Normal distribution

IB Mathematics: Analysis and Approaches SL & HL | Section 4.9

50 shared questions + 10 HL-exclusive challenges

Learning goal	Interpret normal models; calculate probabilities, percentiles and cutoffs.
Syllabus link	AA SL 4.9, shared by SL and HL: properties, diagrams, the empirical rule, normal and inverse-normal calculations with given mean and standard deviation.
Structure	1–15 short response; 16–30 extended response; 31–50 original IB exam-style; 51–60 HL-exclusive challenge; answer key at the back.
Technology	NC = no calculator. GDC = calculator required. Numerical normal probabilities and inverse-normal values use technology. Questions 31–35 are Paper 1 style; 36–50 are Paper 2 style.

Instructions. Work on separate paper and show your method. Use $X \sim \mathrm{N}(\mu, \sigma^2)$: the second parameter is the *variance*. Give exact answers where appropriate; otherwise use 3 significant figures unless instructed differently. Retain unrounded values in working; round estimated counts to the nearest whole number. Use 68%, 95% and 99.7% only when the empirical rule is requested.
Scope. The HL section deepens 4.9; it does not require z-scores or finding unknown means or standard deviations (4.12). Sketches may be drawn on separate paper unless axes are supplied.

1. Short response

1. NC

Let $X \sim \mathrm{N}(42, 3.5^2)$. Write down:

(a) the mean;

(b) the standard deviation;

(c) the variance.

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2. NC

A normal random variable has mean 68 and standard deviation 4.

(a) Write down its median and mode.

(b) State the equation of the vertical line of symmetry of its density curve.

(c) State the total area under the density curve.

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3. NC

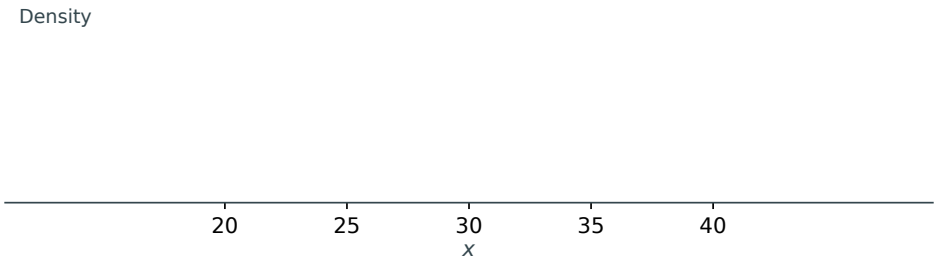
Let $A \sim \mathrm{N}(20, 4)$ and $B \sim \mathrm{N}(20, 16)$.

(a) Find the standard deviation of each variable.

(b) Which density curve is narrower and has a higher peak? Give a reason.

4. NC

Let $X \sim \mathrm{N}(30, 5^2)$. Use the supplied axes.



(a) Sketch its density curve and shade the region representing $\mathrm{P}(X > 35)$.

(b) Write down $\mathrm{P}(X < 30)$.

5. NC

Let $X \sim \mathrm{N}(100, 8^2)$. Use the 68–95–99.7 rule to estimate:

(a) $\mathrm{P}(92 < X < 108)$;

(b) $\mathrm{P}(84 < X < 116)$;

(c) $\mathrm{P}(X > 124)$.

6. NC

Let $T \sim \mathrm{N}(50, 6^2)$. Use the 68–95–99.7 rule to estimate:

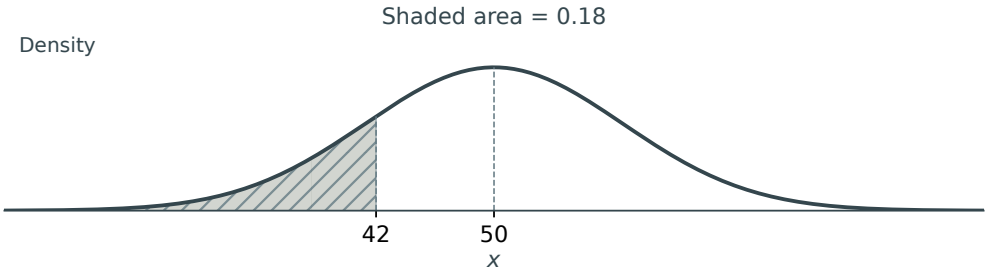
(a) $\mathrm{P}(50 < T < 56)$;

(b) $\mathrm{P}(56 < T < 62)$;

(c) $\mathrm{P}(T < 38)$.

7. NC

X is normally distributed with mean 50. The diagram shows that $\mathrm{P}(X < 42) = 0.18$.



(a) Find $\mathrm{P}(X > 58)$.

(b) Find $\mathrm{P}(42 < X < 58)$.

8. NC

Let $X \sim \text{N}(12, 2^2)$.

(a) Write down $\text{P}(X = 12)$.

(b) Explain why $\text{P}(X < 15) = \text{P}(X \leq 15)$.

9. GDC

Let $X \sim \text{N}(70, 9^2)$. Find $\text{P}(X < 76)$.

10. GDC

Let $X \sim \text{N}(120, 15^2)$. Find $\text{P}(X > 142)$.

11. GDC

Let $X \sim \text{N}(24, 3^2)$. Find $\text{P}(20 < X < 28)$.

