Quadratic Functions

IB Mathematics: Analysis and Approaches | SL & HL | SL 2.6

60 original questions ▪ all three forms ▪ parabolas, vertices, intercepts and modelling

At a glance

Learning goal	Interpret, sketch and reconstruct quadratic graphs; convert fluently among expanded, factorised and vertex forms.
Syllabus link	IB AA SL and AA HL, **SL 2.6**. The discriminant and quadratic inequalities are reserved for SL 2.7.
Structure	Section 1: 15 fluency questions; Section 2: 15 extended questions; Section 3: 20 original IB-style questions; Section 4: 10 higher-challenge questions for AA HL.
Calculator	Designed primarily for **non-calculator / Paper 1** practice. Exact values expected.

Instructions. Show working for conversions and determining unknown coefficients. On sketches, clearly label axes, intercepts, the vertex and axis of symmetry. Read points carefully from the diagrams; sketches are not always drawn to scale. Questions 51–60 are optional AA HL stretch questions, using the same SL 2.6 content.

Expanded	**Factorised**	**Vertex**
ax^2+bx+c	$a(x-p)(x-q)$	$a(x-h)^2+k$
immediately gives $(0, c)$	immediately gives roots p, q	immediately gives (h, k)

1. Short response

Recognise forms, intercepts and basic features.

1. For each quadratic, identify the feature that can be read directly from its form: (a) the y-intercept of $f(x) = 2x^2 - 8x + 3$; (b) the x-intercepts of $g(x) = -3(x+2)(x-4)$; (c) the vertex of $h(x) = \frac{1}{2}(x-3)^2 + 5$.

2. For each function, state whether its parabola opens upwards or downwards, and whether it has a minimum or maximum: (a) $2(x-1)^2 + 4$; (b) $-\frac{1}{3}(x+3)^2 - 1$; (c) $-5x^2 + 2x - 7$.

3. For $f(x) = x^2 - 6x + 8$, find the axis of symmetry, the vertex, and the y-intercept.

4. For $f(x) = -2(x-1)(x-5)$, state the x-intercepts and axis of symmetry, then calculate the vertex and y-intercept.

5. For $f(x) = 3(x+2)^2 - 12$, state the vertex, axis of symmetry, minimum value and range.

6. Expand and simplify $f(x) = -2(x-3)(x+1)$ into the form $ax^2 + bx + c$.

7. Express $f(x) = x^2 - 7x + 10$ in factorised form.

8. Complete the square to write $f(x) = x^2 + 8x + 9$ in vertex form.

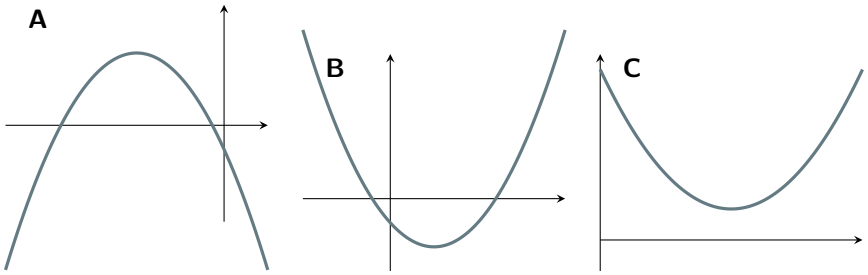
9. Write $f(x) = 2x^2 - 12x + 11$ in vertex form by completing the square.

10. Expand $f(x) = -3(x + 1)^2 + 12$ and identify the y-intercept.

11. The vertex of a quadratic is $(2, -3)$ and its graph passes through $(0, 5)$. Find its equation in vertex form.

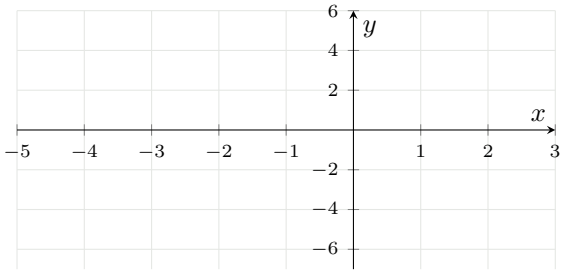
12. A parabola has x-intercepts $(-1, 0)$ and $(3, 0)$ and passes through $(0, -6)$. Find its factorised equation.

