = \frac{\sqrt{x+5}}{x-2}$.

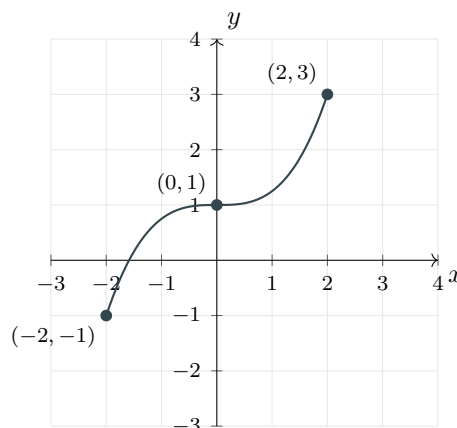
- (a) Find the largest real domain, showing both restrictions.
- (b) Find $g(-5)$ and $g(4)$ exactly.
- (c) A student writes the domain as $[-5, \infty)$. Explain what is missing.

18. Let $h(x) = 6 - (x + 1)^2$, for $-4 < x \leq 1$.

- (a) State the domain and find the range.
- (b) Solve $h(x) = 2$, keeping only allowed inputs.
- (c) Explain why -3 is not in the range but 6 is.

19. The complete graph of a one-to-one function f is shown. Both endpoints are included.

- State the domain and range of f .
- Find $f^{-1}(1)$.
- On the supplied grid, sketch $y = f^{-1}(x)$. Label the three reflected points and the mirror line.
- State the domain and range of f^{-1} .



20. A liquid cools from 86°C at a constant rate of 3°C per minute until it reaches 20°C . Let $T(t)$ be its temperature t minutes after cooling begins.

- Write a model for $T(t)$ and state its domain and range during cooling.
- Find $T(12)$ and interpret it.
- Find $T^{-1}(35)$ and interpret it in context.

21. A club charges €25 membership plus €6 for each session attended, with a maximum of 14 sessions.

- Define $C(n)$ and give its domain and range as discrete sets.
- Can a total charge of €95 occur? Justify your answer.
- A member pays €79. How many sessions did they attend?

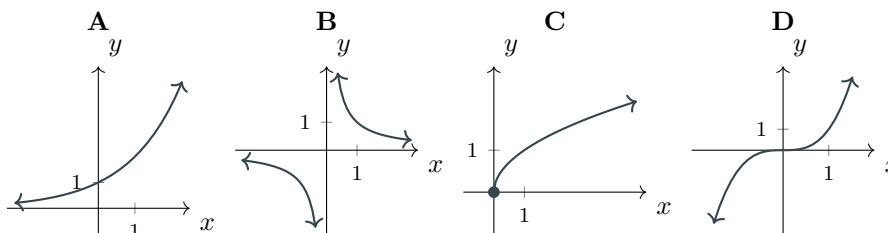
22. Inverse rule: 2.5

Let $f(x) = \frac{3x-4}{2}$, for $-2 \leq x \leq 6$.

- Find the range of f .
- Find $f^{-1}(x)$ and state its domain and range.
- Explain the order of the operations used to undo f .

23. Parent-function review

Match graphs A–D to $y = x^3$, $y = \sqrt{x}$, $y = 2^x$ and $y = 1/x$. For each, state its domain and range. State which graph is unchanged by reflection in $y = x$.



**24.** Range and translations: 2.11

Find the domain and range of each function. Explain what happens to the range in each case.

- (a) $g(x) = 2^x - 4$.
- (b) $h(x) = \sqrt{x} + 3$.
- (c) $p(x) = f(x) - 5$, where f has domain $[-2, 6]$ and range $(-1, 7]$.

25. A student claims: “For $f(x) = (x - 2)^2$ on $[-1, 5]$, both endpoint values are 9, so the range is $\{9\}$.”

- (a) Explain the error and give the correct range.
- (b) Give two different inputs with output 4.
- (c) Explain why f has no inverse function on its range.

26. Logarithmic connection

For $p(x) = \ln(5 - 2x)$, a student writes $5 - 2x \geq 0$, hence $x \geq \frac{5}{2}$.

- (a) Identify both errors and state the correct domain.
- (b) Find $p(2)$ exactly.
- (c) Explain why $p(\frac{5}{2})$ is undefined.

27. CALCULATOR REQUIRED (GDC)

Let $f(x) = x^3 - 2x^2 - 3x + 2$, for $-1.5 \leq x \leq 3.2$.

- (a) Use your GDC to find the minimum and maximum values on this domain, and hence the range.
- (b) Find every solution of $f(x) = 1$ in the domain.
- (c) Does f have an inverse function on its range? Explain using your graph.

28. CALCULATOR REQUIRED (GDC)

A culture is modelled by $N(t) = 240(1.18)^t$, for $0 \leq t \leq 10$, where t is time in hours and N is an estimated population.

- (a) Find $N(6)$ and state the range of this continuous model.
- (b) Find the time when $N(t) = 900$.
- (c) Explain the meaning of $N^{-1}(900)$, including units.