12. GDC

Let $X \sim \text{N}(500, 18^2)$. Find the value of a such that $\text{P}(X < a) = 0.90$.

13. GDC

Let $T \sim \text{N}(45, 7^2)$. Find b such that $\text{P}(T > b) = 0.12$.

14. GDC

Let $X \sim \text{N}(60, 8^2)$. Find the endpoints of the central 90% interval.

15. NC

Give one reason why a normal model might be unsuitable for each variable.

(a) The number of messages received in one hour.

(b) The waiting time for a bus when many waits are short but a few are very long.

2. Extended response

16. GDC

The mass M grams of a packet is modelled by $M \sim \mathrm{N}(500, 6^2)$.

(a) Find the probability that a packet has mass below 490 g.

(b) Find the probability that its mass is between 494 g and 508 g.

(c) In a batch of 2000 packets, estimate the number with mass below 490 g.

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17. GDC

A journey time T minutes is modelled by $T \sim \mathrm{N}(32, 4.5^2)$.

(a) Find $\mathrm{P}(T > 40)$.

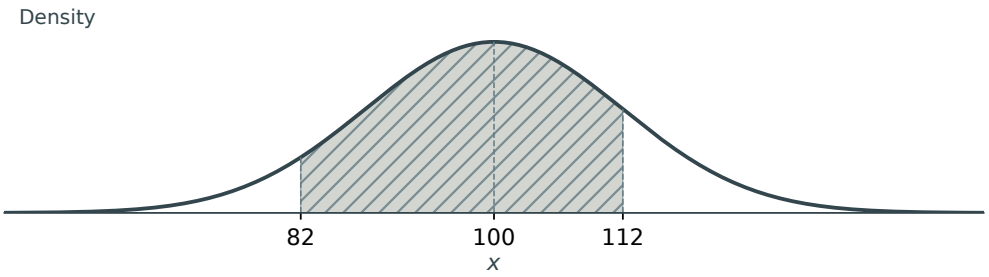
(b) Find the time exceeded by only 5% of journeys.

(c) A traveller allows 40 minutes. Does this give at least a 95% chance of arriving on time? Justify your answer.

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18. GDC

Let $X \sim \mathrm{N}(100, 12^2)$. The shaded region is shown below.



(a) Write the shaded area as a probability.

(b) Calculate this probability.

(c) Find the probability represented by the unshaded area under the curve.

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19. GDC

The heights H centimetres of plants are modelled by $H \sim \mathrm{N}(72, 8^2)$.

(a) Find the lower quartile and the upper quartile.

(b) Hence find the interquartile range.

(c) Interpret the interquartile range in this context.

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20. NC

Test scores are modelled by $S \sim \mathrm{N}(64, 10^2)$. There are 800 candidates. Use the 68–95–99.7 rule.

(a) Estimate the number with scores between 54 and 74.

(b) Estimate the number with scores above 84.

(c) Explain why the model does not guarantee these exact counts.

21. GDC

Bottle volumes V millilitres are modelled by $V \sim \mathrm{N}(330, 4^2)$. Bottles with volume below 324 ml or above 337 ml are rejected.

(a) Find the probability that a bottle is rejected.

(b) Estimate the number accepted in a production run of 5000 bottles.

(c) Which rejection tail is larger? Support your answer with probabilities.

22. GDC

The lifetime L hours of a component is modelled by $L \sim \mathrm{N}(1500, 180^2)$.

(a) Find the probability that it lasts longer than 1750 hours.

(b) A warranty is set so that only 2% of components fail before it expires. Find the warranty duration.

(c) If the duration must be a whole number of hours and the failure probability must not exceed 2%, state the longest allowable warranty.

23. GDC

Two machines produce rods with lengths (in mm) modelled by $A \sim \mathrm{N}(50, 0.4^2)$ and $B \sim \mathrm{N}(50, 0.7^2)$.

(a) Without calculating, state which machine has a higher proportion of lengths between 49.5 and 50.5 mm. Explain.

(b) Find both proportions.

(c) For a 1000-rod run from each machine, estimate the difference in the numbers outside this tolerance.

24. NC

A calculator gives the following probabilities for a normal random variable X.

x	12	18	24
$\mathrm{P}(X \le x)$	0.10	0.50	0.90

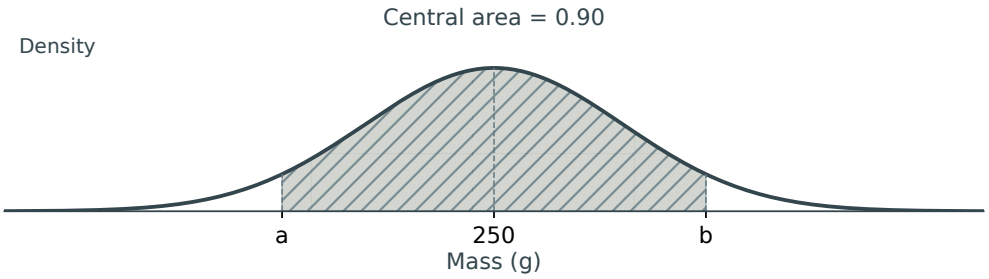
(a) Find $\mathrm{P}(12 < X < 24)$.

(b) Find $\mathrm{P}(X > 18)$.

(c) State the mean of X and justify your answer.

25. GDC

Product masses M grams follow $M \sim \mathrm{N}(250, 12^2)$. The central 90% of masses lie between a and b, as shown.



