

Binomial Distribution

IB Mathematics: Analysis and Approaches SL & HL — SL 4.8

60 questions: 50-question core worksheet + 10-question HL-only challenge section

Learning goal	Recognize binomial settings, calculate binomial probabilities using technology, and use the mean and variance of a binomial distribution.
Syllabus link	IB Mathematics AA SL/HL: SL 4.8 — binomial distribution; mean and variance; situations where the binomial distribution is an appropriate model.
Structure	Section 1: 15 short response; Section 2: 15 extended response; Section 3: 20 IB exam-style technology questions; Section 4: 10 HL-only challenge questions.
Calculator / technology	A GDC or appropriate calculator is required for binomial probability calculations . Give probabilities to 4 significant figures unless instructed otherwise.

Instructions. For a binomial random variable, write

$$X \sim B(n, p), \quad E(X) = np, \quad \text{Var}(X) = np(1 - p).$$

A binomial model requires a fixed number of trials, two outcomes per trial, a constant probability of success, and independent trials. Use technology for cumulative and individual binomial probabilities. Formal proofs of the mean and variance are not required.

Section 1: Short response

Questions 1–15

1. A random variable is modelled by $X \sim B(12, 0.30)$. State the number of trials and the probability of success on each trial.
2. For $X \sim B(18, 0.40)$, find $E(X)$.
3. For $X \sim B(20, 0.35)$, find $\text{Var}(X)$.
4. Let $X \sim B(10, 0.40)$. Find $P(X = 3)$.
5. Let $X \sim B(8, 0.25)$. Find $P(X \leq 2)$.
6. Let $X \sim B(15, 0.10)$. Find $P(X \geq 1)$.
7. If X counts the number of successes in 14 trials, write the event “at least 4 successes” using an inequality involving X .
8. A fair coin is tossed 20 times and X is the number of heads. State whether a binomial model is appropriate, giving a brief reason.

9. Five cards are drawn without replacement from a standard deck and X is the number of aces drawn. State whether X is exactly binomial, giving a brief reason.

10. For $X \sim B(12, p)$, it is given that $E(X) = 3.6$. Find p .

11. For $X \sim B(n, 0.25)$, it is given that $E(X) = 7.5$. Find n .

12. For $X \sim B(20, 0.35)$, find the mean and variance.

13. Let $X \sim B(6, 0.20)$. Find $P(X = 0)$.

14. The probability that a randomly selected item is defective is 0.07. In a sample of 40 independent items, find the expected number of defective items.

15. For $X \sim B(10, p)$, $\text{Var}(X) = 1.6$ and $E(X) < 5$. Find p .

Section 2: Extended response

Questions 16–30

16. A basketball player makes each free throw independently with probability 0.70. The player takes 8 free throws. Let X be the number made.

- (a) State the distribution of X .
- (b) Find $P(X = 6)$.
- (c) Find $P(X \geq 6)$.
- (d) Find the mean and variance of X .

17. Each light bulb produced by a factory is defective with probability 0.04, independently of the others. A box contains 20 bulbs. Let X be the number of defective bulbs.

- (a) Find $P(X = 0)$.
- (b) Find $P(X \geq 1)$.
- (c) Find $E(X)$ and $\text{Var}(X)$.

18. A bag contains 4 red and 6 blue counters.

- (a) Eight counters are selected one at a time *with replacement*. Explain why the number of red counters selected can be modelled by a binomial distribution, and state its parameters.
- (b) Eight counters are instead selected *without replacement*. Explain why the number of red counters selected is not exactly binomial.

19. For $X \sim B(15, p)$, the mean is 4.2.

- (a) Find p .
- (b) Hence find $P(X \geq 6)$.

20. A binomial random variable X has mean 6 and variance 4.2.

- (a) Find p .
 - (b) Find n .
 - (c) Hence find $P(X \leq 4)$.
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21. For $X \sim B(10, p)$, the variance is 1.6 and the mean is less than 5.

- (a) Find p .
 - (b) Find $P(X \geq 3)$.
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22. A binomial random variable has mean 4 and variance 3.

- (a) Find n and p .
 - (b) Find $P(3 \leq X \leq 6)$.
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23. A multiple-choice quiz contains 10 questions, each with four possible answers and exactly one correct answer. A student guesses independently on every question. Let X be the number of correct answers.

