Finding sine and cosine coefficients

IB Mathematics: Analysis and Approaches | SL & HL

10 questions ▪ Read the curve ▪ Determine a, b, c and d

Learning goal	Recover a sine or cosine equation from its graph or key features.
Syllabus link	AA SL 3.7 (shared SL/HL): circular functions and transformations.
Structure	Questions 1–8: shared practice; Questions 9–10: HL challenge.
Calculator	Not required. A graphing calculator may be used to check your answers.

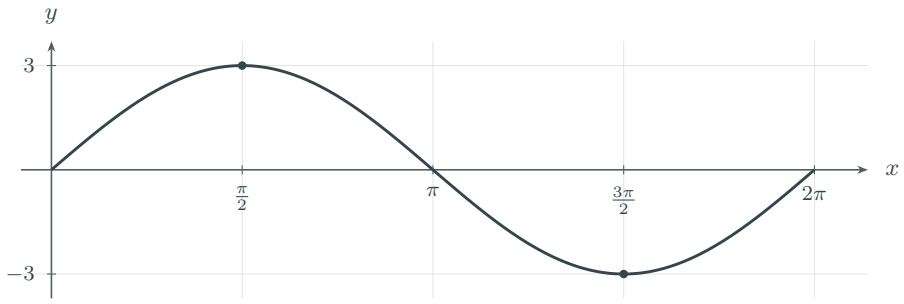
For every question: state the period T, find a, b, c and d, and write the equation in the specified form. Show your working.

Unless stated otherwise, use $a > 0$, $b > 0$ and $-\frac{T}{2} \le c < \frac{T}{2}$. Angles are in **radians**, except in Question 6. Give exact values.

1 Reading the curves

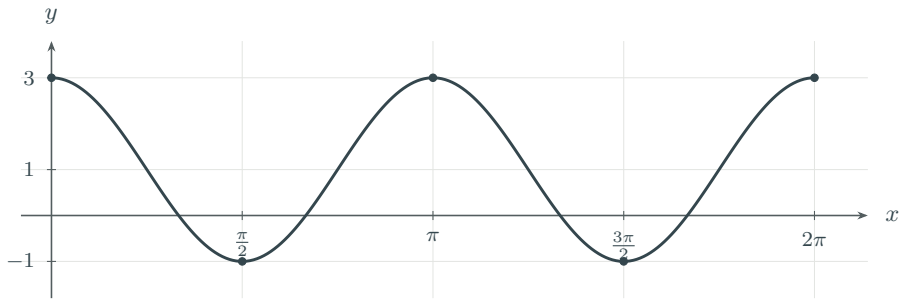
AA SL & HL

Question 1 Find the coefficients in $f(x) = a\sin\big(b(x-c)\big) + d$.



WORKING

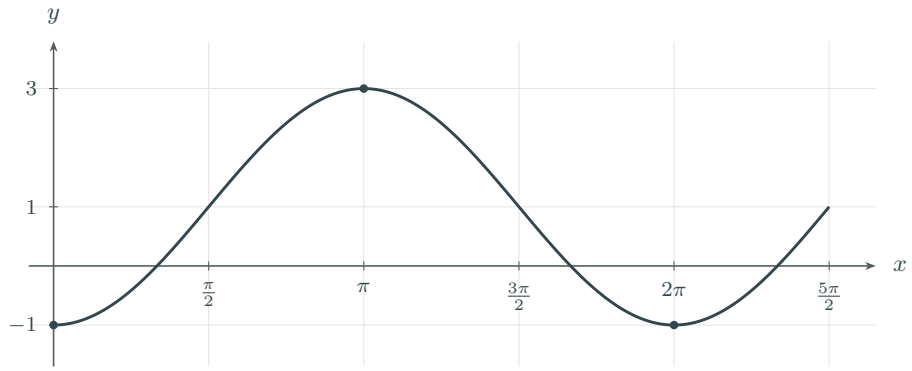
Question 2 Find the coefficients in $g(x) = a\cos\big(b(x-c)\big) + d$.



