Key Features of Graphs

IB Mathematics: Analysis and Approaches SL & HL

Section 2.4 | 60 carefully graded questions | 18 original graph/table illustrations

Learning goal	Identify, calculate and interpret key graph features and intersections.
Syllabus link	IB Mathematics AA SL/HL: SL 2.4. Maxima/minima, intercepts, symmetry, vertices, roots, vertical/horizontal asymptotes and intersections using technology.
Structure	Q1–15 short response; Q16–30 extended response; Q31–50 original Paper 1 practice; Q51–60 **AA HL exclusive**.
Technology	Calculator/GDC encouraged in Q1–30; **no calculator** in Q31–60.

Instructions. Show reasoning where possible; a graph alone is not a justification when algebra is requested. Label coordinates carefully. Use exact values unless a decimal approximation is explicitly requested. Q51–60 are designed exclusively for HL students and extend the difficulty of the same SL 2.4 content.

Section 1 • Short response (Questions 1–15)

1. For $f(x) = (x-3)^2 - 5$, state the coordinates of its vertex, its axis of symmetry, its minimum value and its y-intercept.

2. The function $g(x) = -2(x+1)^2 + 8$ is defined for $x \in \mathbb{R}$. Find its maximum value and the coordinates of all its intercepts.

3. For $f(x) = x^2 - 5x + 4$, determine the zeros, the y-intercept and the coordinates of the vertex.

4. The parabola $y = 3 - (x-2)^2$ has a vertex on a vertical line L. State the equation of L, the maximum value and the y-intercept.

5. Find all zeros of $f(x) = x^3 - 4x$. Is the graph symmetric about the y-axis, the origin, or neither?

6. Find the four zeros of $f(x) = x^4 - 5x^2 + 4$ and determine whether f is even, odd or neither.

7. For $f(x) = \dfrac{2}{x-3} + 1$, state the vertical and horizontal asymptotes, and find both intercepts.

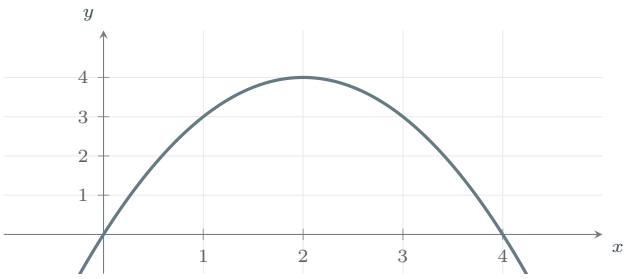
8. For $g(x) = \dfrac{3x+1}{x-2}$, determine both asymptotes and the coordinates where the graph crosses the axes.

9. Consider $f(x) = 4^x - 1$. Find its x- and y-intercepts, state its horizontal asymptote, and explain whether it has a maximum or minimum on $\mathbb{R}$.

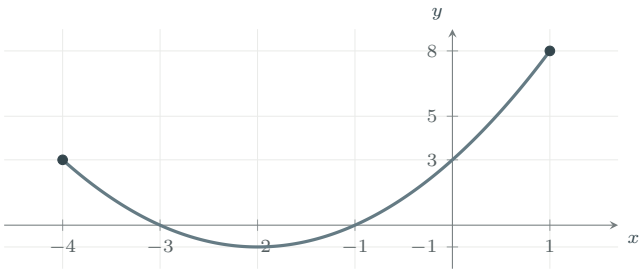
10. For $g(x) = \ln(x + 2)$, state the domain, the vertical asymptote and the x-intercept.

11. Classify each function as even, odd, or neither. Justify your answers using $f(-x)$.
(a) $f(x) = x^2 - 4$
(b) $g(x) = x^3 + x$
(c) $h(x) = x^2 + x$

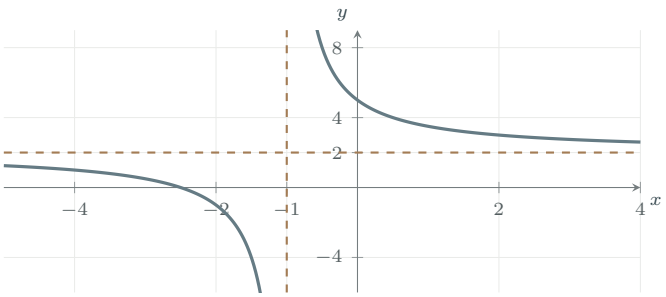
12. The curve shown is a parabola. By reading the graph, state its zeros, vertex, axis of symmetry and maximum value.



13. The graph represents $f(x) = (x + 2)^2 - 1$ on the restricted domain $-4 \leq x \leq 1$. Find the minimum and maximum values on this interval, with the x-coordinates at which they occur.