13. Match each function to graph A, B or C: (i) $f(x) = (x-1)^2 - 2$; (ii) $g(x) = -(x+2)^2 + 3$; (iii) $h(x) = \frac{1}{2}(x-3)^2 + 1$. *The graphs are schematic, not to a common scale.*



14. Compare $f(x) = (x - 2)^2 - 3$ and $g(x) = -2(x - 2)^2 + 5$. Give their common axis, each vertex, and each range.

15. Sketch $f(x) = (x + 3)(x - 1)$ on the grid, labelling both x-intercepts, the y-intercept, the vertex and the axis of symmetry.



2. Extended response

Convert between forms, recover equations and interpret diagrams.

16. For each quadratic, give **all three forms**: expanded, factorised and vertex form. Show sufficient working.

(a) $x^2 - 2x - 8$

(b) $-x^2 + 6x - 5$

(c) $2x^2 + 8x + 6$

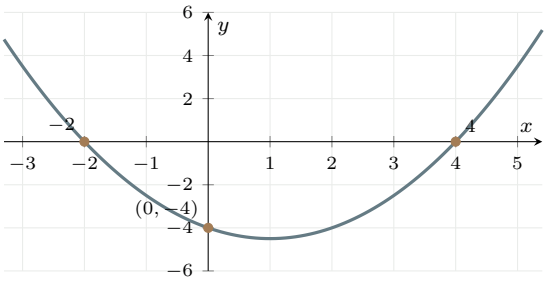
17. For $f(x) = 2x^2 + 4x - 6$: (a) write it in factorised form; (b) write it in vertex form; (c) state its intercepts and vertex; (d) sketch its graph.

18. Let $f(x) = -\frac{1}{2}(x + 4)(x - 2)$. Find its expanded form, axis of symmetry, vertex and range.

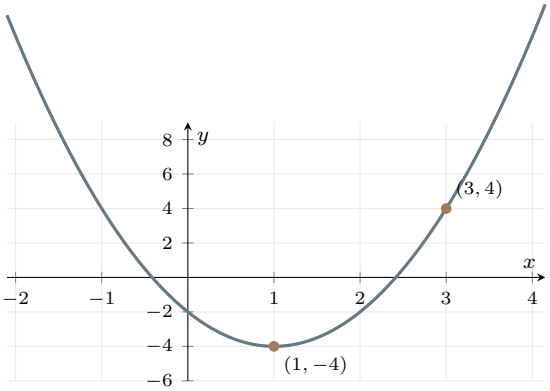
19. Let $f(x) = 3(x - 2)^2 + 1$. (a) Expand f. (b) State its vertex and minimum value. (c) Explain from the graph why it has no x-intercepts.

20. A quadratic has vertex $(1, 4)$ and passes through the points $(0, 2)$ and $(3, -4)$. Determine its vertex form and expanded form.

21. The parabola in the diagram passes through $(-2, 0)$, $(4, 0)$ and $(0, -4)$. Find its equation in factorised form, and determine its vertex.



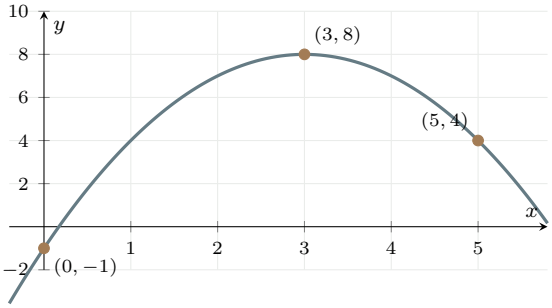
22. The graph has vertex $(1, -4)$ and passes through $(3, 4)$, as shown. Find the quadratic in vertex and expanded form, and state its y-intercept.



23. The graph of $f(x) = a(x-1)^2 - 3$ passes through $(-1, 5)$. Find a and the coordinates of the point on the graph that is the reflection of $(-1, 5)$ in its axis of symmetry.

24. A quadratic is given by $f(x) = x^2 + bx + 9$ and has axis of symmetry $x = 4$. Find b and express f in vertex form.

25. A function is defined by $f(x) = -x^2 + 6x - 1$ for $0 \le x \le 5$. Use the graph to justify its **exact range**, checking the vertex and both endpoints.