- (a) State the distribution of X .
 - (b) Find $P(X = 3)$.
 - (c) Find $P(X \geq 1)$.
 - (d) Find the mean and variance of X .
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24. The probability that a randomly observed car is speeding is 0.12. Assume the observations are independent. Among 40 cars, let X be the number speeding.

- (a) Find $P(X \leq 3)$.
 - (b) Find the mean and variance of X .
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25. A bag contains 4 red and 6 blue counters. A counter is drawn, its colour recorded, and then it is replaced. This is repeated 8 times. Let X be the number of red counters drawn.

- (a) State the distribution of X .
 - (b) Find $P(X = 3)$.
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26. A treatment is successful for a patient with probability 0.85, independently from patient to patient. It is given to 12 patients. Let X be the number for whom it is successful.

- (a) Find $P(X \geq 10)$.
 - (b) Find $E(X)$ and $\text{Var}(X)$.
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27. For $X \sim B(n, 0.60)$, the expected number of successes is 18.

- (a) Find n .
- (b) Find $P(X \geq 20)$.

28. Let $X \sim B(5, p)$. It is given that $P(X = 0) = 0.16807$.

- (a) Find p .
- (b) Find the mean and variance of X .
- (c) Find $P(X \leq 1)$.

29. Let $X \sim B(6, p)$. It is given that $P(X = 6) = 0.117649$.

- (a) Find p .
- (b) Find $P(X \geq 5)$.

30. Two binomial random variables are defined by $X \sim B(20, 0.30)$ and $Y \sim B(12, 0.50)$.

- (a) Show that X and Y have the same mean.
- (b) Find the variance of each distribution.
- (c) State which distribution is more spread out according to variance.

Section 3: IB exam-style / technology

Questions 31–50 Original questions

31. A hotel records that each booking is cancelled independently with probability 0.08. On a particular day there are 25 bookings. Let X be the number cancelled.

- (a) State the distribution of X . [1 marks]
- (b) Find the probability that no bookings are cancelled. [2 marks]
- (c) Find the probability that at least 3 bookings are cancelled. [2 marks]
- (d) Find the mean and variance of X . [2 marks]

32. A player scores a penalty with probability 0.65, independently on each attempt. The player takes 10 penalties. Let X be the number scored.

- (a) Find $P(X = 7)$. [2 marks]
- (b) Find $P(X > 7)$. [2 marks]
- (c) Find $E(X)$ and $\text{Var}(X)$. [2 marks]
- (d) State one assumption, other than a fixed number of attempts, required for the binomial model. [1 marks]

33. A binomial random variable X has mean 7.2 and variance 5.04.

- (a) Show that $p = 0.30$. [2 marks]
- (b) Find n . [1 marks]
- (c) Find $P(5 \leq X \leq 9)$. [3 marks]

34. A manufacturer produces components independently, each with probability 0.03 of being defective. A sample of 50 components is inspected. Let X be the number defective.

- (a) Find $P(X \leq 2)$. [2 marks]
- (b) Find $P(X \geq 1)$. [2 marks]
- (c) Find the mean and variance of X . [2 marks]

35. A test contains 15 independent multiple-choice questions. Each question has five options, exactly one of which is correct. A candidate guesses every answer. Let X be the number correct.

- (a) State the distribution of X . [1 marks]
 - (b) Find the probability that the candidate answers at least 5 questions correctly. [2 marks]
 - (c) Find the mean and variance of X . [2 marks]
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36. A particular type of seed germinates independently with the same probability p . In a packet of 20 seeds, the expected number that germinate is 16.

- (a) Find p . [2 marks]
 - (b) Find the probability that at least 18 seeds germinate. [2 marks]
 - (c) Find the variance of the number that germinate. [1 marks]
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37. At a call centre, each customer independently requires a follow-up call with probability 0.35. During one period 18 customers call. Let X be the number requiring follow-up.

- (a) Find $P(X = 6)$. [2 marks]
 - (b) Find $P(X \leq 5)$. [2 marks]
 - (c) Find the mean and variance of X . [2 marks]
 - (d) Explain why a changing follow-up probability from one customer to another would invalidate the binomial model. [1 marks]
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38. In a large population, 40% of people support a proposal. A random sample of 30 people is modelled as independent selections. Let X be the number who support the proposal.