29. CALCULATOR REQUIRED (GDC)

Let $g(x) = x + e^x$, for $-2 \leq x \leq 2$. Both terms increase as x increases.

- (a) State why g is one-to-one. Find its range.
- (b) Find $g^{-1}(4)$ numerically.
- (c) Write down a point on $y = g^{-1}(x)$ corresponding to your answer.

**30. CALCULATOR REQUIRED (GDC)**

Let $h(x) = 2 + \frac{1}{(x-1)^2}$, on its largest real domain.

- (a) Graph h in two different viewing windows. State its exact domain and range, supported by algebra.
- (b) Find both solutions of $h(x) = 2.3$ to three significant figures.
- (c) Explain why neither the edge of a viewing window nor the apparent joining of pixels determines an endpoint of the domain or range.

3. IB exam-style questions

Shared SL / HL

Q31–47: Paper 1 style (non-calculator). Q48–50: Paper 2 style (GDC required). These are original practice questions, not official IB questions.

31. [Maximum mark: 7]

A function is defined by $f(x) = ax + b$, for $x \in \mathbb{R}$. Given that $f(-2) = -7$ and $f(3) = 13$:

- (a) Find a and b . [3 marks]
- (b) Find $f(\frac{1}{2})$. [1 mark]
- (c) Find $f^{-1}(9)$. [2 marks]
- (d) State why f has an inverse function. [1 mark]

32. [Maximum mark: 6]

Let $g(x) = \sqrt{18 - 2x}$, on its largest real domain.

- (a) State the domain and range. [3 marks]
- (b) Solve $g(x) = 5$. [2 marks]
- (c) Write your answer to part (b) using inverse notation. [1 mark]

33. [Maximum mark: 6]

The table gives every input of a function f .

x	-4	-1	2	6
$f(x)$	2	k	5	8

- (a) State all values of k for which f is one-to-one. [2 marks]
- (b) For $k = -3$, state the domain and range of f^{-1} . [2 marks]
- (c) For $k = -3$, find $f^{-1}(5) + f^{-1}(-3)$. [2 marks]

34. [Maximum mark: 7]

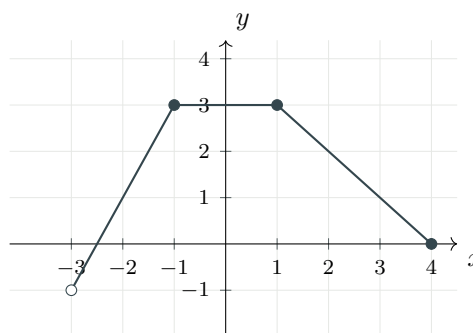
A printer charges €42 for setup plus €0.08 per page. One order may contain 0 to 600 pages.

- (a) Write $C(n)$ for the total cost and state its domain. [2 marks]
- (b) State the range as a discrete set. [2 marks]
- (c) An order costs €66. Find the number of pages. [2 marks]
- (d) Explain why €106 cannot be the cost of one order. [1 mark]

35. [Maximum mark: 8]

The complete graph of f consists of three straight segments. An open circle means an excluded endpoint.

- State the domain and range. [2 marks]
- Find $f(0)$. [1 mark]
- Solve $f(x) = 2$. [2 marks]
- Solve $f(x) = 0$. [2 marks]
- Explain why f^{-1} is not a function. [1 mark]



36. [Maximum mark: 7]

Let $f(x) = (x - 1)^2$, for $-2 < x \leq 4$.

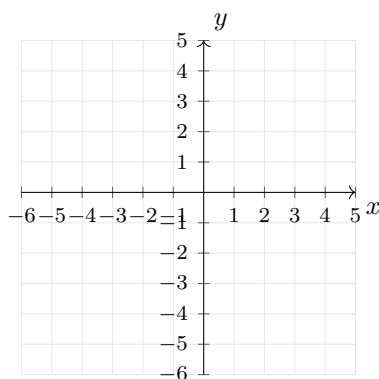
- Find the range of f . [3 marks]
- Solve $f(x) = 9$, taking account of the domain. [2 marks]
- Explain why the open endpoint at $x = -2$ does not exclude 9 from the range. [2 marks]

37. [Maximum mark: 8]

Inverse rule: 2.5

Let $f(x) = 2x - 3$, for $-1 \leq x \leq 3$.

- Find the range of f . [2 marks]
- Find $f^{-1}(x)$ and state its domain. [3 marks]
- Sketch f and f^{-1} on the supplied equal-scale axes. Label both pairs of endpoints and the line of reflection. [3 marks]



38. [Maximum mark: 7]

Reciprocal and translation connection

Let $g(x) = \frac{3}{x+2} - 1$.

- State the largest real domain. [1 mark]
- Show that $g(x)$ can never equal -1 . [2 marks]
- State the range and both asymptote equations. [3 marks]
- Find $g^{-1}(0)$ by solving an equation. [1 mark]

**39. [Maximum mark: 6]**

Let $p(x) = \frac{\sqrt{7-x}}{x+1}$.