WORKING

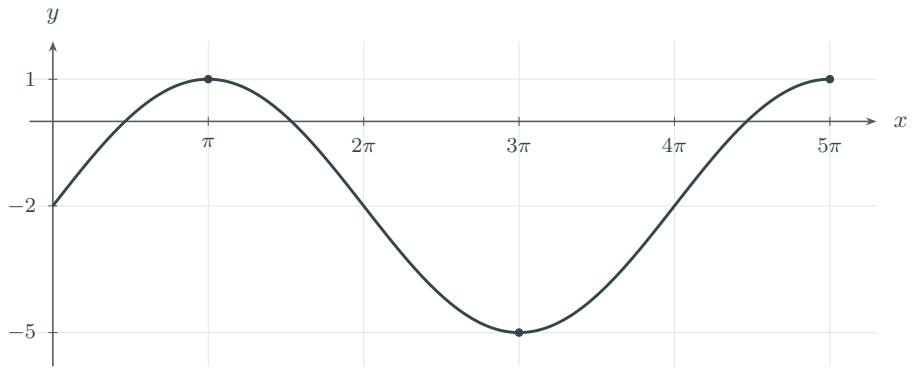
1 Reading the curves (continued) AA SL & HL

Question 3 Find the coefficients in $f(x) = a\sin\big(b(x-c)\big) + d$.



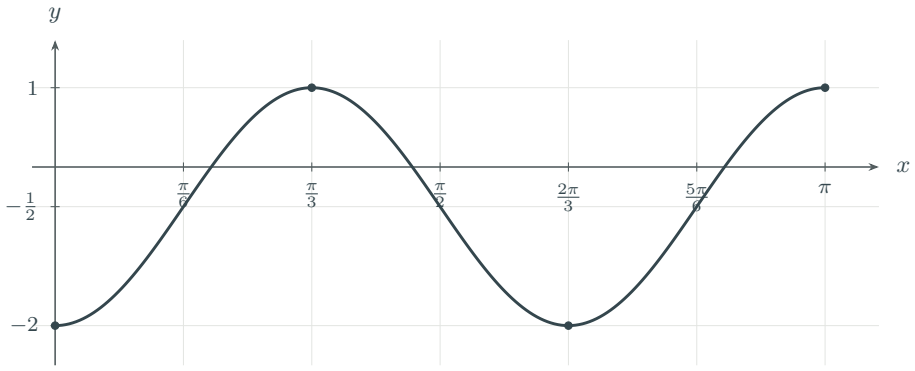
WORKING

Question 4 Find the coefficients in $g(x) = a\cos\big(b(x-c)\big) + d$.



WORKING

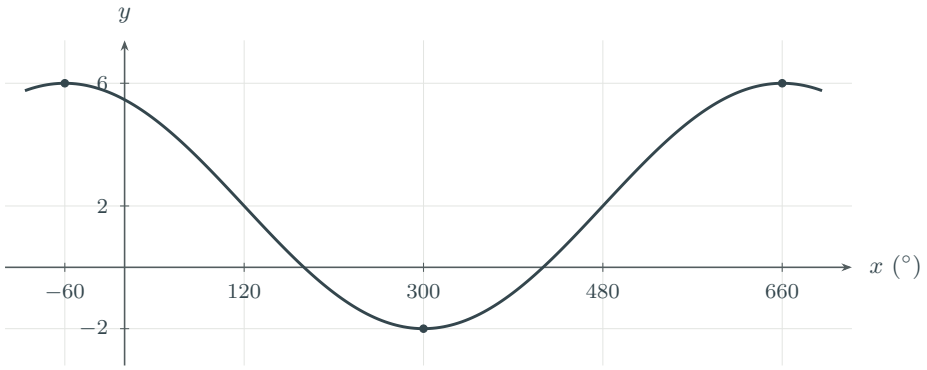
Question 5 Find the coefficients in $f(x) = a\sin\big(b(x-c)\big) + d$.