- (a) Find $P(X \geq 15)$. [2 marks]
 - (b) Find $E(X)$ and $\text{Var}(X)$. [2 marks]
 - (c) Interpret $E(X)$ in this context. [1 marks]
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39. Let $X \sim B(n, 0.25)$. It is given that $\text{Var}(X) = 3.75$.

- (a) Find n . [2 marks]
 - (b) Find $E(X)$. [1 marks]
 - (c) Find $P(X \geq 8)$. [2 marks]
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40. A binomial random variable has mean 9 and variance 6.3.

- (a) Find p and n . [3 marks]
 - (b) Find $P(7 \leq X \leq 11)$. [3 marks]
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41. A machine independently produces a particular fault on any item with probability p . In 14 items, the probability of no faults is 0.2288, correct to 4 decimal places.

- (a) Find p , giving your answer to 3 significant figures. [2 marks]
- (b) Hence find the probability of at least 3 faults. [2 marks]
- (c) Find the expected number of faults. [1 marks]

42. A printer produces a visible fault on each page independently with probability 0.04. In a batch of 60 pages, let X be the number with a visible fault.

- (a) Find $P(X \leq 2)$. [2 marks]
- (b) Find $P(X \geq 1)$. [2 marks]
- (c) Find the mean and variance of X . [2 marks]

43. A box contains 5 green and 7 yellow counters.

- (a) Ten counters are drawn without replacement. Explain why the number of green counters drawn is not exactly binomial. [2 marks]
- (b) The experiment is changed so that each counter is replaced before the next draw. Let X be the number of green counters in 10 draws. State the distribution of X . [1 marks]
- (c) Find $P(X = 4)$. [2 marks]

44. A binomial random variable has mean 5 and variance 4.

- (a) Find p and n . [3 marks]
- (b) Find $P(X \geq 8)$. [2 marks]

45. A company receives a complaint from any given customer with probability 0.06, independently of other customers. From 40 customers, let X be the number who complain.

- (a) Find $P(1 \leq X \leq 4)$. [2 marks]
- (b) Find $P(X \geq 1)$. [2 marks]
- (c) Find the mean and variance of X . [2 marks]

46. On a particular day, each of 120 students is absent independently with probability 0.08. Let X be the number absent.

- (a) Find the expected number absent. [1 marks]
- (b) Find the variance of X . [1 marks]
- (c) Find $P(X \geq 12)$. [2 marks]

47. A fair six-sided die is rolled 12 times. Let X be the number of sixes.

- (a) State the distribution of X . [1 marks]
- (b) Find $P(X \geq 3)$. [2 marks]
- (c) A game pays €20 if $X \geq 3$, and pays nothing otherwise. Find the entry fee that would make the game fair. [2 marks]

48. Each component in a system works independently with probability 0.97. The system contains 10 components and operates successfully if at least 9 components work. Let X be the number of working components.

- (a) Find the probability that the system operates successfully. [2 marks]
- (b) Find the expected number of working components. [1 marks]
- (c) Find the variance of X . [1 marks]
- (d) Find the expected number of failed components. [1 marks]

49. Each of 60 applicants independently accepts an offer with probability 0.40. Let X be the number who accept.
- Find the mean and variance of X . [2 marks]
 - There are 28 places available. Find the probability that more applicants accept than there are places available. [2 marks]

50. For $X \sim B(25, p)$, it is known that $P(X = 0) = 0.0718$, correct to 4 decimal places.
- Find p , giving your answer to 3 significant figures. [2 marks]
 - Find $P(X \geq 3)$. [2 marks]
 - Find the mean and variance of X . [2 marks]
 - State one condition that must hold for the binomial model to be appropriate. [1 marks]

Section 4: HL-only challenge

Questions 51–60 SL 4.8 content at greater depth

HL note. These questions stay within SL 4.8. Their additional difficulty comes from inverse parameter problems, algebraic relationships between binomial probabilities, threshold decisions and model critique, rather than from introducing content outside this syllabus section.