- (a) Find the largest real domain, showing both restrictions. [3 marks]
- (b) Find $p(7)$ and $p(0)$ exactly. [2 marks]
- (c) Explain why $p(-1)$ is undefined. [1 mark]

40. [Maximum mark: 8]

Inverse rule: 2.5

Celsius and Fahrenheit temperatures are related by $F(C) = \frac{9}{5}C + 32$. Consider only $-20 \leq C \leq 50$.

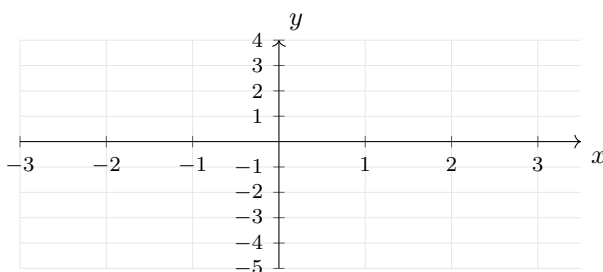
- (a) Find the range of F . [2 marks]
- (b) Find the inverse rule, with its domain and range. [4 marks]
- (c) Interpret $F^{-1}(86)$ in context. [2 marks]

41. [Maximum mark: 7]

Exponential and translation connection

Let $f(x) = 3 - 2^x$, for $x \in \mathbb{R}$.

- (a) State the range and horizontal asymptote. [2 marks]
- (b) Find the y -intercept and solve $f(x) = 1$. [2 marks]
- (c) Sketch $y = f(x)$ on the supplied axes. Label the asymptote, the y -intercept and the point found in part (b). [3 marks]

**42. [Maximum mark: 6]**

Logarithmic connection

Let $g(x) = \ln(2x + 4)$.

- (a) State the largest real domain and the range. [2 marks]
- (b) Find the input giving output 0. [2 marks]
- (c) Find the input giving output $\ln 6$. [2 marks]

**43. [Maximum mark: 7]**

A one-to-one function f has domain $[-4, 3]$ and range $(-2, 6]$. It is known that $f(2) = -1$. Let $g(x) = f(x) + 5$ on the same domain.

- (a) State the domain and range of f^{-1} . [2 marks]
- (b) Find $f^{-1}(-1)$. [1 mark]
- (c) State the domain and range of g . [2 marks]
- (d) Find $g^{-1}(4)$ and state the domain of g^{-1} . [2 marks]

44. [Maximum mark: 8]

A rectangle has perimeter 20 cm. Its width is x cm, and both side lengths must be positive.

- (a) Show that its area is $A(x) = x(10 - x)$ and state a suitable domain. [3 marks]
- (b) Use $A(x) = 25 - (x - 5)^2$ to state the range. [2 marks]
- (c) Find both widths giving area 16 cm^2 . [2 marks]
- (d) Explain why the area does not determine a unique width on this domain. [1 mark]

45. [Maximum mark: 7]

A machine has allowed inputs $\{0, 1, 2, 3\}$. It sends 0 to 5, 1 to 5, 2 to 7 and 3 to 9.

- (a) Explain why this defines a function f , and state its range. [2 marks]
- (b) Find every input for which $f(x) = 5$. [1 mark]
- (c) Explain why reversing all arrows does not define a function. [2 marks]
- (d) One input is removed so that the remaining function is one-to-one. State every possible choice of the removed input. [2 marks]

46. [Maximum mark: 7]

Let $f(x) = \sqrt{9 - x^2}$, using its largest real domain.

- (a) Find the domain and range. [3 marks]
- (b) Explain why the graph is the upper half of the circle $x^2 + y^2 = 9$, including its endpoints. [2 marks]
- (c) Explain why f is a function but is not one-to-one. [2 marks]

47. [Maximum mark: 8]

The function $f(x) = (x - h)^2 + k$ has minimum point $(2, -3)$ and domain $[0, 5]$.

- (a) Write down h and k . [2 marks]
- (b) Find the range of f . [3 marks]
- (c) Solve $f(x) = 1$ on the given domain. [2 marks]
- (d) State whether f^{-1} is a function on the range of f . [1 mark]

**48. CALCULATOR REQUIRED (GDC)****[Maximum mark: 9]**

A ball's height above the ground is modelled by $H(t) = -4.9t^2 + 17t + 1.2$ metres, from its release at $t = 0$ until it first reaches the ground.

- (a) Find when the ball reaches the ground and state the model's domain. [3 marks]
- (b) Find the maximum height and state the range. [3 marks]
- (c) Find both times when $H(t) = 10$. [2 marks]
- (d) Explain why H^{-1} is not a function on the full range. [1 mark]

49. CALCULATOR REQUIRED (GDC)**[Maximum mark: 7]**

Let $f(x) = x^3 + x - 4$, for $x \in \mathbb{R}$. This function is strictly increasing and has range $\mathbb{R}$.

- (a) Explain why f^{-1} exists and state its domain and range. [3 marks]
- (b) Find $f^{-1}(2)$. [2 marks]
- (c) State the corresponding point on each of the graphs of f and f^{-1} . [2 marks]

50. CALCULATOR REQUIRED (GDC)**[Maximum mark: 9]**

Let $p(x) = x^4 - 2.6x^2 + 0.8$, for $-1.8 \leq x \leq 1.2$.