14. The graph of a rational function is shown. Use the dashed guides to read off the equations of its two asymptotes.

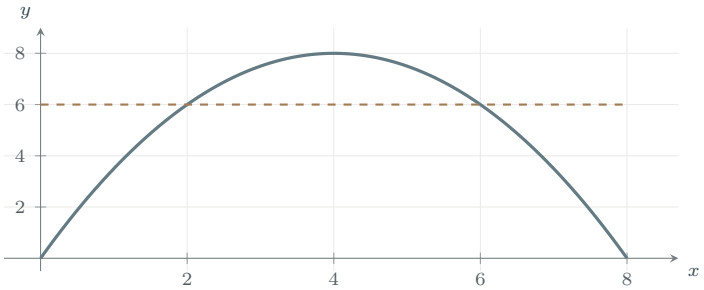


15. Find the coordinates of the intersection of the lines $f(x) = x + 2$ and $g(x) = 5 - x$.

Section 2 • Extended response (Questions 16–30)

16. An arch is modelled by $h(x) = -\frac{1}{2}(x-4)^2 + 8$, where h is height (metres) and $0 \le x \le 8$ is horizontal distance (metres).

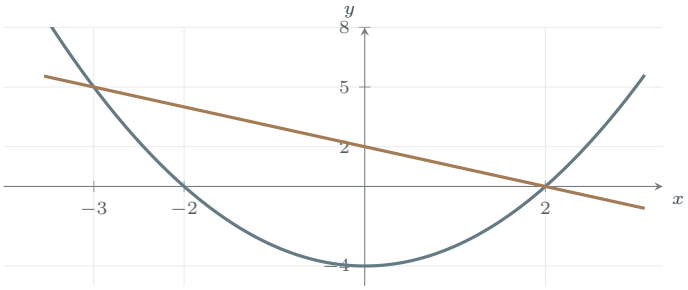
(a) Determine the maximum height and its position.
(b) Find the two positions where the arch is 6 m high.
(c) Hence determine the horizontal width of the arch at height 6 m.



$h(x)$ (metres) against x (metres); the horizontal dashed line is $h = 6$.

17. Consider the graphs of $f(x) = x^2 - 4$ and $g(x) = 2 - x$.

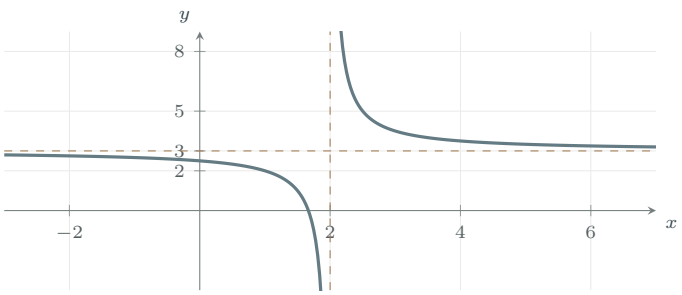
(a) Form an equation for their intersections.
(b) Find the exact coordinates of both intersection points.
(c) State which graph is above the other for $-3 < x < 2$.



Blue: $y = f(x)$; ochre: $y = g(x)$.

18. Two functions are defined by $f(x) = x^2 + 1$ and $g(x) = 3x + 1$. Find all their points of intersection exactly. Explain why the solutions are the same as the roots of $f(x) - g(x) = 0$.

19. The function $f(x) = \dfrac{1}{x-2} + 3$ is shown. Determine its domain, range, asymptotes and intercepts.



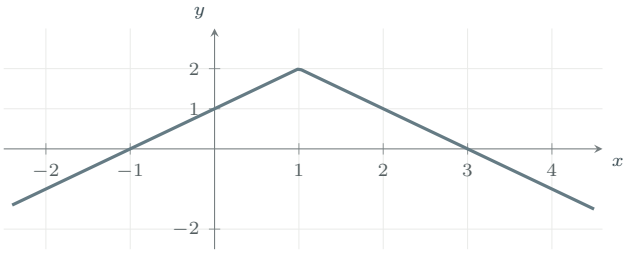
20. Let $f(x) = \dfrac{x^2 - 4}{x - 2}$ for $x \neq 2$.

(a) Simplify $f(x)$ on its domain.

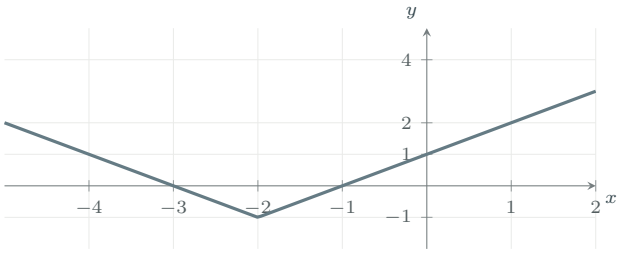
(b) State the coordinates of the missing point in its graph.

(c) Explain why $x = 2$ is *not* a vertical asymptote.

21. For $f(x) = 2 - |x - 1|$, determine the vertex, axis of symmetry, range and all intercepts. Sketch the graph on the axes provided.