26. The values of a quadratic are tabulated below. Determine its equation in (a) vertex form and (b) factorised form. Explain how the table reveals its axis of symmetry.

x	-1	0	1	2	3
$f(x)$	8	3	0	-1	0

27. The height of a toy rocket (metres) is modelled by $h(t) = -5(t-2)^2 + 20$ for $0 \le t \le 4$ seconds. State (a) the maximum height and when it occurs; (b) the heights at $t = 0, 1, 4$; (c) the physically relevant range.

28. The cross-section of an arch is modelled by $h(x) = -\frac{1}{2}(x-6)^2 + 18$ for $0 \le x \le 12$, where x and h are in metres. Find the arch's maximum height, its total ground-level width, and its height 4 metres from the left end.

29. A rectangular garden has perimeter 24 m. If one side is x metres, show that its area is $A(x) = x(12 - x)$, express A in vertex form, and hence find its maximum area and the dimensions that achieve it.

30. A student analyses $f(x) = -2(x+3)^2 + 5$ and writes: "vertex $(3, 5)$, axis $x = 3$, y-intercept 5, minimum 5." Identify and correct **all four** mistakes, explaining how each correct feature is found.

3. Original IB exam-style questions

Algebraic reasoning, labelled sketches and mathematical interpretation.

31. Given $f(x) = x^2 - 4x - 5$: (a) express f in vertex form; (b) express it in factorised form; (c) state its vertex and three intercepts; (d) sketch its graph. [7 marks]

32. The function $g(x) = -2(x-1)(x+3)$ is defined on $\mathbb{R}$. (a) State the x-intercepts; (b) calculate the axis and vertex; (c) find the y-intercept and range. [6 marks]

33. Let $f(x) = \frac{1}{2}(x+2)^2 - 3$. (a) Write f in expanded form; (b) state its vertex and range; (c) calculate the y-intercept. [6 marks]

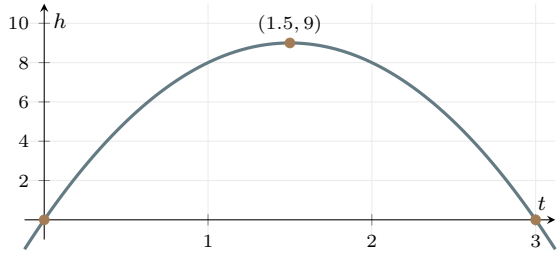
34. A parabola has vertex $(2, 3)$ and passes through $(0, -5)$. Determine its equation in both vertex and expanded form. State whether its vertex is a maximum or minimum. [6 marks]

35. A quadratic has x-intercepts $(1, 0)$ and $(7, 0)$, and y-intercept $(0, -14)$. Find its factorised form, axis of symmetry, and maximum or minimum value. [7 marks]

36. The graph of $f(x) = 3x^2 + px + 8$ has axis of symmetry $x = -2$. Determine p, the vertex, and the minimum value. [5 marks]

37. The function $f(x) = x^2 - 2x + c$ has a minimum value of -4. Find c, then compare its axis and vertex with those of $g(x) = -2(x-1)^2 + 3$. [5 marks]

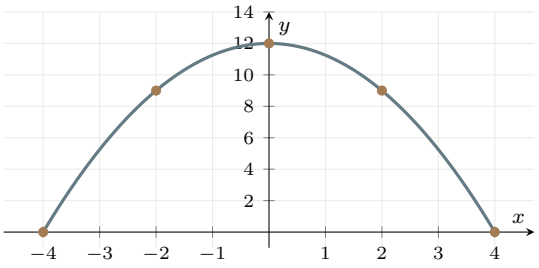
38. A ball's height above the ground is modelled by $h(t) = -4(t-1.5)^2 + 9$ for $0 \le t \le 3$. Determine its maximum height and the time at which it occurs, its initial height, and its height at $t = 1$. Interpret the vertex. [6 marks]



39. A rectangular animal enclosure is built against a straight wall using 24 m of fence for the other three sides. If each perpendicular side is x metres, (a) show $A(x) = x(24 - 2x)$; (b) give the relevant domain; (c) find the largest area and the dimensions. [7 marks]

40. The parabola through $(-2, 0)$, $(4, 0)$ and $(0, 8)$ has equation $f(x) = a(x+2)(x-4)$. Find a, its vertex and range. Explain how the intercepts and vertex would be labelled on a sketch. [8 marks]

41. Five measurements of the height of a parabolic arch are $(x, h) = (-4, 0), (-2, 9), (0, 12), (2, 9), (4, 0)$. Explain its symmetry and derive a quadratic rule $h(x)$ that fits all five points. [6 marks]