- (a) Find the minimum and maximum outputs, and hence the range. [3 marks]
- (b) Find every zero in this domain. [3 marks]
- (c) State whether p has an inverse function on its range. [1 mark]
- (d) A student uses only $p(-1.8)$ and $p(1.2)$ to find the range. Explain why this method fails. [2 marks]

4. AA HL exclusive challenge**HL only**

The shared SL/HL questions end at Q50. This section combines harder 2.2 reasoning with the labelled AHL 2.14 extensions on restricted inverses, self-inverse functions, and even/odd functions.

51. Let $f(x) = \sqrt{\frac{x-2}{x+3}}$.

- (a) Find its largest real domain.
- (b) By writing $\frac{x-2}{x+3} = 1 - \frac{5}{x+3}$, find the range of f . Explain why the output 1 is excluded.

52. Let $g(x) = \frac{x^2 - 4}{x^2 + 1}$, for $x \in \mathbb{R}$.

- (a) Rewrite $g(x)$ in the form $A - \frac{B}{x^2 + 1}$ and hence find its range.
- (b) Solve $g(x) = 0$ and explain why g is not one-to-one.

53. Let $h_k(x) = \frac{\sqrt{k-x}}{x+2}$, where $k \in \mathbb{R}$.

- (a) State the largest real domain in terms of k .
- (b) Find all values of k for which this domain is exactly $(-\infty, k]$.
- (c) Describe the domain when $k = -2$ and when $k = 3$.

**54.** AHL 2.14

Let $f(x) = (x - 3)^2 - 4$, for $x \leq 3$.

- (a) State its range and explain why f is one-to-one.
- (b) Find $f^{-1}(x)$, including its domain and range. Explain your choice of square-root branch.
- (c) Find $f^{-1}(5)$ and check your answer in the original rule.

55. AHL 2.14

Let $f(x) = ax + b$, for $x \in \mathbb{R}$, with $a \neq 0$. Determine every pair (a, b) for which f is self-inverse, meaning $f^{-1} = f$.

56. AHL 2.14

Let $f(x) = \frac{2x + 3}{x - 2}$.

- (a) State its domain and find its range.
- (b) Find $f^{-1}(x)$ and show that f is self-inverse.
- (c) Find the exact coordinates of the points on the graph which are unchanged by reflection in $y = x$.

57. AHL 2.14

Let $f(x) = \frac{x^2}{x^2 + 1}$ and $g(x) = \frac{x}{x^2 + 1}$, both on $\mathbb{R}$.

- (a) Determine whether each is even, odd or neither.
- (b) Find the range of f .
- (c) Use $(x - 1)^2 \geq 0$ and $(x + 1)^2 \geq 0$ to find the range of g .

58. AHL 2.14

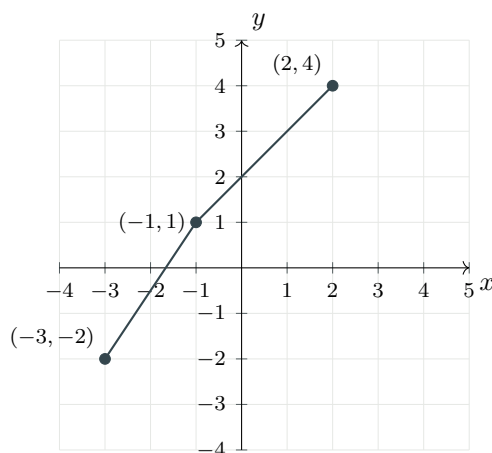
Let $f(x) = \frac{a}{x - b} + c$, where $a \neq 0$. Its largest real domain is $\mathbb{R} \setminus \{4\}$, its range is $\mathbb{R} \setminus \{-2\}$ and its graph passes through $(1, 0)$.

- (a) Find a , b and c .
- (b) Find $f^{-1}(x)$ and state its domain and range.
- (c) State the asymptotes of f^{-1} and the point corresponding to $(1, 0)$.

59. AHL 2.14

The complete graph of f consists of the two segments shown, with all three marked points included.

- Write a piecewise rule for f .
- Find a piecewise rule for f^{-1} , stating the interval for each branch.
- Sketch the inverse on the supplied grid and label its three corresponding points.


60. CALCULATOR REQUIRED (GDC)

AHL 2.14

Let $f(x) = x^3 - 3x + 1$, for $-2 \leq x \leq 2$.

- Find the coordinates of both turning points using your GDC.
- Give the three maximal intervals of this domain on which f is one-to-one, and state the range on each. Here maximal means that the interval cannot be enlarged within $[-2, 2]$ while preserving one-to-one behaviour.
- Let g be the restriction of f to $[1, 2]$. Find $g^{-1}(0.5)$ to three significant figures.
- Explain why one value of $f^{-1}(0.5)$ cannot be assigned on the original domain.