(a) Find a and b.

(b) Find the width $b - a$ of this interval.

(c) State the percentage of products with mass greater than b.

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26. GDC

The time T seconds for a runner to complete a race follows $T \sim \mathrm{N}(14.2, 0.6^2)$.

(a) Find the time below which the fastest 8% of performances lie.

(b) Find the time above which the slowest 15% lie.

(c) Find the percentage of performances between these two cutoffs.

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27. GDC

Let $X \sim \mathrm{N}(80, 5^2)$. A student uses 0.07 as the left-tail input to find c in $\mathrm{P}(X > c) = 0.07$.

(a) Explain the error and state the correct left-tail input.

(b) Find the correct value of c.

(c) Explain, without further calculation, where the student's incorrect cutoff lies relative to 80.

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28. GDC

The daily output X kilograms of a process is modelled by $X \sim \mathrm{N}(18, 2.5^2)$.

(a) Find $\mathrm{P}(15 < X < 22)$.

(b) Find the output exceeded on 90% of days.

(c) Estimate the number of days in a 240-day year with output above 22 kg.

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29. GDC

Let $X \sim \mathrm{N}(40, 6^2)$. Find the positive number d such that $\mathrm{P}(40 - d < X < 40 + d) = 0.92$.

(a) State the probability in each tail.

(b) Find d.

(c) Find $\mathrm{P}(X < 40 - d \text{ or } X > 40 + d)$.

30. GDC

A model for the mass M kilograms of a small parcel is $M \sim \mathrm{N}(0.40, 0.25^2)$.

(a) Find $\mathrm{P}(M < 0)$.

(b) Explain why this probability reveals a limitation of the model.

(c) Would this model be convincing for estimating very small parcel masses? Justify your answer.

3. IB exam-style questions

All questions are original. Marks are shown in brackets. **31–35: NC / Paper 1 style. 36–50: GDC / Paper 2 style.**

31. NC Maximum mark: 5

A measurement X is normally distributed with mean 24. It is given that $\mathrm{P}(X < 19) = 0.12$.

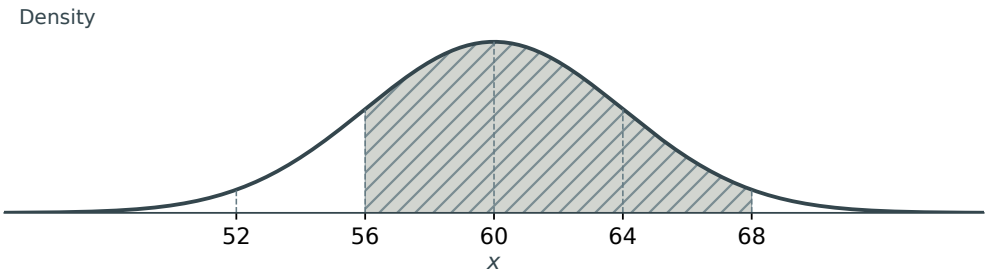
(a) Find $\mathrm{P}(X > 29)$. [1]

(b) Find $\mathrm{P}(19 < X < 29)$. [2]

(c) Find $\mathrm{P}(24 < X < 29)$. [2]

32. NC Maximum mark: 6

Let $X \sim \mathrm{N}(60, 4^2)$. Use the 68–95–99.7 rule.



(a) Estimate the probability represented by the shaded region. [3]

(b) Estimate $\mathrm{P}(X > 68)$. [2]

(c) For 400 observations, estimate the number lying between 56 and 68. [1]

33. NC Maximum mark: 5

A normal random variable X has mean μ and standard deviation $\sigma > 0$.

(a) Write down $\mathrm{P}(X < \mu)$. [1]

(b) Explain why $\mathrm{P}(X = \mu) = 0$, although the density curve is highest at μ. [2]

(c) Use the 68–95–99.7 rule to estimate $\mathrm{P}(\mu + \sigma < X < \mu + 2\sigma)$. [2]

34. NC Maximum mark: 6

The models $A \sim \mathrm{N}(100, 9)$ and $B \sim \mathrm{N}(100, 36)$ describe two sets of measurements.

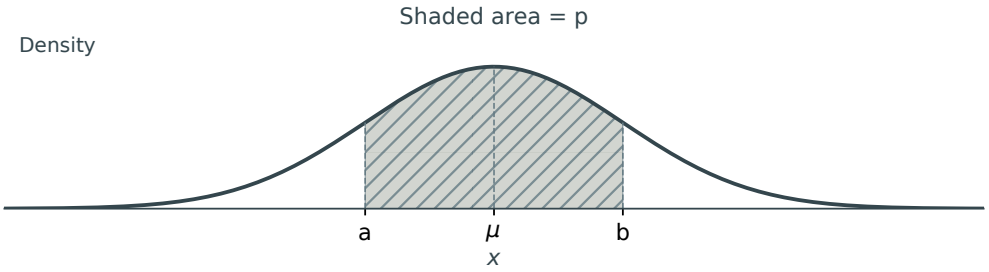
(a) Write down both standard deviations. [2]

(b) Sketch both density curves on the same axes, labelling the common mean and identifying each curve. [2]

(c) Use the 68–95–99.7 rule to estimate $\mathrm{P}(94 < A < 106)$ and $\mathrm{P}(94 < B < 106)$. [2]

35. NC Maximum mark: 4

The diagram shows a normal density curve. The points a and b are equally far from the mean μ. The shaded area is p.