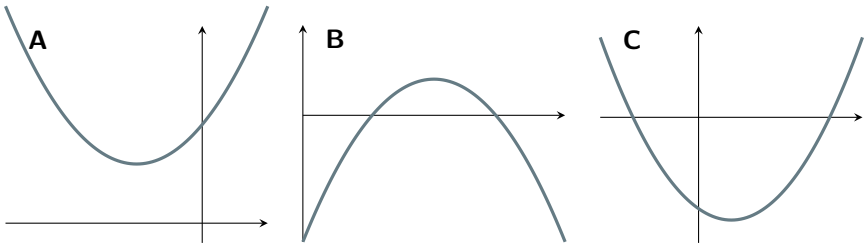


42. For $f(x) = 2x^2 + 8x - 10$, obtain factorised and vertex forms. Use them to state both x-intercepts, the vertex and the range. [8 marks]

43. A parabola has maximum point $(-1, 6)$ and passes through $(1, -2)$. Determine its equation and y-intercept, giving exact values. [6 marks]

44. Consider $f(x) = 2(x-3)^2 - 5$ and $g(x) = -(x-3)^2 + 5$. Give both vertices, their common axis, both ranges, and compare $f(1)$ with $g(1)$. [6 marks]

45. Match graphs A, B and C to (i) $f(x) = (x+2)(x-4)$; (ii) $g(x) = -(x-3)^2 + 2$; (iii) $h(x) = 2(x+1)^2 + 3$. Justify each choice by stating the vertex or intercepts. [6 marks]



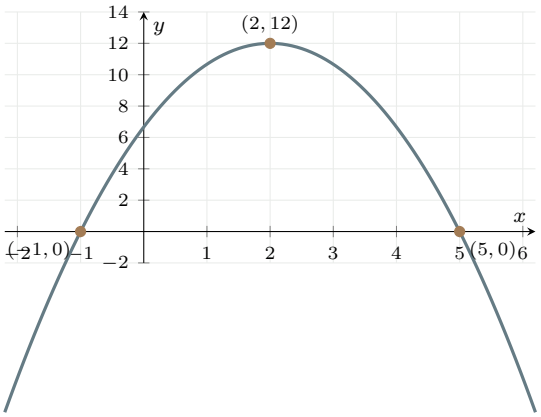
46. A quadratic $f(x) = ax^2 + bx + c$ has vertex $(2, -7)$ and y-intercept $(0, 1)$. Determine a, b and c. [6 marks]

47. The function $f(x) = x^2 - 6x + 2$ is defined only for $1 \le x \le 5$. Express it in vertex form and find the range, justifying the maximum and minimum on the restricted domain. [5 marks]

48. The vertex form of a quadratic is $f(x) = -2(x-1)^2 + 8$. Express it in factorised form and use this to state its x-intercepts. Calculate its y-intercept. [6 marks]

49. Show algebraically that $f(x) = 2x^2 - 4x + 3$ and $g(x) = -x^2 + 2x + 3$ have both the same y-intercept and axis of symmetry. Explain why their vertices are nevertheless different. [6 marks]

50. A parabola has roots -1 and 5, and its maximum value is 12. The graph is shown below. Determine its equation in factorised, vertex and expanded forms, and state its y-intercept. [9 marks]



4. AA HL challenge

Harder algebra and parameter reasoning using *only* SL 2.6 content.

51. A quadratic has vertex $(3, -2)$ and passes through $(0, 16)$. Determine its equation in each of the three standard forms, verifying that all forms agree.

52. The function $f(x) = ax^2 + bx + c$ has maximum point $(-2, 7)$ and passes through $(1, -20)$. Find a, b and c, and give its range.

53. It is known that $f(1) = f(7) = 5$ for a quadratic with minimum value -3. Determine its axis of symmetry, its vertex form, and $f(0)$.

54. A quadratic has one root at $x = -1$, axis of symmetry $x = 2$, and y-intercept -12. Find the other root, the quadratic rule in factorised form and its vertex.

55. A family of quadratics is defined by $f_k(x) = 2x^2 - 8x + k$, where $k \in \mathbb{R}$. Prove that all graphs have the same axis, express the minimum in terms of k, and find k when the minimum is -3.