Answer key and solution outlines

Check after attempting

Numerical interval endpoints are rounded; use the unrounded values when checking boundary cases. In sketch answers, solid slate curves are originals and brick curves are inverses; dashed diagonals show $y = x$.

1. Short response

1. (a) Yes: each input has exactly one output. (b) No: input 1 has two outputs, 4 and 6.
2. (a) $2(9) + 3 + 1 = 22$. (b) $2(a-1)^2 - (a-1) + 1 = 2a^2 - 5a + 4$.
3. $(x+2)^2 = 9 \Rightarrow x+2 = \pm 3$, so $x = -5$ or $x = 1$.
4. (a) -1 ; (b) $x = -3, 2$; (c) $\{-1, 4, 7\}$. Repeated outputs appear once in the range set.
5. (a) $[-4, 2)$; (b) $(3, \infty)$; (c) $x \leq 1$ or $x > 5$.
6. $x^2 - 9 = (x-3)(x+3) \neq 0$, so $D = \mathbb{R} \setminus \{-3, 3\}$.
7. $12 - 3x \geq 0 \Rightarrow x \leq 4$. Hence $D = (-\infty, 4]$, $R = [0, \infty)$.
8. The root must be defined and nonzero: $x - 6 > 0$. Thus $D = (6, \infty)$; $x = 6$ gives a zero denominator.
9. Outputs are 2, -1 , -1 , 7, so $R = \{-1, 2, 7\}$. No inverse function: $s(-1) = s(1) = -1$.
10. $R_f = [0, \infty)$; $R_g = (4, 25]$.
11. A: not a function (fails the vertical line test). B: a function, but not one-to-one (fails the horizontal line test). C: a one-to-one function (passes both tests).
12. $f^{-1}(7) = 2$; $D_{f^{-1}} = (1, 9]$, $R_{f^{-1}} = [-3, 4)$; point $(7, 2)$.
13. $5x - 2 = 8 \Rightarrow f^{-1}(8) = 2$, whereas $1/f(8) = 1/38$.
14. $g^{-1}(0) = 2$; $D_{g^{-1}} = \{0, 3, 6\}$.
15. $C(n) = 12 + 7n$, $D = \{0, 1, \dots, 18\}$.

2. Extended response

16. (a) $D = \mathbb{R} \setminus \{4\}$ and $f(x) = x + 4$ for $x \neq 4$. (b) $R = \mathbb{R} \setminus \{8\}$: output 8 requires excluded input 4. The original denominator is still zero at 4. (c) No; the graph has a hole there.
17. (a) $x + 5 \geq 0$ and $x \neq 2$, so $D = [-5, 2) \cup (2, \infty)$. (b) 0 and $3/2$. (c) The input 2 must be removed because it makes the denominator zero.
18. (a) $D = (-4, 1]$, $R = (-3, 6]$. (b) $(x+1)^2 = 4 \Rightarrow x = -3, 1$, both allowed. (c) Output -3 needs $x = -4$ or 2 , neither allowed; $h(-1) = 6$.
19. (a) $D_f = [-2, 2]$, $R_f = [-1, 3]$. (b) $f^{-1}(1) = 0$. (c) Reflect in $y = x$: points $(-1, -2)$, $(1, 0)$, $(3, 2)$; see the graphical answer checks. (d) $D_{f^{-1}} = [-1, 3]$, $R_{f^{-1}} = [-2, 2]$.
20. (a) $T(t) = 86 - 3t$; $86 - 3t = 20 \Rightarrow t = 22$. Domain $[0, 22]$ min, range $[20, 86]$ °C. (b) 50°C after 12 min. (c) $86 - 3t = 35 \Rightarrow T^{-1}(35) = 17$ min: time to reach 35°C .
21. (a) $C(n) = 25 + 6n$; $D = \{0, 1, \dots, 14\}$, $R = \{25, 31, \dots, 109\}$. (b) $n = (95 - 25)/6 = 35/3$ is not an integer, so no. (c) $(79 - 25)/6 = 9$ sessions.
22. (a) $f(-2) = -5$, $f(6) = 7$, so $R_f = [-5, 7]$. (b) $x = (3y - 4)/2 \Rightarrow y = (2x + 4)/3$; $D_{f^{-1}} = [-5, 7]$, $R_{f^{-1}} = [-2, 6]$. (c) Multiply by 2, add 4, then divide by 3.
23. A: 2^x , $D = \mathbb{R}$, $R = (0, \infty)$. B: $1/x$, $D = R = \mathbb{R} \setminus \{0\}$. C: $\sqrt{x}$, $D = R = [0, \infty)$. D: x^3 , $D = R = \mathbb{R}$. Only B is unchanged by reflection in $y = x$.
24. (a) $D = \mathbb{R}$, $R = (-4, \infty)$: subtract 4 from positive outputs. (b) $D = [0, \infty)$, $R = [3, \infty)$. (c) $D = [-2, 6]$, $R = (-6, 2]$. Vertical translations shift outputs and preserve the domain.
25. (a) Endpoint values alone omit interior outputs. The minimum is $f(2) = 0$, so $R = [0, 9]$. (b) $x = 0, 4$. (c) Different inputs share an output, so the inverse relation would have more than one output for some inputs.
26. (a) A logarithm needs $5 - 2x > 0$; division by -2 reverses the inequality. Thus $x < 5/2$. (b) $\ln 1 = 0$. (c) Its argument is 0; $\ln 0$ is undefined.
27. (a) Minimum -4.06 at $x \approx 1.87$; maximum 4.69 at $x = 3.2$. Range approximately $[-4.06, 4.69]$. (b) $x \approx -1.20, 0.286, 2.91$. (c) No: the line $y = 1$ meets the graph three times.
28. (a) $N(6) \approx 648$; $R \approx [240, 1260]$ (upper endpoint $240(1.18)^{10}$). (b) $t = \ln(900/240)/\ln 1.18 \approx 7.99$ h. (c) The time, in hours, when the model predicts 900 organisms; the same 7.99 h.