(a) Write down an expression for $a + b$ in terms of μ. [1]

(b) Find $\mathrm{P}(X > b)$ in terms of p. [2]

(c) Find $\mathrm{P}(\mu < X < b)$ when $p = 0.84$. [1]

36. GDC Maximum mark: 6

The net mass M grams of a cereal packet follows $M \sim \mathrm{N}(750, 12^2)$.

(a) Find $\mathrm{P}(M < 735)$. [2]

(b) Find $\mathrm{P}(740 < M < 770)$. [2]

(c) Estimate how many packets in a batch of 2400 have mass below 735 g. [2]

37. GDC Maximum mark: 7

The arrival time T minutes after 08:00 of a train follows $T \sim \mathrm{N}(6, 2.5^2)$.

(a) Find the probability that the train arrives before 08:05. [2]

(b) Find the probability that it arrives between 08:04 and 08:09. [2]

(c) Find the arrival time that is later than 95% of arrivals. Give your answer to the nearest second. [3]

38. GDC Maximum mark: 6

The diameter D millimetres of a manufactured part follows $D \sim \mathrm{N}(8.00, 0.06^2)$. A part is acceptable if $7.90 \leq D \leq 8.08$.

(a) Find the probability that a part is acceptable. [2]

(b) Find the probability that a part is rejected because it is too large. [2]

(c) Estimate the total number rejected in 10 000 parts. [2]

39. GDC Maximum mark: 5

The time T minutes needed to complete a task follows $T \sim \mathrm{N}(28, 5^2)$.

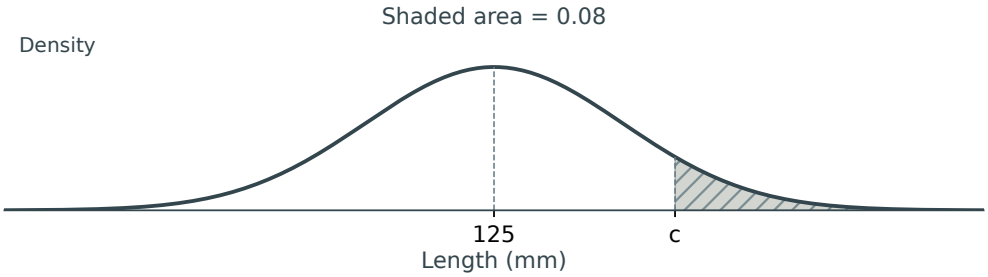
(a) Find the time below which the quickest 20% of completions lie. [2]

(b) Find the time above which the slowest 10% lie. [2]

(c) Find the percentage of completions between these two times. [1]

40. GDC Maximum mark: 6

The length L millimetres of a leaf follows $L \sim \mathrm{N}(125, 8^2)$. Leaves longer than c are classified as large. The shaded area is 0.08.



(a) Find c. [3]

(b) Find $\mathrm{P}(120 < L < c)$. [2]

(c) In a sample of 350 leaves, estimate the number classified as large. [1]

41. GDC Maximum mark: 7

A battery lifetime L hours follows $L \sim \mathrm{N}(11.5, 1.2^2)$.

(a) Find the probability that a battery lasts at least 13 hours. [2]

(b) A manufacturer claims that at least 90% last 10 hours or longer. Determine whether the model supports the claim. [3]

(c) Find the lifetime exceeded by 95% of batteries. [2]

42. GDC Maximum mark: 5

Let $X \sim \mathrm{N}(35, 4^2)$. The interval $[35 - k, 35 + k]$ contains 98% of the distribution, where $k > 0$.

(a) Find the probability that $X > 35 + k$. [1]

(b) Find k. [3]

(c) Find $\mathrm{P}(35 < X < 35 + k)$. [1]

43. GDC Maximum mark: 6

Adult plant heights H centimetres follow $H \sim \mathrm{N}(90, 11^2)$. A nursery labels the shortest 15% as small and the tallest 10% as tall.

(a) Find the upper height limit for the small group. [2]

(b) Find the lower height limit for the tall group. [2]

(c) Find the percentage in neither group. [1]

(d) Estimate how many of 600 plants are in neither group. [1]

44. GDC Maximum mark: 5

A normally distributed measurement X has mean 200 and standard deviation 16. A calculator displays $\mathrm{P}(X \leq 184) = 0.158655\ldots$.

(a) Use symmetry to write down $\mathrm{P}(X \geq 216)$. [1]

(b) Hence find $\mathrm{P}(184 < X < 216)$. [2]

(c) Compare your answer with the empirical-rule estimate, and explain the small difference. [2]

45. GDC Maximum mark: 6

The time T seconds for a chemical process follows $T \sim \mathrm{N}(52, 6^2)$. A time limit is set at a whole number m of seconds.

(a) Find the probability that the process is finished within 60 seconds. [2]

(b) Find the 97th percentile of T. [2]

(c) Hence determine the least integer m for which $\mathrm{P}(T \leq m) \geq 0.97$. [2]

46. GDC Maximum mark: 7

Two filling machines have models $A \sim \mathrm{N}(1000, 5^2)$ and $B \sim \mathrm{N}(1002, 4^2)$, where volumes are in millilitres. The acceptable interval is $[995, 1008]$.