WORKING

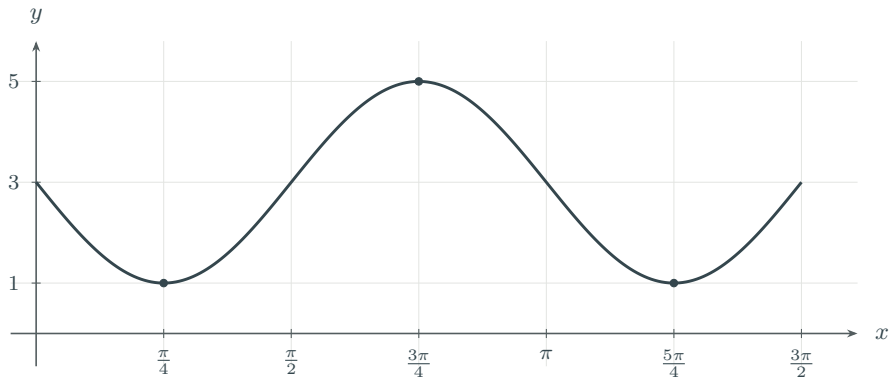
2 Signs, units and representations AA SL & HL

Question 6 Find the coefficients in $g(x) = a\cos\big(b(x-c)\big) + d$.
Here x and c are measured in **degrees**; interpret the cosine in degree mode.



WORKING

Question 7 Find the coefficients in $f(x) = a\sin\big(b(x-c)\big) + d$.
In this question, $c = 0$ **and** a **may be negative**. Also state the amplitude.



WORKING

Question 8 A cycle recorded in a table.
The first and last entries below are **consecutive maxima**. The middle entry is the minimum between them.

x	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$
$f(x)$	6	2	-2	2	6

(a) Find a, b, c and d for $f(x) = a\cos\big(b(x-c)\big) + d$.

(b) Write the same function in sine form, using the stated restrictions on the coefficients.

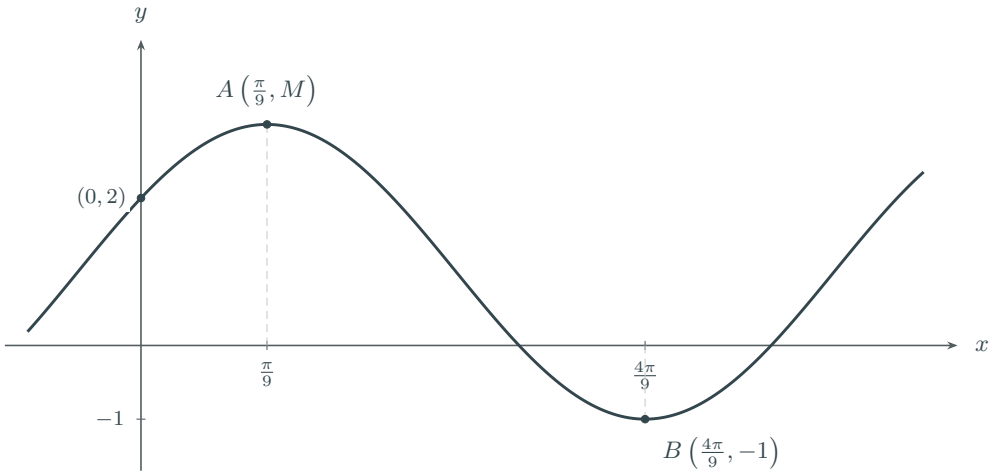
3 AA HL-exclusive challenge

Questions 9–10

These questions extend the algebra and interpretation within the same section 3.7 content.

Question 9 An unknown maximum.

The graph represents $f(x) = a\cos\big(b(x-c)\big) + d$, where $a > 0$ and $b > 0$. The points $A\big(\frac{\pi}{9}, M\big)$ and $B\big(\frac{4\pi}{9}, -1\big)$ are a maximum and the **next minimum**. It is also known that $f(0) = 2$.



The remaining vertical scale is omitted. Use the given coordinates to find exact values.

(a) Find a, b, c and d, and hence find M.