56. The quadratic $f(x) = a(x - h)^2 + k$ passes through $(h + 2, k + 6)$. Determine a and hence find $f(h - 1)$ in terms of k. Explain why h does not appear in the answer.

57. A parabola crosses the x-axis at $x = -4$ and $x = 2$, and its minimum value is -27. Determine $f(x)$ in factorised form and the y-intercept.

58. A graph has roots -3 and 1, vertex $(-1, -4)$, and y-intercept -3. Determine its equation in factorised and vertex forms. Show algebraically that the two forms describe the same function.

59. A quadratic is zero at $x = 1$ and $x = 5$, and has value -18 at the midpoint of those roots. Determine $f(x)$ in factorised form, then find $f(0)$ and $f(7)$.

60. The quadratic $f(x) = a(x-h)^2 + k$ has vertex $(3, -8)$ and $f(1) = -4$. Determine its rule in vertex and expanded forms, its y-intercept, and the point other than $(1, -4)$ with the same y-coordinate. Justify the last step using symmetry.

Answer key and solution outlines

Exact results, algebraic justification and key graph features

Section 1 | Short response

1. (a) $(0,3)$; (b) $(-2,0)$ and $(4,0)$; (c) $(3,5)$.

2. (a) Upwards, minimum; (b) downwards, maximum; (c) downwards, maximum. The sign of the coefficient of x^2 determines the direction.

3. $x=\frac{6}{2}=3$, $f(3)=9-18+8=-1$; vertex $(3,-1)$, y-intercept $(0,8)$.

4. x-intercepts $(1,0)$ and $(5,0)$, axis $x=3$, $f(3)=8$, vertex $(3,8)$, $f(0)=-10$.

5. Vertex $(-2,-12)$; axis $x=-2$; minimum -12; range $[-12,\infty)$.

6. $(x-3)(x+1)=x^2-2x-3$, hence $f(x)=-2x^2+4x+6$.

7. $f(x)=(x-2)(x-5)$.

8. $f(x)=(x+4)^2-16+9=(x+4)^2-7$.

9. $f(x)=2(x^2-6x)+11=2[(x-3)^2-9]+11=2(x-3)^2-7$.

10. $f(x)=-3x^2-6x+9$, so the y-intercept is $(0,9)$.

11. $f(x)=a(x-2)^2-3$; $5=4a-3$, so $a=2$. Hence $f(x)=2(x-2)^2-3$.

12. $f(x)=a(x+1)(x-3)$; $-6=-3a$, so $a=2$. Thus $f(x)=2(x+1)(x-3)$.

13. (i) B; (ii) A; (iii) C. Read the vertex and the sign of a.

14. Common axis $x=2$. f: vertex $(2,-3)$, range $[-3,\infty)$. g: vertex $(2,5)$, range $(-\infty,5]$.

15. x-intercepts $(-3,0)$ and $(1,0)$; axis $x=-1$; vertex $(-1,-4)$; y-intercept $(0,-3)$. Upward parabola.

Section 2 | Extended response

16. (a) $x^2-2x-8=(x-4)(x+2)=(x-1)^2-9$.
(b) $-x^2+6x-5=-(x-1)(x-5)=-(x-3)^2+4$.
(c) $2x^2+8x+6=2(x+1)(x+3)=2(x+2)^2-2$.

17. $2(x+3)(x-1)=2(x+1)^2-8$. Intercepts $(-3,0)$, $(1,0)$, $(0,-6)$; vertex $(-1,-8)$. Sketch opens upwards.

18. $f(x)=-\frac{1}{2}x^2-x+4$; axis $x=-1$; $f(-1)=\frac{9}{2}$, vertex $(-1,\frac{9}{2})$; range $(-\infty,\frac{9}{2}]$.

19. $f(x)=3x^2-12x+13$. Vertex $(2,1)$, minimum 1. Since $3(x-2)^2\geq 0$, $f(x)\geq 1>0$, so the graph lies above the x-axis.

20. $f(x)=a(x-1)^2+4$. From $(0,2)$, $2=a+4$, so $a=-2$. Hence $f(x)=-2(x-1)^2+4=-2x^2+4x+2$. Check $f(3)=-4$.

21. $f(x)=a(x+2)(x-4)$. At $x=0$, $-4=-8a$, so $a=\frac{1}{2}$. Axis $x=1$ and $f(1)=\frac{1}{2}(3)(-3)=-\frac{9}{2}$; vertex $(1,-\frac{9}{2})$.