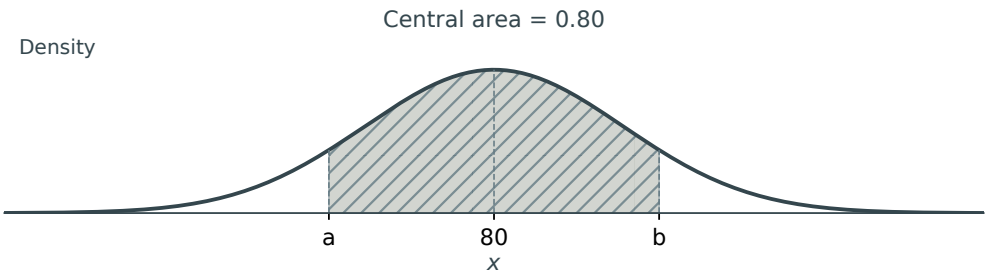
(a) Find the acceptance probability for each machine. [4]

(b) Determine which machine is expected to produce fewer rejected bottles. [1]

(c) For 4000 bottles from each machine, estimate the difference in the numbers rejected. [2]

47. GDC Maximum mark: 7

Let $X \sim \mathrm{N}(80, 9^2)$. The points a and b bound the central 80% of the distribution.



(a) State $\mathrm{P}(X < a)$ and $\mathrm{P}(X > b)$. [1]

(b) Find a and b. [4]

(c) Find $\mathrm{P}(X < 70 \text{ or } X > b)$. [2]

48. GDC Maximum mark: 7

For each delivery, the delay D minutes relative to the scheduled time follows $D \sim \mathrm{N}(2, 3^2)$. A negative delay means an early arrival.

(a) Find the probability of an early arrival. [2]

(b) Find the probability of an arrival within 2 minutes of the scheduled time. [2]

(c) Find the delay exceeded by only 1% of deliveries. [2]

(d) Explain why negative model values are meaningful here. [1]

49. GDC Maximum mark: 6

The breaking force F newtons of a wire follows $F \sim \mathrm{N}(420, 25^2)$. A wire is suitable if its breaking force is at least 380 N.

(a) Find the probability that a wire is suitable. [2]

(b) Find the breaking force below which the weakest 3% of wires lie. [2]

(c) Estimate the number unsuitable in 1200 wires. [2]

50. GDC Maximum mark: 8

A test score S is modelled by $S \sim \mathrm{N}(58, 14^2)$, although actual scores range from 0 to 100.

(a) Find the probability assigned by the model to scores outside $[0, 100]$. [3]

(b) Find the score that marks the top 12% under this model. [2]

(c) Estimate the number outside $[0, 100]$ predicted by the model in 2000 candidates. [2]

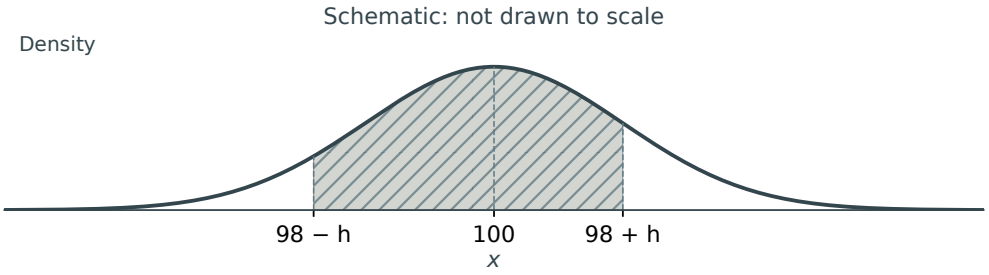
(d) Explain why your answer to part (c) is a model warning rather than a possible observed count. [1]

4. AA HL exclusive challenge

Shared SL/HL questions end at Question 50. This section is intended for HL students: less scaffolded applications within 4.9, with all distribution parameters given. Use technology throughout.

51. GDC Maximum mark: 7

Let $X \sim \mathrm{N}(100, 10^2)$. The interval $[98-h, 98+h]$ contains 80% of the distribution, where $h > 0$.



(a) Explain why the two excluded tails do not have equal probability. [2]

(b) Use a numerical or graphical method to find h. [3]

(c) Find the probability in each excluded tail. [2]

52. GDC Maximum mark: 6

Let $X \sim \mathrm{N}(70, 8^2)$. An acceptance interval $[a, b]$ has $\mathrm{P}(a < X < b) = 0.90$ and $\mathrm{P}(X < a) = 3\mathrm{P}(X > b)$. Find a and b. [6]

53. GDC Maximum mark: 8

Two machines produce lengths in millimetres with models $A \sim \mathrm{N}(100, 5^2)$ and $B \sim \mathrm{N}(104, 7^2)$.

(a) Use technology to find the cutoff c for which $\mathrm{P}(A > c) = \mathrm{P}(B > c)$. [3]

(b) Find the common probability. [2]

(c) At a cutoff of 110 mm, determine which machine produces the larger proportion above the cutoff. [3]

54. GDC Maximum mark: 9

The true mass M grams of an object follows $M \sim \mathrm{N}(50, 0.8^2)$. A display rounds the mass to the nearest whole gram; exact half-gram ties have probability zero.