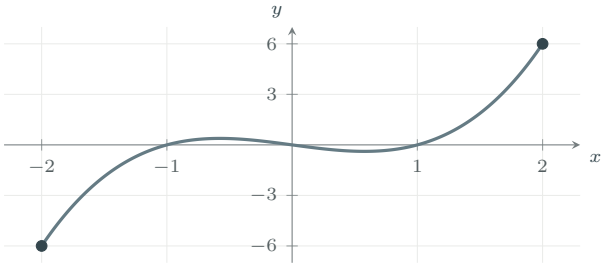
22. The graph of $g(x) = |x + 2| - 1$ is shown. Find its minimum point, axis of symmetry and zeros.



23. The graph of $f(x) = x^3 - x$ is restricted to $-2 \leq x \leq 2$.

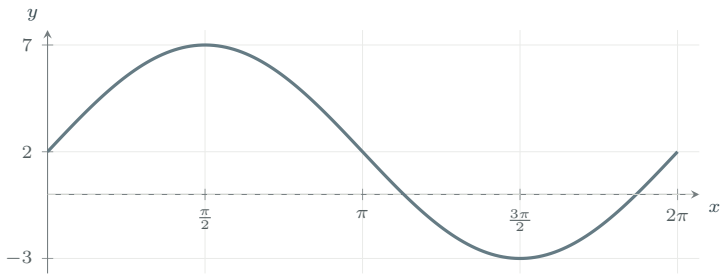
(a) Explain its symmetry and give its three zeros.

(b) With graphing technology, find the absolute maximum and minimum values on the stated domain.



24. Graph $y = 2^x$ and $y = 3$ with technology. Determine their intersection, giving the x-coordinate to three significant figures. State the exact x-coordinate using logarithms.

25. For $f(x) = 5\sin x + 2$, $0 \le x \le 2\pi$ (radians):
(a) State its maximum and minimum values.
(b) Using the graph, determine how many zeros occur on the given interval.
(c) State its y-intercept.

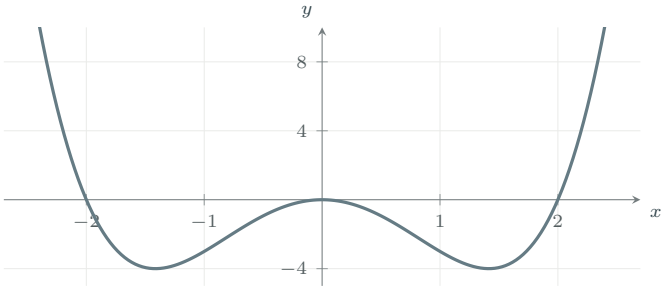


Horizontal scale in radians: $0,\ \pi/2,\ \pi,\ 3\pi/2,\ 2\pi$ (in order).

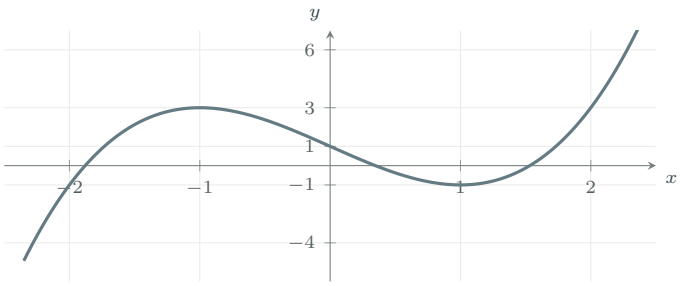
26. Consider $f(x) = -x^2 + 6x - 5$ for $0 \le x \le 6$. Determine the vertex, zeros, maximum value and the interval on which $f(x) \ge 0$.

27. For $f(x) = \dfrac{2x-1}{x+1}$, find both asymptotes and both intercepts. Use technology to check your results on the graph.

28. Using graphing technology, investigate $f(x) = x^4 - 4x^2$.
(a) State the zeros and any symmetry.
(b) Give the coordinates of its three turning points.
(c) Explain why the local maximum is not an absolute maximum on $\mathbb{R}$.



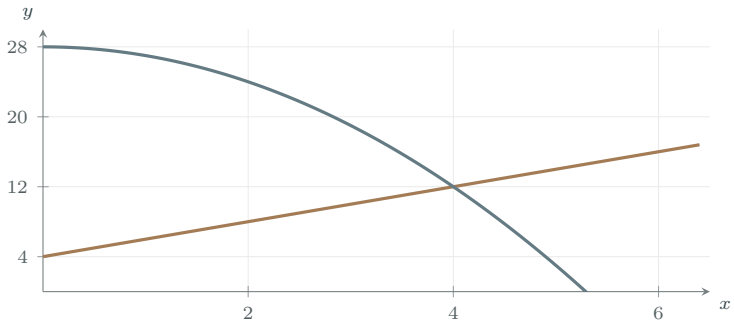
29. For $f(x) = x^3 - 3x + 1$, use a graphing calculator to:
(a) find all zeros to three decimal places;
(b) determine the local maximum and minimum points;
(c) state the interval on which f decreases.



30. A simplified market model has supply price $S(x) = 2x + 4$ and demand price $D(x) = 28 - x^2$ where $x \geq 0$ is quantity and price is in euros.