- 29.** (a) The sum is strictly increasing. $R = [-2 + e^{-2}, 2 + e^2] \approx [-1.86, 9.39]$. (b) Solve $x + e^x = 4$: $g^{-1}(4) \approx 1.07$. (c) (4, 1.07) approximately.
- 30.** (a) $D = \mathbb{R} \setminus \{1\}$; $1/(x-1)^2 > 0$, so $R = (2, \infty)$. All values above 2 occur. (b) $(x-1)^2 = 10/3$, giving $x \approx -0.826, 2.83$. (c) The screen shows only a finite window and uses finite resolution; it cannot include the undefined input 1 or the unattained output 2.

3. IB exam-style questions

- 31.** (a) $-2a + b = -7$, $3a + b = 13 \Rightarrow 5a = 20$, so $a = 4, b = 1$. (b) 3. (c) $4x + 1 = 9 \Rightarrow f^{-1}(9) = 2$. (d) A line with nonzero gradient is one-to-one.
- 32.** (a) $18 - 2x \geq 0 \Rightarrow D = (-\infty, 9]$; $R = [0, \infty)$. (b) $18 - 2x = 25 \Rightarrow x = -7/2$. (c) $g^{-1}(5) = -7/2$.
- 33.** (a) $k \in \mathbb{R} \setminus \{2, 5, 8\}$. (b) $D_{f^{-1}} = \{-3, 2, 5, 8\}$, $R_{f^{-1}} = \{-4, -1, 2, 6\}$. (c) $2 + (-1) = 1$.
- 34.** (a) $C(n) = 42 + 0.08n$, $D = \{0, 1, \dots, 600\}$. (b) $R = \{42, 42.08, 42.16, \dots, 90\}$. (c) $n = (66 - 42)/0.08 = 300$. (d) It would require 800 pages, above the limit (or exceeds the maximum cost €90).
- 35.** (a) $D = (-3, 4]$, $R = (-1, 3]$. (b) 3. (c) On the rising segment $y = 2x + 5$, $x = -3/2$; on the falling segment $y = 4 - x$, $x = 2$. (d) $x = -5/2$ and 4. (e) Many inputs give output 3, so f is not one-to-one.
- 36.** (a) Minimum $f(1) = 0$; $f(4) = 9$ is the maximum, so $R = [0, 9]$. (b) $x = -2$ or 4 algebraically, but only $x = 4$ is allowed. (c) The other allowed input 4 still produces 9.
- 37.** (a) $R_f = [-5, 3]$. (b) $x = 2y - 3 \Rightarrow f^{-1}(x) = (x + 3)/2$, with domain $[-5, 3]$. (c) Original endpoints $(-1, -5)$, $(3, 3)$; inverse endpoints $(-5, -1)$, $(3, 3)$, all included. Mirror line $y = x$; see sketch.
- 38.** (a) $D = \mathbb{R} \setminus \{-2\}$. (b) $3/(x+2) - 1 = -1$ would give $3/(x+2) = 0$, impossible. (c) $R = \mathbb{R} \setminus \{-1\}$; asymptotes $x = -2$, $y = -1$. Every other output is possible. (d) $3/(x+2) = 1 \Rightarrow g^{-1}(0) = 1$.
- 39.** (a) $x \leq 7$ and $x \neq -1$, so $D = (-\infty, -1) \cup (-1, 7]$. (b) 0 and $\sqrt{7}$. (c) The denominator is zero.
- 40.** (a) $[-4, 122]^\circ\text{F}$. (b) $x = 9y/5 + 32 \Rightarrow F^{-1}(x) = 5(x - 32)/9$; domain $[-4, 122]^\circ\text{F}$ and range $[-20, 50]^\circ\text{C}$. (c) $F^{-1}(86) = 30$: 86°F equals 30°C .
- 41.** (a) Since $2^x > 0$, $R = (-\infty, 3)$; asymptote $y = 3$. (b) $f(0) = 2$, so $(0, 2)$; $2^x = 2 \Rightarrow x = 1$, giving $(1, 1)$. (c) Strictly decreasing, approaching $y = 3$ from below to the left; see sketch.
- 42.** (a) $2x + 4 > 0 \Rightarrow D = (-2, \infty)$; $R = \mathbb{R}$. (b) $2x + 4 = 1 \Rightarrow x = -3/2$. (c) $2x + 4 = 6 \Rightarrow x = 1$.
- 43.** (a) $D_{f^{-1}} = (-2, 6]$, $R_{f^{-1}} = [-4, 3)$. (b) 2. (c) $D_g = [-4, 3)$, $R_g = (3, 11]$. (d) $g(2) = 4$, so $g^{-1}(4) = 2$; $D_{g^{-1}} = (3, 11]$.
- 44.** (a) $2x + 2l = 20 \Rightarrow l = 10 - x$, so $A = x(10 - x)$; $D = (0, 10)$ cm. (b) $R = (0, 25]$ cm². (c) $x^2 - 10x + 16 = (x - 2)(x - 8) = 0$, so $x = 2, 8$ cm. (d) Distinct widths x and $10 - x$ give the same area, except at 5.
- 45.** (a) Each allowed input has one output; $R = \{5, 7, 9\}$. (b) 0, 1. (c) The reversed input 5 would have two outputs, 0 and 1. (d) Remove either 0 or 1.
- 46.** (a) $9 - x^2 \geq 0 \Rightarrow D = [-3, 3]$; $R = [0, 3]$. (b) Squaring gives the circle, while the root requires $y \geq 0$. (c) One nonnegative output per allowed input, but $f(-x) = f(x)$, for example $f(-3) = f(3) = 0$.
- 47.** (a) $h = 2, k = -3$. (b) Minimum -3 ; endpoint values 1 and 6, hence $R = [-3, 6]$. (c) $(x - 2)^2 = 4 \Rightarrow x = 0, 4$. (d) No, because f is not one-to-one.
- 48.** (a) Positive root $t \approx 3.54$ s; $D \approx [0, 3.54]$. (b) Maximum 15.9 m at $t \approx 1.73$ s; $R \approx [0, 15.9]$ m. (c) $t \approx 0.633, 2.84$ s. (d) Some heights occur once on the way up and again on the way down.
- 49.** (a) Strict increase makes f one-to-one; $D_{f^{-1}} = R_{f^{-1}} = \mathbb{R}$. (b) Solve $x^3 + x - 4 = 2$: $f^{-1}(2) \approx 1.63$. (c) (1.63, 2) on f , (2, 1.63) on its inverse (approximately).
- 50.** (a) Minimum -0.890 at $x = \pm\sqrt{1.3} \approx \pm 1.14$; maximum 2.87 at $x = -1.8$. Range approximately $[-0.890, 2.87]$. (b) $x \approx -1.50, -0.597, 0.597$; the positive root near 1.50 is outside the domain. (c) No. (d) The lowest values occur inside the interval; endpoint checks miss them.