51. A binomial random variable X has mean μ and variance v , where $0 < v < \mu$.
- Using $\mu = np$ and $v = np(1 - p)$, show that
$$p = 1 - \frac{v}{\mu}.$$
 [2 marks]
 - Hence obtain an expression for n in terms of μ and v . [2 marks]
 - For $\mu = 12$ and $v = 9$, find n , p , and $P(X \geq 15)$. [4 marks]

52. Let $X \sim B(n, p)$, where $0 < p < 1$. It is given that $P(X = 0) = P(X = 1)$.
- Show that $p = \frac{1}{n + 1}$. [4 marks]
 - For $n = 9$, find the mean and variance of X . [2 marks]
 - Find $P(X \geq 2)$. [2 marks]

53. Let $X \sim B(n, p)$, with $n > 3$. It is given that $P(X = 2) = P(X = 3)$.
- Show that $p = \frac{3}{n + 1}$. [4 marks]
 - For $n = 11$, find the mean and variance of X . [2 marks]
 - Find $P(X \geq 4)$. [2 marks]

54. Two binomial random variables X and Y both have mean 6. It is given that $\text{Var}(X) = 4.5$ and $Y \sim B(12, 0.5)$.
- Find the parameters of the distribution of X . [3 marks]
 - Find $\text{Var}(Y)$. [1 marks]
 - Which distribution is more concentrated around its mean, according to variance? [1 marks]
 - Find $P(4 \leq X \leq 8)$. [2 marks]

55. A particular event occurs independently on each trial with probability 0.20. Find the smallest integer n such that the probability of at least one occurrence in n trials is at least 0.95. Justify your answer. [5 marks]

56. For independent trials with success probability 0.40, let X be the number of successes in n trials. Find the smallest integer n such that

$$P(X \geq 3) \geq 0.90.$$

Show sufficient calculator evidence to justify that your value of n is the smallest possible. [5 marks]

57. Let $X \sim B(8, p)$. It is given that $P(X \geq 1) = 0.90$.

- (a) Find p . [3 marks]
- (b) Find the mean and variance of X . [2 marks]
- (c) Find $P(X \geq 4)$. [2 marks]

58. Let $X \sim B(10, p)$. It is given that $P(X \leq 1) = 0.85$.

- (a) Use technology to find p , giving your answer to 4 significant figures. [3 marks]
- (b) Find the mean and variance of X . [2 marks]
- (c) Explain why $P(X \geq 2) = 0.15$ follows immediately from the information given. [1 marks]

59. Let $X \sim B(10, p)$, where $0 < p < 1$. It is given that $P(X = 0) = P(X = 2)$.

- (a) Show that

$$\frac{p}{1-p} = \sqrt{\frac{1}{45}}.$$

- (b) Hence find p . [4 marks]
- (c) Find $P(X \geq 3)$. [2 marks]

60. A basketball player takes 10 shots. Historical data suggest that the probability of scoring is 0.80 for each of the first 5 shots and 0.60 for each of the final 5 shots. Assume shot outcomes are independent.

- (a) Explain why the total number of successful shots is *not* binomial. [2 marks]
- (b) An analyst nevertheless models the total as $Y \sim B(10, 0.70)$. Find $E(Y)$, $\text{Var}(Y)$, and $P(Y \geq 8)$. [4 marks]
- (c) Find the true expected total number of successful shots from the information given, and explain why matching the expected value does not make the binomial model valid. [3 marks]

Answer key and solution outlines

Questions 1–60

Section 1: Short response

1. $n = 12$, $p = 0.30$. 2. $E(X) = 18(0.40) = 7.2$. 3. $\text{Var}(X) = 20(0.35)(0.65) = 4.55$.
4. $P(X = 3) = 0.2150$. 5. $P(X \leq 2) = 0.6785$. 6. $P(X \geq 1) = 1 - P(X = 0) = 0.7941$.
7. $X \geq 4$. 8. Yes: fixed 20 trials, two outcomes, constant $p = 0.5$, independent tosses.
9. No: without replacement, the success probability changes and trials are not independent.
10. $12p = 3.6 \Rightarrow p = 0.30$. 11. $0.25n = 7.5 \Rightarrow n = 30$.
12. Mean = 7, variance = 4.55. 13. $P(X = 0) = 0.8^6 = 0.2621$. 14. $40(0.07) = 2.8$.
15. $10p(1 - p) = 1.6 \Rightarrow p = 0.2$ or 0.8 . Since $E(X) = 10p < 5$, $p = 0.20$.