(a) Determine the equilibrium quantity and price.

(b) Explain why one algebraic solution must be rejected.



Ochre: supply $y = S(x)$; blue: demand $y = D(x)$.

Section 3 • IB Paper 1 style, non-calculator (Questions 31–50)

Original exam-style questions; mark allocations are indicative. No calculator.

31. The function f is defined by $f(x) = -(x-3)^2 + 9$, $x \in \mathbb{R}$.

(a) State the coordinates of the maximum point. [2 marks]

(b) Find the coordinates of the points where the graph meets the axes. [3 marks]

32. Let $f(x) = x^2 - 4x + 1$.

(a) Express f in vertex form. [2 marks]

(b) Hence state the range of f. [1 marks]

(c) Find the exact zeros of f. [2 marks]

33. The graph of $f(x) = -x^2 + 2x + 8$ intersects the x-axis at A and B, where A is to the left of B.

(a) Find the coordinates of A and B. [2 marks]

(b) Determine the equation of the axis of symmetry. [1 marks]

(c) Hence find the maximum value of f. [2 marks]

34. The quadratic function $g(x) = (x-1)(x-5)$ has two distinct zeros.

(a) State the zeros and y-intercept. [2 marks]

(b) Find the axis of symmetry and minimum point. [3 marks]

35. The function $f(x) = \dfrac{2x+1}{x-3}$, $x \neq 3$, has two asymptotes.

(a) State the equation of each asymptote. [2 marks]

(b) Find both intercepts. [3 marks]

36. Consider $f(x) = 3 + \dfrac{4}{x-1}$, where $x \neq 1$.

(a) State both asymptotes. [2 marks]

(b) Find the x-intercept. [2 marks]

(c) Find the x-coordinate where $f(x) = 5$. [2 marks]

37. Consider $g(x) = x^4 - 9x^2$, where $x \in \mathbb{R}$.

(a) Show that g is even. [2 marks]

(b) Find its distinct zeros. [2 marks]

38. The function $f(x) = x^3 - 9x$ is defined on $\mathbb{R}$.

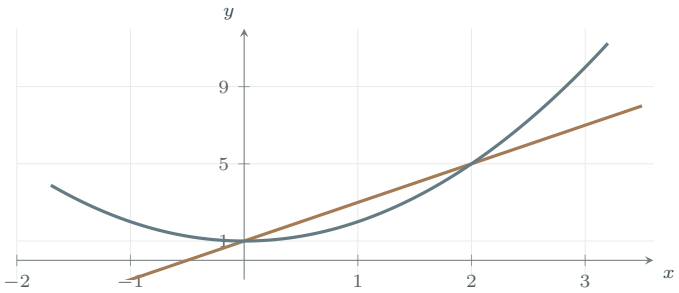
(a) Show that its graph has symmetry about the origin. [2 marks]

(b) Determine every x-intercept. [2 marks]

39. The graphs of $f(x) = 2x + 1$ and $g(x) = x^2 + 1$ are shown.

(a) Write down an equation whose solutions are the x-coordinates of the intersections. [1 marks]

(b) Determine the exact coordinates of both intersections. [4 marks]



Ochre: $y = f(x)$; blue: $y = g(x)$.

40. Consider $f(x) = 5 - (x + 2)^2$.

(a) Find the maximum value. [1 marks]

(b) Solve $f(x) = 1$ exactly. [2 marks]

41. Let $f(x) = 2x^2 - 8x + 3$.

(a) Find its vertex coordinates by completing the square. [3 marks]

(b) Determine the points where it intersects the horizontal line $y = 3$. [2 marks]

42. Let $f(x) = \dfrac{x + 4}{x - 2}$ for $x \neq 2$.

(a) State the vertical and horizontal asymptotes. [2 marks]

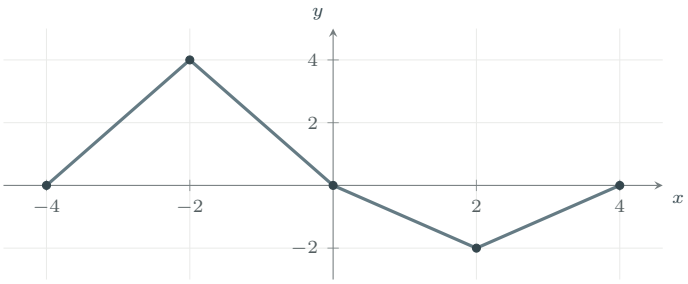
(b) Find where the graph intersects the line $y = 2$. [2 marks]

43. The graph shows a piecewise linear function f on $-4 \leq x \leq 4$.

(a) State the x-intercepts. [2 marks]

(b) Identify the absolute maximum and absolute minimum points. [2 marks]

(c) Find $f(0)$. [1 marks]



44. For $f(x) = x^3 - x$:

(a) Find all intercepts with the axes. [2 marks]

(b) Show algebraically that f is odd. [1 marks]

