

Definite integrals and areas

IB Mathematics AA SL/HL

50 shared questions + 10 HL challenges | Signed area, exact values and areas between curves

Learning goal	Evaluate definite integrals and calculate areas using correct bounds, signs and upper minus lower.
Syllabus link	SL 5.11: analytical and technological evaluation; areas with the x -axis and between curves. Uses integration skills from SL 5.10.
Structure	15 short + 15 extended + 20 original IB-style questions + 10 HL challenges.
Technology	Non-calculator unless marked  GDC. Unmarked exam questions are Paper 1-style; marked questions are Paper 2-style.

Instructions. Write a correct definite-integral expression before calculating each area. Split where the sign or upper curve changes. Give exact answers unless a decimal is requested; GDC answers to 3 significant figures, except money to the nearest cent. Use radians and retain unrounded values in later calculations. Areas are in square units unless otherwise stated. SL students: Q1–50; HL students: also Q51–60. Work on separate paper.

1. Short response

1. Evaluate $\int_{-1}^2 (3x^2 - 2x + 1) dx$.

2. Evaluate $\int_1^4 \left(3\sqrt{x} + \frac{2}{x^2} \right) dx$.

3. Evaluate $\int_0^{\ln 3} e^{2x} dx$.

4. Evaluate $\int_1^{e^2} \frac{2}{x} dx$.

5. Evaluate $\int_0^{\pi/6} \cos(2x) dx$.

6. Evaluate $\int_{\pi/6}^{\pi/2} \sin x dx$.

7. Evaluate $\int_1^0 (2x + 1) dx$. Explain the effect of reversing the limits.

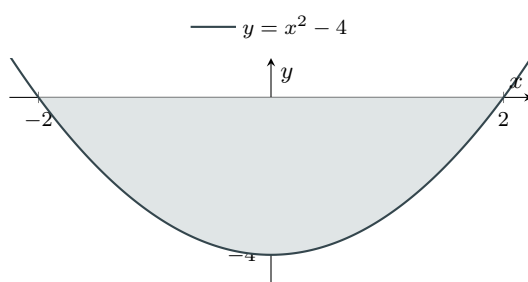
8. The table gives values of a differentiable function g .

x	-1	1	3
$g(x)$	4	-2	7

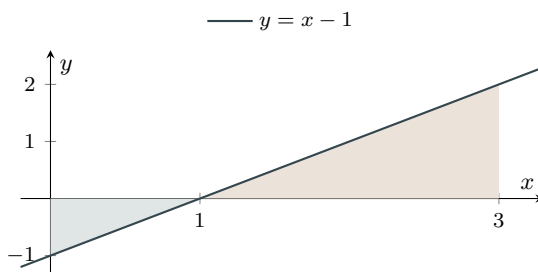
Find (a) $\int_{-1}^3 g'(x) dx$; (b) $\int_3^1 g'(x) dx$.

9. It is given that $\int_0^2 f(x) dx = 5$ and $\int_2^5 f(x) dx = -3$. Find $\int_0^5 [2f(x) + 1] dx$.

10. The shaded region is enclosed by $y = x^2 - 4$ and the x -axis. Write an integral for its area and evaluate it.



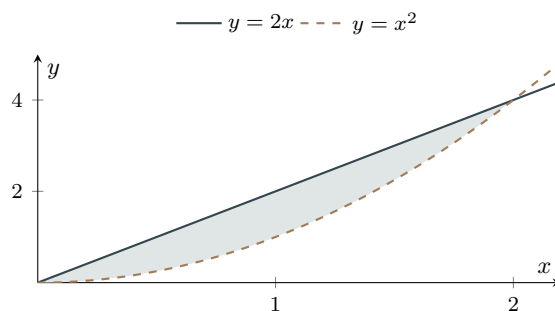
11. For $f(x) = x - 1$, find (a) $\int_0^3 f(x) dx$; (b) the total area between the graph and the x -axis on $0 \leq x \leq 3$.



12. F is an antiderivative of f , with $F(1) = 6$ and $F(4) = 2$. Find $\int_1^4 f(x) dx$. Explain why adding a constant to F does not change your answer.

13. Evaluate $\int_0^1 6x(x^2 + 1)^2 dx$.

14. Write an integral for the area enclosed by $y = 2x$ and $y = x^2$, then evaluate it.



15. Find (a) $\int_0^\pi \sin(2x) dx$; (b) the total area between $y = \sin(2x)$ and the x -axis on $[0, \pi]$.

2. Extended response

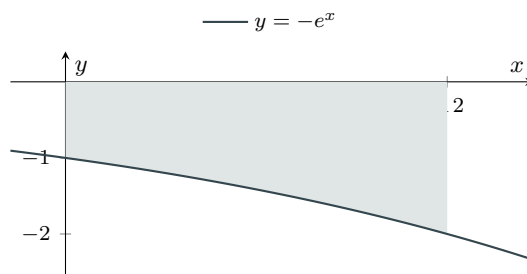
16. Use $u = 3x - 1$ to evaluate $\int_0^2 (3x - 1)^2 dx$. State the transformed limits.