Section 2: Extended response

16. $X \sim B(8, 0.70)$. $P(X = 6) = 0.2965$; $P(X \geq 6) = 0.5518$; mean = 5.6, variance = 1.68.
17. $P(X = 0) = 0.4420$; $P(X \geq 1) = 0.5580$; mean = 0.8, variance = 0.768.
18. (a) With replacement gives fixed $n = 8$, two outcomes, constant $p = 0.4$, independent trials: $X \sim B(8, 0.4)$.
(b) Without replacement, p changes from draw to draw and draws are dependent.
19. $15p = 4.2 \Rightarrow p = 0.28$; $P(X \geq 6) = 0.2220$.
20. Since $4.2 = 6(1 - p)$, $p = 0.30$; then $n = 6/0.30 = 20$. $P(X \leq 4) = 0.2375$.
21. $p = 0.20$; $P(X \geq 3) = 0.3222$.
22. $3 = 4(1 - p) \Rightarrow p = 0.25$; $n = 4/0.25 = 16$. $P(3 \leq X \leq 6) = 0.7233$.
23. $X \sim B(10, 0.25)$; $P(X = 3) = 0.2503$; $P(X \geq 1) = 0.9437$; mean = 2.5, variance = 1.875.
24. $P(X \leq 3) = 0.2768$; mean = 4.8, variance = 4.224.
25. $X \sim B(8, 0.4)$; $P(X = 3) = 0.2787$.
26. $P(X \geq 10) = 0.7358$; mean = 10.2, variance = 1.53.
27. $0.6n = 18 \Rightarrow n = 30$; $P(X \geq 20) = 0.2915$.
28. $(1 - p)^5 = 0.16807 = 0.7^5 \Rightarrow p = 0.30$. Mean = 1.5, variance = 1.05; $P(X \leq 1) = 0.5282$.
29. $p^6 = 0.117649 = 0.7^6 \Rightarrow p = 0.70$; $P(X \geq 5) = 0.4202$.
30. Both means are 6. $\text{Var}(X) = 4.2$, $\text{Var}(Y) = 3$; X is more spread out by variance.

Section 3: IB exam-style / technology

31. $X \sim B(25, 0.08)$; $P(X = 0) = 0.1244$; $P(X \geq 3) = 0.3232$; mean = 2, variance = 1.84.
32. $P(X = 7) = 0.2522$; $P(X > 7) = 0.2616$; mean = 6.5, variance = 2.275. Assumption: constant success probability and/or independence.
33. $5.04 = 7.2(1 - p) \Rightarrow p = 0.30$; $n = 24$; $P(5 \leq X \leq 9) = 0.7361$.
34. $P(X \leq 2) = 0.8108$; $P(X \geq 1) = 0.7819$; mean = 1.5, variance = 1.455.
35. $X \sim B(15, 0.20)$; $P(X \geq 5) = 0.1642$; mean = 3, variance = 2.4.
36. $20p = 16 \Rightarrow p = 0.80$; $P(X \geq 18) = 0.2061$; variance = 3.2.
37. $P(X = 6) = 0.1941$; $P(X \leq 5) = 0.3550$; mean = 6.3, variance = 4.095. A changing probability violates the constant- p condition.
38. $P(X \geq 15) = 0.1754$; mean = 12, variance = 7.2. Interpretation: over many comparable samples, the average number supporting the proposal would be 12 per sample of 30.
39. $0.1875n = 3.75 \Rightarrow n = 20$; mean = 5; $P(X \geq 8) = 0.1018$.
40. $6.3 = 9(1 - p) \Rightarrow p = 0.30$; $n = 30$; $P(7 \leq X \leq 11) = 0.6812$.
41. $(1 - p)^{14} = 0.2288 \Rightarrow p \approx 0.100$; $P(X \geq 3) = 0.1584$; expected faults = 1.4.
42. $P(X \leq 2) = 0.5676$; $P(X \geq 1) = 0.9136$; mean = 2.4, variance = 2.304.
43. Without replacement, success probability changes and draws are dependent. With replacement, $X \sim B(10, 5/12)$; $P(X = 4) = 0.2494$.
44. $4 = 5(1 - p) \Rightarrow p = 0.20$; $n = 25$; $P(X \geq 8) = 0.1091$.
45. $P(1 \leq X \leq 4) = 0.8263$; $P(X \geq 1) = 0.9158$; mean = 2.4, variance = 2.256.
46. Mean = 9.6; variance = 8.832; $P(X \geq 12) = 0.2523$.
47. $X \sim B(12, 1/6)$; $P(X \geq 3) = 0.3226$. Fair fee = $20(0.3225738) \approx \text{€}6.45$.
48. $P(X \geq 9) = 0.9655$; mean working = 9.7; variance = 0.291; expected failures = $10 - 9.7 = 0.3$.
49. Mean = 24, variance = 14.4; $P(X > 28) = 0.1184$.
50. $(1 - p)^{25} = 0.0718 \Rightarrow p \approx 0.100$; $P(X \geq 3) = 0.4629$; mean = 2.5, variance = 2.25. One required condition: independent trials, constant p , two outcomes, or fixed n .