(c) Find the exact x-coordinates where the curve meets $y = x$. [2 marks]

45. The straight lines $f(x) = 3x - 2$ and $g(x) = -x + 6$ meet at P. Find the coordinates of P. [3 marks]

46. The function $h(x) = |2x - 6| - 4$ has a V-shaped graph.

(a) Find its minimum point and symmetry axis. [2 marks]

(b) Determine the x-intercepts. [2 marks]

(c) Find the y-intercept. [1 marks]

47. For $f(x) = (x - 3)^2 + 2$, determine the number of real solutions to $f(x) = k$ for (i) $k < 2$, (ii) $k = 2$, and (iii) $k > 2$. Justify using the vertex. [4 marks]

48. The graph of $f(x) = a(x - 2)^2 + 1$ passes through $(0, 9)$.

(a) Find a. [2 marks]

(b) Hence state the range of f. [1 marks]

(c) Explain why f has no x-intercepts. [2 marks]

49. A quadratic function is known to be symmetric about the y-axis. The table shows five values.

x	-2	-1	0	1	2
$f(x)$	3	0	-1	0	3

(a) State the vertex and zeros. [2 marks]

(b) Determine an equation of the parabola. [3 marks]

50. A quadratic function is $f(x) = 2x^2 + bx + 8$ with a symmetry axis $x = 3$.

(a) Find b. [2 marks]

(b) Determine its minimum value. [2 marks]

(c) Find its zeros exactly. [3 marks]

Section 4 • AA HL EXCLUSIVE CHALLENGE (Questions 51–60)

HL-only: These tasks demand deeper algebraic and graphical reasoning while remaining centred on Section 2.4.

51. A family of quadratic functions is defined by $f_a(x) = x^2 - 2ax + 5$, with $a \in \mathbb{R}$.

(a) Express f_a in vertex form and state its minimum. [3 marks]

(b) Determine the ranges of a for which f_a has zero, one or two distinct x-intercepts. [4 marks]

52. Two graphs are defined by $f(x) = \dfrac{x-1}{x+2}$ $(x \neq -2)$ and $g(x) = x - 1$.

(a) Determine the asymptotes of f. [2 marks]

(b) Find every point of intersection of the graphs, checking domain restrictions. [4 marks]

53. Let $f(x) = \dfrac{x^2 - 9}{x - 3}$, where $x \neq 3$.

(a) Describe its graph, including any excluded point. [3 marks]

(b) Determine all $k \in \mathbb{R}$ for which the line $y = k$ fails to intersect the graph. [2 marks]

54. A line $y = mx + 1$ is tangent to the parabola $y = x^2 - 2x + 5$.

(a) Show that the x-coordinates of intersections satisfy $x^2 - (m+2)x + 4 = 0$. [2 marks]

(b) Find all possible values of m. [3 marks]

(c) For each, determine the point of contact. [3 marks]

55. Consider $f(x) = \dfrac{2x + 3}{x - 1}$, $x \neq 1$.

(a) State the vertical and horizontal asymptotes. [2 marks]

(b) Solve $f(x) = k$ for x, giving conditions on k. [3 marks]

(c) Hence state the range of f. [1 marks]

56. For $f(x) = x^4 - 5x^2 + 4$, $x \in \mathbb{R}$:

(a) factorize $f(x)$ completely over the real numbers; [2 marks]

(b) hence solve $f(x) \geq 0$; [4 marks]

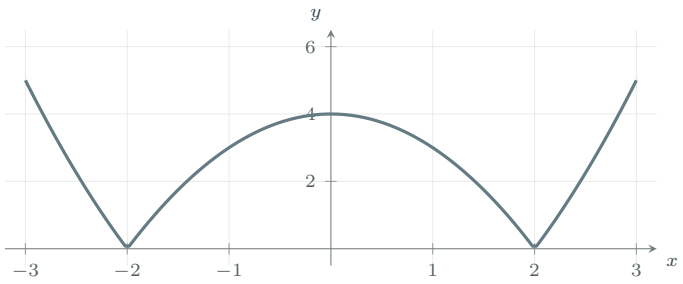
(c) state a graph symmetry. [1 marks]

57. The function $f(x) = |x^2 - 4|$ is defined on $\mathbb{R}$.

(a) Find all zeros and determine the y-intercept. [2 marks]

(b) Identify all local maximum and minimum points. [3 marks]

(c) State the range and justify the symmetry. [2 marks]



58. Two curves are given by $f(x) = x^2 - 4$ and $g(x) = -x^2 + 2x + 2$.

(a) Show that their intersection x-coordinates satisfy $x^2 - x - 3 = 0$. [2 marks]

(b) Find the exact coordinates of both intersections. [5 marks]

59. Consider $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$ for $x \in \mathbb{R}$.

(a) Show that $f(x) = 1 - \dfrac{2}{x^2 + 1}$ and hence state the horizontal asymptote. [2 marks]