22. $f(x)=a(x-1)^2-4$. Substitute $(3,4)$: $4=4a-4$, hence $a=2$. So $f(x)=2(x-1)^2-4=2x^2-4x-2$ and $f(0)=-2$.

23. $5=4a-3$ gives $a=2$. Reflection in the axis $x=1$ maps $x=-1$ to $x=3$; the corresponding point is $(3,5)$.

24. $-b/2=4$ gives $b=-8$. Hence $f(x)=x^2-8x+9=(x-4)^2-7$.

25. $f(x)=-(x-3)^2+8$. Vertex gives maximum 8. Endpoints: $f(0)=-1$ and $f(5)=4$; the minimum on $[0,5]$ is -1. Range $[-1,8]$.

26. Equal values $f(1)=f(3)=0$ give axis $x=2$; vertex $(2,-1)$. Let $f(x)=a(x-2)^2-1$; $f(1)=0$ gives

$a = 1$. Thus $(x-2)^2 - 1 = (x-1)(x-3)$.

27. Vertex $(2, 20)$ gives maximum height 20 m at 2 s. $h(0) = 0$, $h(1) = 15$, $h(4) = 0$ metres. Range $[0, 20]$ m.

28. Maximum 18 m at $x = 6$. $h(0) = h(12) = 0$, so width 12 m. $h(4) = -\frac{1}{2}(4) + 18 = 16$ m.

29. Other side $12 - x$ since $2x + 2(12 - x) = 24$. $A = x(12 - x) = -(x-6)^2 + 36$. Maximum area $36\,\text{m}^2$ for a 6×6 m square.

30. Vertex $(-3, 5)$ (because $x + 3 = x - (-3)$); axis $x = -3$; y-intercept $f(0) = -13$; the graph opens downwards ($a = -2$), so it has a *maximum* of 5, not a minimum.

Section 3 | IB exam-style

31. $f = (x-2)^2 - 9 = (x-5)(x+1)$. Vertex $(2, -9)$; intercepts $(-1, 0)$, $(5, 0)$, $(0, -5)$. Upwards.

32. Intercepts $(1, 0), (-3, 0)$. Axis $x = -1$, vertex $(-1, 8)$, y-intercept $(0, 6)$; range $(-\infty, 8]$.

33. $f = \frac{1}{2}x^2 + 2x - 1$. Vertex $(-2, -3)$; range $[-3, \infty)$; y-intercept $(0, -1)$.

34. $f = a(x-2)^2 + 3$, so $-5 = 4a + 3 \Rightarrow a = -2$. $f = -2(x-2)^2 + 3 = -2x^2 + 8x - 5$; maximum $(2, 3)$.

35. $f = a(x-1)(x-7)$, $-14 = 7a \Rightarrow a = -2$. Axis $x = 4$. $f(4) = 18$, a maximum.

36. $-p/6 = -2 \Rightarrow p = 12$. $f(-2) = 12 - 24 + 8 = -4$. Vertex $(-2, -4)$; minimum -4.

37. $f = (x-1)^2 + c - 1$; $c - 1 = -4 \Rightarrow c = -3$. Both have axis $x = 1$; vertices $f : (1, -4)$ (minimum), $g : (1, 3)$ (maximum).

38. Maximum height 9 m at $t = 1.5$ s; $h(0) = 0$ m; $h(1) = -4(0.5)^2 + 9 = 8$ m. Vertex means the ball is highest after 1.5 s.

39. The open side length is $24 - 2x$, so $A = x(24 - 2x) = -2(x-6)^2 + 72$, with $0 < x < 12$. Max $72\,\text{m}^2$ at $x = 6$; third side 12 m.

40. $8 = -8a \Rightarrow a = -1$. $f = -(x+2)(x-4)$, axis $x = 1$, vertex $(1, 9)$; range $(-\infty, 9]$. Label $(-2, 0), (4, 0), (0, 8), (1, 9)$.

41. By symmetry the axis is $x = 0$ and the vertex is $(0, 12)$. Write $h(x) = ax^2 + 12$. Using $(4, 0)$, $0 = 16a + 12 \Rightarrow a = -\frac{3}{4}$. Check $h(2) = 9$.

42. $f = 2(x^2 + 4x - 5) = 2(x+5)(x-1) = 2(x+2)^2 - 18$. Intercepts $(-5, 0), (1, 0)$; vertex $(-2, -18)$; range $[-18, \infty)$.