4. AA HL exclusive challenge

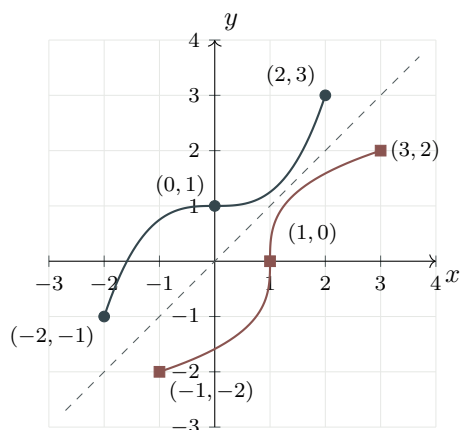
51. (a) $(x-2)/(x+3) \geq 0$, $x \neq -3$, giving $D = (-\infty, -3) \cup [2, \infty)$. (b) The radicand takes values $[0, 1) \cup (1, \infty)$, so the root has the same two intervals as its range. Output 1 would require $5/(x+3) = 0$, impossible. See the graphical answer checks.
52. (a) $g(x) = 1 - 5/(x^2 + 1)$. Since $0 < 1/(x^2 + 1) \leq 1$, $R = [-4, 1)$. (b) $x = \pm 2$; both give 0, so g is not one-to-one.
53. (a) $\{x : x \leq k, x \neq -2\}$. (b) Exactly when $k < -2$, so the forbidden input is already outside the root's domain. (c) For $k = -2$: $(-\infty, -2)$. For $k = 3$: $(-\infty, -2) \cup (-2, 3]$.
54. (a) $R_f = [-4, \infty)$; f is strictly decreasing for $x \leq 3$. (b) $x = (y-3)^2 - 4 \Rightarrow y = 3 \pm \sqrt{x+4}$. Outputs of the inverse must be ≤ 3 , so $f^{-1}(x) = 3 - \sqrt{x+4}$; $D = [-4, \infty)$, $R = (-\infty, 3]$. (c) $f^{-1}(5) = 0$ and $f(0) = 5$.
55. $f^{-1}(x) = (x-b)/a$. Equality with $ax+b$ for all x gives $a^2 = 1$ and $b(a+1) = 0$. Thus $(a, b) = (1, 0)$, or $a = -1$ with any real b .
56. (a) $f(x) = 2 + 7/(x-2)$, so $D = R = \mathbb{R} \setminus \{2\}$. (b) $x = (2y+3)/(y-2) \Rightarrow y = (2x+3)/(x-2)$, the original rule on the same domain. (c) Set $f(x) = x$: $x^2 - 4x - 3 = 0$, so points $(2 + \sqrt{7}, 2 + \sqrt{7})$ and $(2 - \sqrt{7}, 2 - \sqrt{7})$.