17. Evaluate $\int_0^1 \frac{4x}{2x^2 + 1} dx$ by substitution. Check that the logarithm argument stays positive.

18. Given $b > 1$ and $\int_1^b 2x dx = 15$, find b exactly and interpret the integral as an area.

19. Find k if $\int_0^2 (kx + 3) dx = 14$. Is 14 also the total area between the resulting graph and the x -axis on $[0, 2]$? Justify your answer.

20. The graph shows $y = -e^x$. Find the signed integral and the area bounded by this curve, the x -axis, $x = 0$ and $x = \ln 2$. Explain the difference.

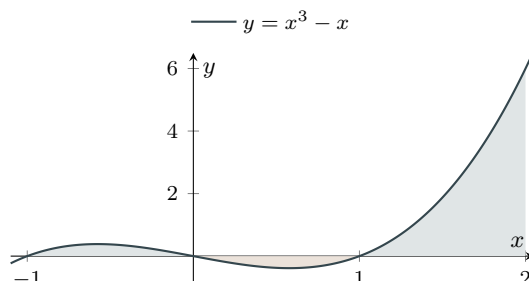


21. Let $f(x) = x^3 - x$ on $-1 \leq x \leq 2$.

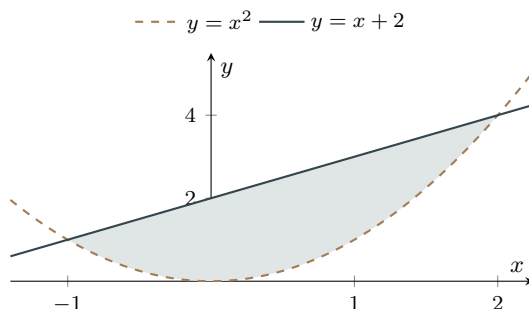
(a) Find all zeros of f in this interval.

(b) Find $\int_{-1}^2 f(x) dx$.

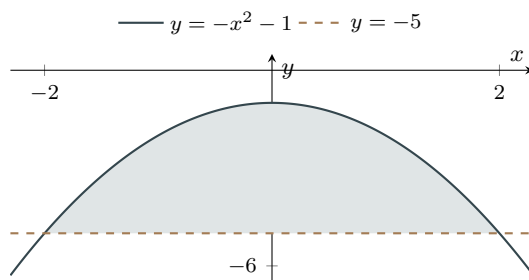
(c) Find the total area between the curve and the x -axis on the interval.



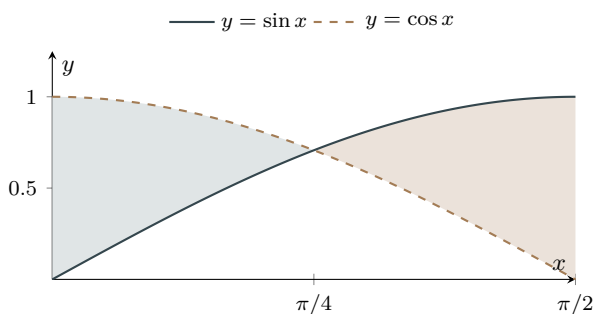
22. Find the points of intersection of $y = x^2$ and $y = x + 2$. Hence find the exact area enclosed by the two curves.



23. Find the area enclosed by $y = -x^2 - 1$ and $y = -5$. Explain why the fact that both graphs are below the x -axis does not change the upper-minus-lower method.



24. Find the total area between $y = \sin x$ and $y = \cos x$ on $0 \leq x \leq \pi/2$. State where the upper curve changes.



25. Use symmetry to evaluate $\int_{-2}^2 (x^3 + 3x^2 - x + 1) dx$. Verify your result by evaluating an antiderivative at both limits.

26. The vertical line $x = c$, where $0 < c < 4$, divides the region under $y = x$ above the x -axis on $[0, 4]$ into two equal areas. Find c exactly.

27. For $k > 0$, the area enclosed by $y = kx(3 - x)$ and the x -axis is 18. Find k , stating the limits and sign of the integrand.

28. F is differentiable with continuous derivative. The table gives some values of F .

x	-1	0	2	4
$F(x)$	2	5	1	6

(a) Find $\int_{-1}^4 F'(x) dx$ and $\int_0^4 F'(x) dx$.