(a) Find the probability that the display reads 50 g. [3]

(b) An object is accepted if the display reads 49, 50 or 51 g. Find the acceptance probability. [3]

(c) Compare this with the probability that the true mass lies in $[49, 51]$. Explain the difference. [3]

55. GDC Maximum mark: 7

Let $X \sim \mathrm{N}(40, 5^2)$. Two disjoint intervals, $[a, 38]$ and $[42, b]$, each contain 30% of the distribution, where $a < 38 < 42 < b$. Find a and b, and explain their relationship. [7]

56. GDC Maximum mark: 7

Let $X \sim \mathrm{N}(120, 15^2)$. A cutoff $c > 120$ satisfies $\mathrm{P}(120 < X < c) = 4\mathrm{P}(X > c)$.

(a) Find $\mathrm{P}(X > c)$. [3]

(b) Find c. [2]

(c) Find the reflected cutoff $d < 120$ such that $\mathrm{P}(X < d) = \mathrm{P}(X > c)$. [2]

57. GDC Maximum mark: 8

A population has model $X \sim \mathrm{N}(65, 9^2)$. A classification scheme requires $\mathrm{P}(X < a) = 0.08$ and $\mathrm{P}(a < X < b) = 0.75$.

(a) Find a and b. [5]

(b) Find the percentage above b. [1]

(c) Determine whether b is above or below the upper quartile, without calculating that quartile. [2]

58. GDC Maximum mark: 8

Two batches contain 600 and 400 rods respectively. Their length models (in mm) are shown below.

Batch	Mean	Standard deviation
A	100	2
B	101	3

A rod is accepted if its length lies in $[97, 104]$.

(a) Find the acceptance probability in each batch. [4]

(b) Estimate the total number accepted. [2]

(c) Explain why modelling all 1000 rods by a single normal distribution with an averaged standard deviation is not justified by the information given. [2]

59. GDC Maximum mark: 8

Let $X \sim \mathrm{N}(200, 20^2)$. An interval begins at 180 and must contain 85% of the distribution.

(a) Determine whether any finite upper endpoint can meet this requirement. [3]

(b) Find the upper endpoint if the required probability is reduced to 80%. [3]

(c) Explain why the upper endpoint in part (b) is much farther from the mean than 180 is. [2]

60. GDC Maximum mark: 10

A process has measurement $X \sim \mathrm{N}(30, 4^2)$. Measurements below a are labelled low, those between a and b standard, and those above b high. There are twice as many standard measurements as low measurements, and three times as many standard measurements as high measurements, in the long-run model.

(a) Find the model probability of each label. [4]

(b) Find a and b. [4]

(c) In 1100 measurements, estimate the number with each label. [2]

Answer key and solution outlines

Decimal answers are normally rounded to 3 significant figures. Counts use unrounded probabilities. invNorm(p, μ, σ) below denotes inverse normal using **left-tail** area p. Graph descriptions specify the required features.

1. Short response | Questions 1–15

1.
(a) $\mu = 42$.
(b) $\sigma = 3.5$.
(c) $\sigma^2 = 12.25$.

2.
(a) Median $= 68$; mode $= 68$.
(b) $x = 68$.
(c) 1.

3.
(a) $\sigma_A = 2$; $\sigma_B = 4$.
(b) A: its standard deviation is smaller and both curves have total area 1.

4.
(a) Bell-shaped curve symmetric about $x = 30$; shade the area to the right of 35.
(b) 0.5.

5.
(a) 0.68.
(b) 0.95.
(c) $(1 - 0.997)/2 = 0.0015$.

6.
(a) $0.68/2 = 0.34$.
(b) $(0.95 - 0.68)/2 = 0.135$.
(c) $(1 - 0.95)/2 = 0.025$.

7.
(a) 0.18, by symmetry about 50.
(b) $1 - 2(0.18) = 0.64$.

8.
(a) 0, since X is continuous.
(b) $\mathrm{P}(X = 15) = 0$, so including the endpoint adds no probability.

9. Left-tail calculation: $\mathrm{P}(X < 76) = 0.748$.

10. $1 - \mathrm{P}(X \leq 142) = 0.0712$.

11. Area between 20 and 28: 0.818.

12. Inverse normal with left-tail area 0.90: $a = 523$.

13. Left-tail area $= 0.88$; $b = 53.2$.

14. Each tail has area 0.05. Endpoints: 46.8 and 73.2.

15.
(a) The count is discrete; a normal model is continuous. A suitable approximation would need justification.
(b) The stated distribution is right-skewed, whereas a normal distribution is symmetric.

2. Extended response | Questions 16–30

16.
(a) $\mathrm{P}(M < 490) = 0.0478$.
(b) $\mathrm{P}(494 < M < 508) = 0.750$.
(c) $2000\mathrm{P}(M < 490) = 95.6$, so about 96 packets.