(b) Write an equivalent sine equation with a positive amplitude coefficient and $-\dfrac{T}{2} \le c < \dfrac{T}{2}$.

Question 10 A tidal model from partial information.

The depth of water, $h(t)$ metres, is modelled by

$$h(t) = a\cos\big(b(t-c)\big) + d,$$

where t is the number of hours after midnight and the cosine argument is in radians. Consecutive high tides occur at **02:00** and **14:00**. Two depth measurements are recorded:

Time	04:00	06:00
t (hours)	4	6
$h(t)$ (metres)	6.5	3.5

(a) Determine a, b, c and d, and write the model.

(b) State the maximum and minimum water depths predicted by the model.

Answer key and solution outlines

Questions 1–8

In the stated form, the amplitude is $|a|$, the principal axis is $y = d$, and c is the horizontal shift. With $b > 0$, $b = 2\pi/T$ in radians and $b = 360^\circ/T$ in degrees.

1 Reading the curves

1 Maximum 3, minimum -3: $a = 3$, $d = 0$. One cycle has length $T = 2\pi$, so $b = 1$. The rising crossing of the principal axis is at $x = 0$, giving $c = 0$.

$$\boxed{(a, b, c, d) = (3, 1, 0, 0)}, \qquad \boxed{f(x) = 3\sin x}.$$

2 Maximum 3, minimum -1: $a = 2$, $d = 1$. Consecutive maxima occur at $x = 0$ and $x = \pi$, so $T = \pi$, $b = 2$ and $c = 0$.

$$\boxed{(a, b, c, d) = (2, 2, 0, 1)}, \qquad \boxed{g(x) = 2\cos(2x) + 1}.$$

3 Maximum 3, minimum -1: $a = 2$, $d = 1$. The minima at 0 and 2π give $T = 2\pi$, so $b = 1$. The curve crosses $y = 1$ upwards at $x = \pi/2$.

$$\boxed{(a, b, c, d) = \left(2, 1, \frac{\pi}{2}, 1\right)}, \qquad \boxed{f(x) = 2\sin\left(x - \frac{\pi}{2}\right) + 1}.$$

4 Maximum 1, minimum -5: $a = 3$, $d = -2$. Maxima at π and 5π give $T = 4\pi$, so $b = \frac{1}{2}$. For positive cosine, use the maximum at $c = \pi$.

$$\boxed{(a, b, c, d) = \left(3, \frac{1}{2}, \pi, -2\right)}, \qquad \boxed{g(x) = 3\cos\left(\frac{1}{2}(x - \pi)\right) - 2}.$$

5 $a = \dfrac{1 - (-2)}{2} = \dfrac{3}{2}$, $d = \dfrac{1 + (-2)}{2} = -\dfrac{1}{2}$. Consecutive minima give $T = \frac{2\pi}{3}$, so $b = 3$. The rising crossing of $y = -\frac{1}{2}$ is at $c = \frac{\pi}{6}$.

$$\boxed{(a, b, c, d) = \left(\frac{3}{2}, 3, \frac{\pi}{6}, -\frac{1}{2}\right)}, \qquad \boxed{f(x) = \frac{3}{2}\sin\left(3\left(x - \frac{\pi}{6}\right)\right) - \frac{1}{2}}.$$

2 Signs, units and representations

6 Maximum 6, minimum -2: $a = 4$, $d = 2$. The maxima give $T = 660^\circ - (-60^\circ) = 720^\circ$, so $b = \frac{1}{2}$. The permitted phase shift is $c = -60^\circ$.

$$\boxed{(a, b, c, d) = \left(4, \frac{1}{2}, -60^\circ, 2\right)}, \qquad \boxed{g(x) = 4\cos\left(\frac{1}{2}(x + 60^\circ)\right) + 2}.$$

7 The amplitude is 2 and $d = 3$. The period is $T = \pi$, so $b = 2$. With $c = 0$, the curve crosses $y = 3$ downwards at $x = 0$, so $a = -2$.

$$\boxed{(a, b, c, d) = (-2, 2, 0, 3)}, \qquad \boxed{f(x) = -2\sin(2x) + 3}.$$

8 $a = 4$, $d = 2$ and $T = \frac{7\pi}{4} - \left(-\frac{\pi}{4}\right) = 2\pi$, so $b = 1$.