Section 4: HL-only challenge

51. $v = \mu(1 - p) \Rightarrow p = 1 - v/\mu$. Then

$$n = \frac{\mu}{p} = \frac{\mu^2}{\mu - v}.$$

For $\mu = 12, v = 9$: $p = 0.25$, $n = 48$, and $P(X \geq 15) = 0.1999$.

52. $(1 - p)^n = np(1 - p)^{n-1} \Rightarrow 1 - p = np \Rightarrow p = 1/(n + 1)$. For $n = 9$, $p = 0.1$, mean = 0.9, variance = 0.81, and $P(X \geq 2) = 0.2252$.

53. Equating

$$\binom{n}{2} p^2 (1 - p)^{n-2} = \binom{n}{3} p^3 (1 - p)^{n-3}$$

gives $3(1 - p) = (n - 2)p$, hence $p = 3/(n + 1)$. For $n = 11$, $p = 0.25$, mean = 2.75, variance = 2.0625, and $P(X \geq 4) = 0.2867$.

54. $4.5 = 6(1 - p) \Rightarrow p = 0.25$, so $n = 24$. $\text{Var}(Y) = 12(0.5)(0.5) = 3$. Therefore Y is more concentrated. $P(4 \leq X \leq 8) = 0.7637$.

55. Need $1 - 0.8^n \geq 0.95$, so $0.8^n \leq 0.05$. Thus

$$n \geq \frac{\ln(0.05)}{\ln(0.8)} \approx 13.43,$$

so the smallest integer is 14.

56. Technology gives $P(X \geq 3) = 0.8811$ for $n = 11$ and 0.9166 for $n = 12$. Therefore the smallest value is 12.

57. $1 - (1 - p)^8 = 0.90 \Rightarrow p = 1 - 0.1^{1/8} \approx 0.2501$. Mean ≈ 2.001 , variance ≈ 1.500 , and $P(X \geq 4) \approx 0.1140$.

58. Solving $P(X \leq 1) = 0.85$ for $X \sim B(10, p)$ gives $p \approx 0.06951$. Mean ≈ 0.6951 , variance ≈ 0.6468 . By complement, $P(X \geq 2) = 1 - 0.85 = 0.15$.

59. From $P(X = 0) = P(X = 2)$,

$$(1 - p)^{10} = 45p^2(1 - p)^8 \Rightarrow \left(\frac{p}{1 - p} \right)^2 = \frac{1}{45}.$$

Since $p > 0$, $p/(1 - p) = 1/\sqrt{45}$, giving $p \approx 0.1297$. Then $P(X \geq 3) \approx 0.1301$.

60. Not binomial because the success probability is not constant across all 10 shots. For $Y \sim B(10, 0.70)$, mean = 7, variance = 2.1, and $P(Y \geq 8) = 0.3828$. The true expected total is $5(0.8) + 5(0.6) = 7$. Equal expected values do not restore the constant- p condition, so the binomial model is still invalid.