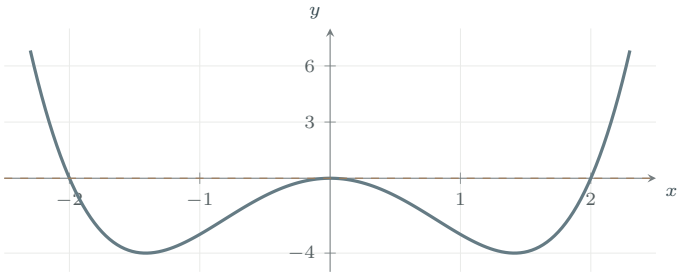
(b) Find the zeros and the minimum value. [3 marks]

(c) State the range of f and justify why the endpoint 1 is excluded. [2 marks]

60. Let $f(x) = x^4 - 4x^2$. A horizontal line with equation $y = k$ can intersect its graph in different numbers of distinct points.

(a) Express $f(x)$ in the form $(x^2 - a)^2 + b$. [2 marks]

(b) Determine the number of distinct real solutions to $f(x) = k$ in each case: $k < -4$, $k = -4$, $-4 < k < 0$, $k = 0$, and $k > 0$. Justify. [6 marks]



Answer key and solution outlines

Exact results are given unless technology or the question specifically calls for a decimal approximation.

Section 1: Short response

1. Vertex $(3,-5)$; axis $x=3$; minimum value -5; $f(0)=4$, giving $(0,4)$.

2. Maximum 8 at $x=-1$. Solve $-2(x+1)^2+8=0$: $x=-3,1$. Intercepts $(-3,0)$, $(1,0)$, and $(0,6)$.

3. $f(x)=(x-1)(x-4)$, so roots $1,4$; y-intercept $(0,4)$. Since $x_v=5/2$, $f(5/2)=-9/4$; vertex $\left(\frac{5}{2},-\frac{9}{4}\right)$.

4. $L: x=2$; maximum 3; $f(0)=3-4=-1$ so $(0,-1)$.

5. $x(x-2)(x+2)=0 \Rightarrow x=-2,0,2$. As $f(-x)=-f(x)$, the graph has origin symmetry.

6. $(x^2-1)(x^2-4)=0$, hence $x=-2,-1,1,2$. Since $f(-x)=f(x)$, f is even (symmetric about the y-axis).

7. VA: $x=3$; HA: $y=1$. Setting $f(x)=0$ gives $x=1$, and $f(0)=1/3$. Intercepts $(1,0)$ and $\left(0,\frac{1}{3}\right)$.

8. VA $x=2$; HA $y=3$. The intercepts are $\left(-\frac{1}{3},0\right)$ and $\left(0,-\frac{1}{2}\right)$.

9. Intercept at $(0,0)$. HA $y=-1$ as $x\to-\infty$. It is strictly increasing and $f(x)>-1$; neither maximum nor minimum exists on $\mathbb{R}$.

10. Domain $x>-2$; VA $x=-2$; zero at $\ln(x+2)=0 \Rightarrow x=-1$, so $(-1,0)$.

11. (a) Even: $f(-x)=f(x)$. (b) Odd: $g(-x)=-g(x)$. (c) Neither: $h(-x)=x^2-x$ is neither $h(x)$ nor $-h(x)$.

12. The zeros are $x=0,4$; vertex $(2,4)$; symmetry axis $x=2$; maximum 4.

13. $f(-2)=-1$ gives the absolute minimum. The endpoints give $f(-4)=3$ and $f(1)=8$, hence maximum 8 at $x=1$.

14. Vertical asymptote $x=-1$; horizontal asymptote $y=2$.

15. Set $x+2=5-x$. Then $x=3/2$, $y=7/2$, giving $\left(\frac{3}{2},\frac{7}{2}\right)$.

Section 2: Extended response

16. (a) Vertex $(4,8)$: maximum 8 m at $x=4$ m. (b) $-\frac{1}{2}(x-4)^2+8=6 \Rightarrow (x-4)^2=4$, so $x=2,6$. (c) Width $=6-2=4$ m.

17. (a) $x^2-4=2-x \Rightarrow x^2+x-6=0$. (b) $x=-3,2$, giving $(-3,5)$ and $(2,0)$. (c) g is above f between intersections (for example at $x=0$: $2>-4$).

18. $x^2+1=3x+1 \Rightarrow x(x-3)=0$. Intersections $(0,1)$ and $(3,10)$. Both graphs have the same y-value precisely where $f(x)-g(x)=0$.

19. Domain $\mathbb{R}\setminus\{2\}$; range $\mathbb{R}\setminus\{3\}$. VA $x=2$, HA $y=3$. x-intercept $\left(\frac{5}{3},0\right)$; y-intercept $\left(0,\frac{5}{2}\right)$.

20. (a) $f(x)=x+2$ whenever $x\neq 2$. (b) Missing point $(2,4)$. (c) As $x\to 2$, $f(x)\to 4$ (finite), so $x=2$ is a hole, not a vertical asymptote.