57. (a) $f(-x) = f(x)$: even. $g(-x) = -g(x)$: odd. (b) $f = 1 - 1/(x^2 + 1)$, so $R_f = [0, 1)$. (c) $x^2 + 1 \geq 2x$ and $x^2 + 1 \geq -2x$ give $-1/2 \leq g(x) \leq 1/2$. Endpoints occur at $x = -1, 1$; continuity supplies every intermediate value. $R_g = [-1/2, 1/2]$.
58. (a) $b = 4, c = -2$; $0 = a/(1-4) - 2 \Rightarrow a = -6$. (b) $x+2 = -6/(y-4) \Rightarrow f^{-1}(x) = 4 - 6/(x+2)$; $D = \mathbb{R} \setminus \{-2\}$, $R = \mathbb{R} \setminus \{4\}$. (c) Asymptotes $x = -2, y = 4$; point $(0, 1)$.
59. (a) $f(x) = (3x+5)/2$ for $-3 \leq x \leq -1$, and $f(x) = x+2$ for $-1 < x \leq 2$. (b) $f^{-1}(x) = (2x-5)/3$ for $-2 \leq x \leq 1$, and $f^{-1}(x) = x-2$ for $1 < x \leq 4$. (c) Join $(-2, -3), (1, -1), (4, 2)$; $D_{f^{-1}} = [-2, 4]$, $R_{f^{-1}} = [-3, 2]$.
60. (a) $(-1, 3)$ and $(1, -1)$. (b) $[-2, -1]$: range $[-1, 3]$; $[-1, 1]$: range $[-1, 3]$; $[1, 2]$: range $[-1, 3]$. Each interval is strictly monotonic. (c) Solve $x^3 - 3x + 1 = 0.5$ on $[1, 2]$: $g^{-1}(0.5) \approx 1.64$. (d) The horizontal line $y = 0.5$ meets the original graph three times, so reversing it gives three outputs for one input.

Graphical answer checks

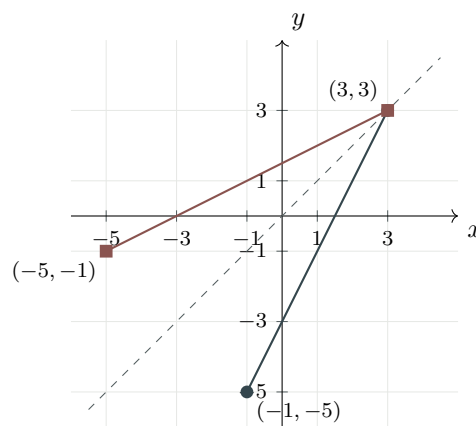
Q19, Q37, Q41, Q51 and Q59

Original graphs: solid slate. Inverse graphs: solid brick with square markers. Dashed diagonals: $y = x$.

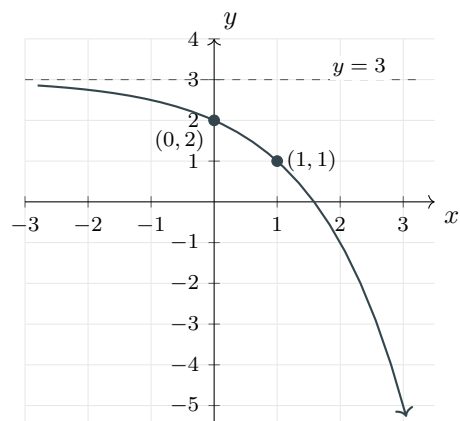
Q19: inverse sketch



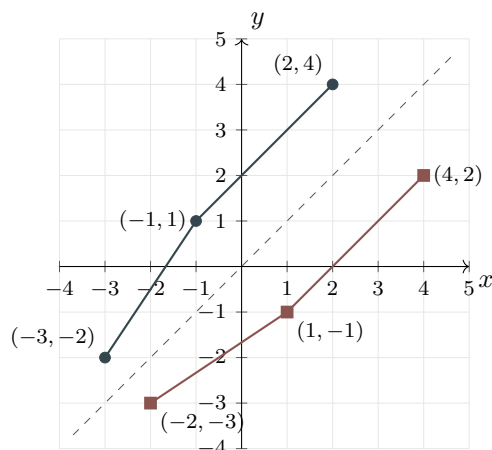
Q37: line and its inverse



Q41: exponential sketch



Q59: piecewise inverse sketch



Q51: domain sign table

x	$-\infty$	-3	2	$+\infty$	
$\frac{x-2}{x+3}$	+	undefined	-	0	+