17.
(a) $\mathrm{P}(T > 40) = 0.0377$.
(b) Left-tail area 0.95 gives 39.4 minutes.
(c) Yes: $\mathrm{P}(T \leq 40) = 0.962 > 0.95$.
18.
(a) $\mathrm{P}(82 < X < 112)$.
(b) 0.775.
(c) $1 - \mathrm{P}(82 < X < 112) = 0.225$.
19.
(a) Left-tail areas 0.25 and 0.75: $Q_1 = 66.6$ cm, $Q_3 = 77.4$ cm.
(b) $Q_3 - Q_1 = 10.8$ cm, using unrounded quartiles.
(c) The middle 50% of heights span about 10.8 cm, from 66.6 to 77.4 cm.
20.
(a) $800(0.68) = 544$.
(b) $800(0.025) = 20$.
(c) The percentages are approximate model probabilities, and observed counts vary.
21.
(a) $\mathrm{P}(V < 324) + \mathrm{P}(V > 337) = 0.107$.
(b) $5000\mathrm{P}(324 \leq V \leq 337) = 4.47 \times 10^3$; about 4466 bottles.
(c) Lower tail: 0.0668; upper tail: 0.0401. The lower tail is larger.
22.
(a) $\mathrm{P}(L > 1750) = 0.0824$.
(b) Left-tail area 0.02: $w = 1.13 \times 10^3$ hours (unrounded 1130.325...).
(c) 1130 hours; round the cutoff down.
23.
(a) Machine A: smaller spread about the same mean, so more mass lies in this centred interval.
(b) $p_A = 0.789$; $p_B = 0.525$.
(c) $1000(p_A - p_B) = 264$; machine B has about 264 more outside tolerance.
24.
(a) $0.90 - 0.10 = 0.80$.
(b) $1 - 0.50 = 0.50$.
(c) 18: the mean and median coincide for a normal distribution.
25.
(a) Use left-tail areas 0.05 and 0.95: $a = 230$ g, $b = 270$ g.
(b) $b - a = 39.5$ g.
(c) 5%.
26.
(a) Left-tail area 0.08: 13.4 s.
(b) Left-tail area 0.85: 14.8 s.
(c) $100(0.85 - 0.08) = 77\%$.
27.
(a) 0.07 is a right-tail area. The left-tail input is $1 - 0.07 = 0.93$.
(b) $c = 87.4$.
(c) Below 80, since a left-tail area below 0.5 gives a cutoff below the mean.
28.
(a) 0.830.
(b) $\mathrm{P}(X > a) = 0.90$ gives left-tail area 0.10; $a = 14.8$ kg.
(c) $240\mathrm{P}(X > 22) = 13.2$; about 13 days.
29.
(a) $(1 - 0.92)/2 = 0.04$.
(b) The upper endpoint is the 96th percentile: $40 + d = 50.5$, so $d = 10.5$.
(c) 0.08.

30.

(a) $\mathrm{P}(M < 0) = 0.0548$.

(b) Negative mass is impossible, yet the model assigns it about 5.48% probability.

(c) No: a substantial part of the lower tail is physically impossible, making lower-tail predictions questionable.

3. IB exam-style questions | Questions 31–50

31.

(a) 0.12, by symmetry.

(b) $1 - 2(0.12) = 0.76$.

(c) $0.76/2 = 0.38$.

32.

(a) $\mathrm{P}(56 < X < 68) \approx 0.68/2 + 0.95/2 = 0.815$.

(b) $(1 - 0.95)/2 = 0.025$.

(c) $400(0.815) = 326$.

33.

(a) 0.5.

(b) Probability is area. A single point has zero width and hence zero area; height is not a point probability.

(c) $(0.95 - 0.68)/2 = 0.135$.

34.

(a) $\sigma_A = 3$; $\sigma_B = 6$.

(b) Both curves are centred on 100; A is narrower and taller, B wider and lower.

(c) For A, the interval is $\mu \pm 2\sigma$: approximately 0.95. For B, it is $\mu \pm \sigma$: approximately 0.68.

35.

(a) $a + b = 2\mu$.

(b) $(1 - p)/2$.

(c) $p/2 = 0.42$.

36.

(a) $\mathrm{P}(M < 735) = 0.106$.

(b) $\mathrm{P}(740 < M < 770) = 0.750$.

(c) $2400\mathrm{P}(M < 735) = 254$; about 254 packets.

37.

(a) $\mathrm{P}(T < 5) = 0.345$.

(b) $\mathrm{P}(4 < T < 9) = 0.673$.

(c) $t = \mathrm{invNorm}(0.95, 6, 2.5) = 10.1121\ldots$ minutes after 08:00, so 08:10:07.

38.

(a) $\mathrm{P}(7.90 \leq D \leq 8.08) = 0.861$.

(b) $\mathrm{P}(D > 8.08) = 0.0912$.

(c) $10\,000[1 - \mathrm{P}(7.90 \leq D \leq 8.08)] = 1.39 \times 10^3$; about 1390 parts.

39.

(a) Left-tail area 0.20: 23.8 minutes.

(b) Left-tail area 0.90: 34.4 minutes.

(c) $100(0.90 - 0.20) = 70\%$.

40.

(a) Left-tail area 0.92: $c = 136$ mm.

(b) $0.92 - \mathrm{P}(L \leq 120) = 0.654$.

(c) $350(0.08) = 28$ leaves.

41.

(a) $\mathrm{P}(L \geq 13) = 0.106$.

(b) $\mathrm{P}(L \geq 10) = 0.894$, which is below 0.90. The model does not support the claim.

(c) Left-tail area 0.05: 9.53 hours.