21. Vertex $(1,2)$; symmetry axis $x=1$; range $y\leq 2$. Zeros $x=-1,3$ and y-intercept $(0,1)$. It is an inverted V.

22. Minimum $(-2,-1)$; symmetry axis $x=-2$; zeros $-3,-1$.

23. (a) $f(-x)=-f(x)$, so origin symmetry. Zeros $-1,0,1$. (b) Endpoints give $f(-2)=-6$, $f(2)=6$; absolute minimum -6, maximum 6.

24. $2^x=3 \Rightarrow x=\log_2 3=\frac{\ln 3}{\ln 2}\approx 1.58$ (3 s.f.). Intersection $(\log_2 3,3)$.

25. (a) Maximum 7, minimum -3. (b) Two zeros ($\sin x=-2/5$ in quadrants III and IV). (c) $(0,2)$.

26. $f(x)=-(x-3)^2+4$: vertex $(3,4)$ and maximum 4. Roots $(x-1)(x-5)=0$, so $x=1,5$. Hence $f(x)\geq 0$ for $1\leq x\leq 5$.

27. $f(x)=2-\frac{3}{x+1}$, so VA $x=-1$ and HA $y=2$. Intercepts $\left(\frac{1}{2},0\right)$ and $(0,-1)$.

28. (a) $x=-2,0,2$ (zero 0 is repeated); even. (b) Turning points $(-\sqrt{2},-4)$, $(0,0)$, $(\sqrt{2},-4)$. (c) $f(x)\to\infty$ as $|x|\to\infty$, so there is no absolute maximum.

29. (a) Roots $x \approx -1.879,\ 0.347,\ 1.532$. (b) Local maximum $(-1, 3)$ and local minimum $(1, -1)$. (c) Decreasing for $-1 < x < 1$.

30. (a) Equate: $x^2 + 2x - 24 = 0 = (x + 6)(x - 4)$, so $x = 4$ and price $S(4) = 12$ euros. (b) $x = -6$ is not a possible quantity since $x \geq 0$.

Section 3: Paper 1 practice

31. (a) Maximum $(3, 9)$. (b) $0 = -(x - 3)^2 + 9 \Rightarrow x = 0, 6$; intercepts $(0, 0)$ and $(6, 0)$; the y-intercept is $(0, 0)$.

32. (a) $f(x) = (x - 2)^2 - 3$. (b) Range $y \geq -3$. (c) $(x - 2)^2 = 3$, so $x = 2 \pm \sqrt{3}$.

33. (a) $-x^2 + 2x + 8 = -(x - 4)(x + 2) = 0$, so $A(-2, 0)$, $B(4, 0)$. (b) $x = 1$. (c) $f(1) = 9$; maximum 9.

34. (a) Zeros $1, 5$ and y-intercept $(0, 5)$. (b) Midpoint of zeros gives $x = 3$; $g(3) = 2(-2) = -4$; minimum $(3, -4)$.

35. (a) Vertical $x = 3$; horizontal $y = 2$. (b) $2x + 1 = 0 \Rightarrow x = -1/2$; $f(0) = -1/3$. Intercepts $\left(-\frac{1}{2}, 0\right)$, $\left(0, -\frac{1}{3}\right)$.

36. (a) $x = 1$, $y = 3$. (b) $4/(x - 1) = -3 \Rightarrow x = -1/3$. (c) $4/(x - 1) = 2 \Rightarrow x = 3$.

37. (a) $g(-x) = (-x)^4 - 9(-x)^2 = g(x)$. (b) $x^2(x - 3)(x + 3) = 0$, so distinct zeros $-3, 0, 3$.

38. (a) $f(-x) = -x^3 + 9x = -f(x)$, so origin symmetry. (b) $x(x - 3)(x + 3) = 0$, giving $(-3, 0), (0, 0), (3, 0)$.

39. (a) $2x + 1 = x^2 + 1$. (b) $x(x - 2) = 0$, so intersections $(0, 1)$ and $(2, 5)$.

40. (a) Maximum 5 at $x = -2$. (b) $(x + 2)^2 = 4 \Rightarrow x = -4$ or $x = 0$.

41. (a) $f(x) = 2(x - 2)^2 - 5$; vertex $(2, -5)$. (b) $2x^2 - 8x + 3 = 3 \Rightarrow 2x(x - 4) = 0$: $(0, 3)$ and $(4, 3)$.

42. (a) $x = 2$ and $y = 1$. (b) $x + 4 = 2(x - 2) \Rightarrow x = 8$, giving $(8, 2)$.

43. (a) $(-4, 0)$, $(0, 0)$, $(4, 0)$. (b) Maximum $(-2, 4)$, minimum $(2, -2)$. (c) $f(0) = 0$.

44. (a) $x = -1, 0, 1$, giving $(-1, 0), (0, 0), (1, 0)$; y-intercept $(0, 0)$. (b) $f(-x) = -f(x)$. (c) $x^3 - x = x \Rightarrow x(x^2 - 2) = 0$, so $x = 0, \pm\sqrt{2}$.