43. $f = a(x+1)^2 + 6$, $-2 = 4a + 6 \Rightarrow a = -2$. $f = -2(x+1)^2 + 6 = -2x^2 - 4x + 4$; y-intercept $(0, 4)$.

44. f vertex $(3, -5)$, range $[-5, \infty)$; g vertex $(3, 5)$, range $(-\infty, 5]$; both axes $x = 3$. $f(1) = 3$ and $g(1) = 1$.

45. (i) C: upward with intercepts $-2, 4$ and vertex $(1, -9)$. (ii) B: downward, vertex $(3, 2)$. (iii) A: upward, vertex $(-1, 3)$.

46. $f = a(x-2)^2 - 7$; $1 = 4a - 7 \Rightarrow a = 2$. Expand to $2x^2 - 8x + 1$, so $(a, b, c) = (2, -8, 1)$.

47. $f = (x-3)^2 - 7$. Vertex $(3, -7)$ is the minimum; $f(1) = f(5) = -3$ are the endpoint maxima. Range $[-7, -3]$.

48. $f = -2[(x-1)^2 - 4] = -2(x-3)(x+1)$. x-intercepts $(3, 0), (-1, 0)$; $f(0) = 6$.

49. Both have y-intercept $(0, 3)$ and axes $x = -b/(2a) = 1$. $f = 2(x-1)^2 + 1$ has minimum $(1, 1)$; $g = -(x-1)^2 + 4$ has maximum $(1, 4)$.

50. Axis $x = 2$; $f = a(x+1)(x-5)$, so $12 = f(2) = -9a$, giving $a = -\frac{4}{3}$. Factorised $-\frac{4}{3}(x+1)(x-5)$; vertex $-\frac{4}{3}(x-2)^2 + 12$; expanded $-\frac{4}{3}x^2 + \frac{16}{3}x + \frac{20}{3}$; y-intercept $(0, \frac{20}{3})$.

Section 4 | HL challenge

51. $f = a(x-3)^2 - 2$, $16 = 9a - 2 \Rightarrow a = 2$. Therefore $f = 2(x-3)^2 - 2 = 2x^2 - 12x + 16 = 2(x-2)(x-4)$.

52. $f = a(x+2)^2 + 7$, $-20 = 9a + 7 \Rightarrow a = -3$. Thus $f = -3x^2 - 12x - 5$ and range $(-\infty, 7]$.

53. Axis $x = 4$; vertex $(4, -3)$. $f(x) = a(x-4)^2 - 3$ and $5 = 9a - 3 \Rightarrow a = \frac{8}{9}$. Hence $f(0) = \frac{8}{9}(16) - 3 = \frac{101}{9}$.

54. Reflection about $x = 2$ sends -1 to 5. $f = a(x+1)(x-5)$; $-12 = -5a \Rightarrow a = \frac{12}{5}$. Vertex $\left(2, -\frac{108}{5}\right)$.

55. $f_k = 2(x-2)^2 + k - 8$. Each axis is $x = 2$. Minimum is $k - 8$; setting $k - 8 = -3$ gives $k = 5$.

56. $k + 6 = 4a + k \Rightarrow a = \frac{3}{2}$; then $f(h-1) = \frac{3}{2}(-1)^2 + k = k + \frac{3}{2}$. Only the distance from the axis matters.

57. Axis $x = (-4+2)/2 = -1$. $f = a(x+4)(x-2)$, and $-27 = f(-1) = -9a \Rightarrow a = 3$. Thus $f = 3(x+4)(x-2)$; $f(0) = -24$.

58. $f = a(x+3)(x-1)$, $-3 = -3a \Rightarrow a = 1$. So $f = (x+3)(x-1) = x^2 + 2x - 3 = (x+1)^2 - 4$.

59. $f = a(x-1)(x-5)$; at $x = 3$, $-18 = -4a \Rightarrow a = \frac{9}{2}$. Thus $f(0) = \frac{45}{2}$ and $f(7) = \frac{9}{2}(6)(2) = 54$.

60. $-4 = 4a - 8 \Rightarrow a = 1$. $f = (x-3)^2 - 8 = x^2 - 6x + 1$; y-intercept $(0, 1)$. Reflect $x = 1$ across $x = 3$: $x = 5$, giving $(5, -4)$.