42.
(a) $(1-0.98)/2 = 0.01$.
(b) Upper endpoint has left-tail area 0.99: $k = \text{invNorm}(0.99, 35, 4) - 35 = 9.31$.
(c) $0.98/2 = 0.49$.
43.
(a) Left-tail area 0.15: 78.6 cm.
(b) Left-tail area 0.90: 104 cm.
(c) $100(1 - 0.15 - 0.10) = 75\%$.
(d) $600(0.75) = 450$ plants.
44.
(a) $0.158655\ldots$.
(b) $1 - 2(0.158655\ldots) = 0.682689\ldots \approx 0.683$.
(c) The empirical rule gives approximately 0.68; 68% is rounded, not an exact normal area.
45.
(a) $\text{P}(T \leq 60) = 0.909$.
(b) Left-tail area 0.97: $63.2848\ldots$ s ≈ 63.3 s.
(c) $m = 64$ s; the unrounded percentile must be rounded up.
46.
(a) $p_A = 0.787$; $p_B = 0.893$.
(b) Machine B, because $p_B > p_A$.
(c) $4000(p_B - p_A) = 426$; machine A has about 426 more rejected bottles.
47.
(a) Each is $(1 - 0.80)/2 = 0.10$.
(b) $a = 68.5$; $b = 91.5$.
(c) The events are disjoint, so $\text{P}(X < 70) + 0.10 = 0.233$.
48.
(a) $\text{P}(D < 0) = 0.252$.
(b) Within 2 minutes means $-2 \leq D \leq 2$: probability 0.409.
(c) Left-tail area 0.99: 8.98 minutes.
(d) A negative delay represents an arrival before the scheduled time.
49.
(a) $\text{P}(F \geq 380) = 0.945$.
(b) Left-tail area 0.03: 373 N.
(c) $1200\text{P}(F < 380) = 65.8$; about 66 wires.
50.
(a) $\text{P}(S < 0) + \text{P}(S > 100) = 0.00137$.
(b) Left-tail area 0.88: 74.4.
(c) $2000[\text{P}(S < 0) + \text{P}(S > 100)] = 2.73$; about 3.
(d) Actual scores cannot lie outside $[0, 100]$. The model's nonzero tail probability shows an approximation limitation.

4. AA HL exclusive challenge | Questions 51–60

51.
(a) The interval is centred on 98, not on the mean 100; its endpoints are not reflections about the mean.
(b) Solve $\text{P}(98 - h < X < 98 + h) = 0.80$ numerically: $h = 13.1$ (unrounded $13.071519\ldots$).
(c) Lower tail: $\text{P}(X < 98 - h) = 0.0659$. Upper tail: $\text{P}(X > 98 + h) = 0.134$. Their sum is 0.20.
52. Let the upper-tail probability be p. Then $3p + p = 0.10$, so $p = 0.025$ and the lower tail is 0.075. Use left-tail areas 0.075 and 0.975: $a = 58.5$, $b = 85.7$.
53.
(a) Solve equality of the two right-tail probabilities numerically: $c = 90$ mm.
(b) $\text{P}(A > 90) = \text{P}(B > 90) = 0.977$.
(c) $\text{P}(A > 110) = 0.0228$; $\text{P}(B > 110) = 0.196$. Machine B has the larger proportion.

54.

(a) The reading 50 represents $49.5 \leq M < 50.5$: probability 0.468.

(b) The three readings represent $48.5 \leq M < 51.5$: probability 0.939.

(c) $\mathrm{P}(49 \leq M \leq 51) = 0.789$. Acceptance by rounded readings covers a wider interval, so has a larger probability.

55. $\mathrm{P}(X < a) = \mathrm{P}(X \leq 38) - 0.30 = 0.0446$. Hence $a = 31.5$. $\mathrm{P}(X \leq b) = \mathrm{P}(X \leq 42) + 0.30 = 0.955$. Hence $b = 48.5$. $a + b = 80$: the intervals are reflections in the mean 40.

56.

(a) Let $p = \mathrm{P}(X > c)$. Then $0.5 - p = 4p$, giving $p = 0.10$.

(b) Left-tail area 0.90: $c = 139$.

(c) $d = 240 - c = 101$.

57.

(a) For a, left-tail area 0.08, so $a = 52.4$. For b, left-tail area $0.08 + 0.75 = 0.83$, so $b = 73.6$.

(b) $100(1 - 0.83) = 17\%$.

(c) Above the upper quartile, since $0.83 > 0.75$ and normal cumulative probabilities increase with the cutoff.

58.

(a) $p_A = 0.910$; $p_B = 0.750$.

(b) $600p_A + 400p_B = 846$; about 846 rods.

(c) The combined population mixes two different normal distributions. Such a mixture is not necessarily normal, and its standard deviation is not a simple average of the two standard deviations.

59.

(a) No. Even an unbounded interval starting at 180 has probability $\mathrm{P}(X > 180) = 0.841 < 0.85$.

(b) $\mathrm{P}(X < b) = \mathrm{P}(X \leq 180) + 0.80 = 0.959$. Thus $b = 235$.

(c) The fixed lower tail is about 0.159 but the remaining upper tail is only about 0.0413. A smaller tail requires a cutoff farther from the mean.

60.

(a) Let p be the standard probability. Then low $= p/2$, high $= p/3$, so $(1 + 1/2 + 1/3)p = 1$. Thus low $= 3/11$, standard $= 6/11$, high $= 2/11$.

(b) Left-tail areas are $3/11$ and $9/11$: $a = 27.6$, $b = 33.6$.

(c) Low: 300; standard: 600; high: 200.