45. $3x - 2 = -x + 6 \Rightarrow 4x = 8 \Rightarrow x = 2$; $y = 4$. Hence $P(2, 4)$.

46. (a) Minimum $(3, -4)$; axis $x = 3$. (b) $|2x - 6| = 4 \Rightarrow 2x - 6 = \pm 4$, so $(1, 0)$ and $(5, 0)$. (c) $h(0) = 2$, so $(0, 2)$.

47. Vertex $(3, 2)$ is the minimum. For $k < 2$: zero intersections; $k = 2$: one (at $x = 3$); $k > 2$: two (at $x = 3 \pm \sqrt{k - 2}$).

48. (a) $9 = 4a + 1 \Rightarrow a = 2$. (b) Range $y \geq 1$. (c) $2(x - 2)^2 + 1 \geq 1 > 0$ for all x, so no zeros.

49. (a) Vertex $(0, -1)$ and zeros $x = -1, 1$. (b) Symmetry and vertex give $f(x) = ax^2 - 1$. Since $f(1) = 0$, $a = 1$; $f(x) = x^2 - 1$.

50. (a) $-b/(2 \cdot 2) = 3$, so $b = -12$. (b) $f(3) = 18 - 36 + 8 = -10$. (c) $2(x^2 - 6x + 4) = 0 \Rightarrow (x - 3)^2 = 5$, so $x = 3 \pm \sqrt{5}$.

Section 4: HL-exclusive challenge

51. (a) $f_a(x) = (x - a)^2 + 5 - a^2$; minimum $5 - a^2$ at $x = a$. (b) Discriminant $4(a^2 - 5)$. No roots if $-\sqrt{5} < a < \sqrt{5}$; one if $a = \pm\sqrt{5}$; two if $a < -\sqrt{5}$ or $a > \sqrt{5}$.

52. (a) Vertical $x = -2$; horizontal $y = 1$. (b) Multiply by $x + 2$ (valid if $x \neq -2$): $x - 1 = (x - 1)(x + 2)$, so $(x - 1)(-x - 1) = 0$. Thus $x = 1$ or -1; intersections $(1, 0)$ and $(-1, -2)$.

53. (a) $f(x) = x + 3$ for $x \neq 3$: straight line with a hole at $(3, 6)$. (b) Only $k = 6$, since the horizontal level $y = 6$ would meet the line solely at the excluded point.

54. (a) Equating and rearranging gives $x^2 - (m + 2)x + 4 = 0$. (b) Tangency means discriminant zero: $(m + 2)^2 - 16 = 0 \Rightarrow m = 2$ or -6. (c) Repeated $x = (m + 2)/2$ gives $(2, 5)$ for $m = 2$, and $(-2, 13)$ for $m = -6$.

55. (a) $x = 1$, $y = 2$. (b) $(k-2)x = k+3$, so $x = (k+3)/(k-2)$ for $k \neq 2$; no solution for $k = 2$ (would require $3 = -2$). (c) Range $\mathbb{R} \setminus \{2\}$.

56. (a) $f = (x-2)(x+2)(x-1)(x+1)$. (b) Set $u = x^2$: $(u-1)(u-4) \geq 0$ if $u \leq 1$ or $u \geq 4$. Thus $x \in (-\infty, -2] \cup [-1, 1] \cup [2, \infty)$. (c) Even, so y-axis symmetry.

57. (a) Zeros $(-2, 0)$ and $(2, 0)$, y-intercept $(0, 4)$. (b) Local minima at $(\pm 2, 0)$; local maximum $(0, 4)$. (c) Range $[0, \infty)$; $f(-x) = f(x)$, so even.

58. (a) $x^2 - 4 = -x^2 + 2x + 2 \Rightarrow 2x^2 - 2x - 6 = 0 \Rightarrow x^2 - x - 3 = 0$. (b) $x = (1 \pm \sqrt{13})/2$; using $x^2 = x + 3$ in $y = x^2 - 4$ gives $y = x - 1$. Thus points $\left(\frac{1\pm\sqrt{13}}{2}, \frac{-1\pm\sqrt{13}}{2}\right)$ with matching signs.

59. (a) $x^2 - 1 = (x^2 + 1) - 2$, so $f = 1 - 2/(x^2 + 1)$; HA $y = 1$. (b) Zeros $x = \pm 1$; since $x^2 + 1 \geq 1$, minimum $f(0) = -1$. (c) Range $[-1, 1)$; $2/(x^2 + 1) > 0$ for every finite x.

60. (a) $f(x) = (x^2 - 2)^2 - 4$. (b) With $u = x^2 \geq 0$, $(u-2)^2 = k + 4$. For $k < -4$: 0 points; $k = -4$: 2 ($x = \pm\sqrt{2}$); $-4 < k < 0$: 4; $k = 0$: 3 ($x = 0, \pm 2$); $k > 0$: 2 (only the positive $u = 2 + \sqrt{k+4}$).