(a) $\boxed{(a, b, c, d) = \left(4, 1, -\dfrac{\pi}{4}, 2\right)}$, $\boxed{f(x) = 4\cos\left(x + \dfrac{\pi}{4}\right) + 2}$.

For sine, move a quarter-period left from the maximum: $c = -\frac{\pi}{4} - \frac{\pi}{2} = -\frac{3\pi}{4}$.

(b) $\boxed{f(x) = 4\sin\left(x + \dfrac{3\pi}{4}\right) + 2}$.

Answer key and solution outlines Questions 9–10

3 AA HL-exclusive challenge

9(a) A maximum and the next minimum are separated by half a period:

$$\frac{T}{2} = \frac{4\pi}{9} - \frac{\pi}{9} = \frac{\pi}{3} \quad \Longrightarrow \quad T = \frac{2\pi}{3}, \qquad b = \frac{2\pi}{T} = 3.$$

For a positive-amplitude cosine model, a maximum occurs at $x = c$. The permitted choice is $c = \frac{\pi}{9}$. At the minimum, $d - a = -1$. Using $f(0) = 2$ gives

$$a\cos\left(-\frac{\pi}{3}\right) + d = 2 \quad \Longrightarrow \quad \frac{a}{2} + d = 2.$$

Subtracting $d - a = -1$ from $\frac{a}{2} + d = 2$ gives $\frac{3a}{2} = 3$, hence $a = 2$ and $d = 1$.

$$\boxed{(a, b, c, d) = \left(2, 3, \frac{\pi}{9}, 1\right)}$$

$$\boxed{f(x) = 2\cos\left(3\left(x - \frac{\pi}{9}\right)\right) + 1}, \qquad \boxed{M = d + a = 3}.$$

9(b) The positive sine model crosses its principal axis upwards one quarter-period before the maximum:

$$c_{\sin} = \frac{\pi}{9} - \frac{T}{4} = \frac{\pi}{9} - \frac{\pi}{6} = -\frac{\pi}{18}.$$

$$\boxed{f(x) = 2\sin\left(3\left(x + \frac{\pi}{18}\right)\right) + 1}.$$

10(a) Consecutive high tides give $T = 14 - 2 = 12$ hours, so $b = \frac{2\pi}{12} = \frac{\pi}{6}$. The maximum at $t = 2$ gives $c = 2$ hours, which lies in $[-6, 6)$. Using the two measured depths:

$$h(4) = 6.5 \quad \Longrightarrow \quad a\cos\left(\frac{\pi}{3}\right) + d = 6.5 \quad \Longrightarrow \quad \frac{a}{2} + d = 6.5,$$

$$h(6) = 3.5 \quad \Longrightarrow \quad a\cos\left(\frac{2\pi}{3}\right) + d = 3.5 \quad \Longrightarrow \quad -\frac{a}{2} + d = 3.5.$$

Subtracting gives $a = 3$; adding gives $2d = 10$, so $d = 5$.

$$\boxed{(a, b, c, d) = \left(3, \frac{\pi}{6}, 2, 5\right)}$$

$$\boxed{h(t) = 3\cos\left(\frac{\pi}{6}(t - 2)\right) + 5}.$$

10(b) Maximum depth: $d + a = \boxed{8\text{ m}}$.
Minimum depth: $d - a = \boxed{2\text{ m}}$.

Checking equivalent answers. Adding any integer multiple of T to c gives the same function. The interval $-T/2 \le c < T/2$ selects one representative. Changing the sign of a and shifting c by $T/2$ also preserves the function, but does not satisfy the stated sign condition unless that condition is changed. In Question 7, $c = 0$ fixes the required negative value of a.

Syllabus reference: supplied *Mathematics: analysis and approaches guide*, SL 3.7, printed page 47. All questions are original focused practice.