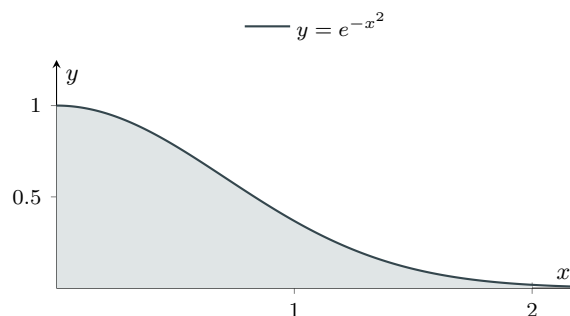
(b) Explain why these data do not determine the total area between $y = F'(x)$ and the x -axis on $[-1, 4]$.

29.  *GDC* Let $f(x) = e^{-x^2}$.

(a) Write an integral for the area under the curve above the x -axis on $0 \leq x \leq 2$.

(b) Use your GDC to evaluate the area to 3 significant figures.

(c) Explain why no splitting at an x -intercept is needed.

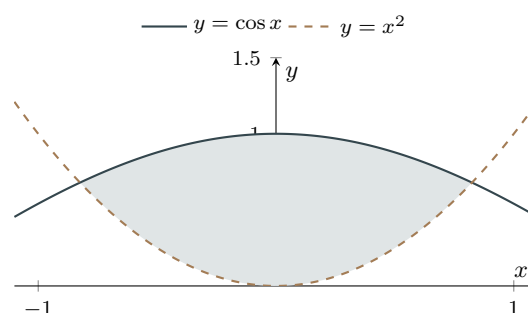


30.  *GDC* The curves $y = \cos x$ and $y = x^2$ enclose a region.

(a) Use your GDC to find the two intersection x -coordinates.

(b) Write a definite integral for the enclosed area.

(c) Calculate this area to 3 significant figures.



3. IB exam-style practice

31. Let $f(x) = 2x^2 - 4x + 3$.

(a) Express $f(x)$ in completed-square form and state its minimum value. [3 marks]

(b) Evaluate $\int_{-1}^2 f(x) dx$. [4 marks]

(c) Explain why this integral equals the total area between the curve and the x -axis on the given interval. [1 mark]

32. For $x > 0$, let $f(x) = (4\sqrt{x} - 3)/\sqrt{x}$.

(a) Express $f(x)$ as a sum of powers of x . [1 mark]

(b) Find $\int_1^9 f(x) dx$. [4 marks]

(c) State whether the integral represents an area above or below the x -axis. Justify your answer. [2 marks]

33. Consider $f(x) = e^{2x} - 1$ on $0 \leq x \leq \ln 2$.

(a) Find an antiderivative of f . [2 marks]

(b) Show that the area between this curve and the x -axis is $\frac{3}{2} - \ln 2$. [4 marks]

34. Let $f(x) = 2\sin(2x) + \cos(2x)$ on $0 \leq x \leq \pi/4$.

(a) Find an antiderivative of f . [3 marks]

(b) Evaluate the definite integral of f over this interval. [3 marks]

(c) Explain why the integral is also a geometrical area. [1 mark]

35. The curve $y = x^2 - 9$ encloses a finite region with the x -axis.

(a) Find the points where the curve meets the axis. [2 marks]

(b) Write a definite integral for the area of this region. [2 marks]

(c) Find the exact area. [3 marks]

36. Let $f(x) = 2x - 3$ for $0 \leq x \leq 4$.

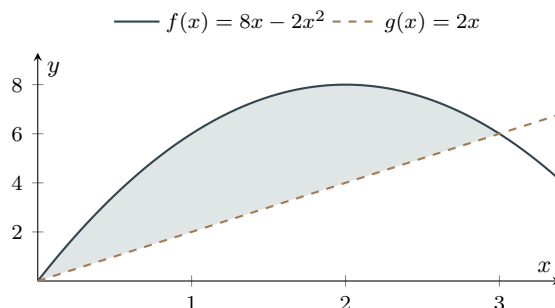
(a) Find the x -intercept. [1 mark]

(b) Calculate $\int_0^4 f(x) dx$. [3 marks]

(c) Find the total area between the graph and the x -axis on the interval. [4 marks]

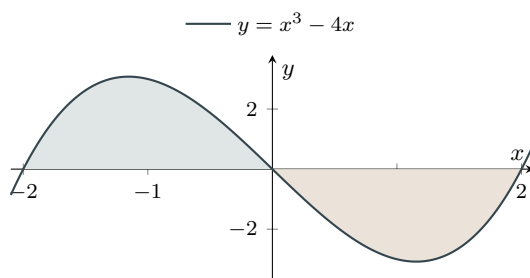
37. The diagram shows $f(x) = 8x - 2x^2$ and $g(x) = 2x$.

- (a) Find the coordinates of their intersections. [3 marks]
- (b) Find the exact area of the shaded region. [4 marks]
- (c) Find the equation of the tangent to f that is parallel to g . [4 marks]



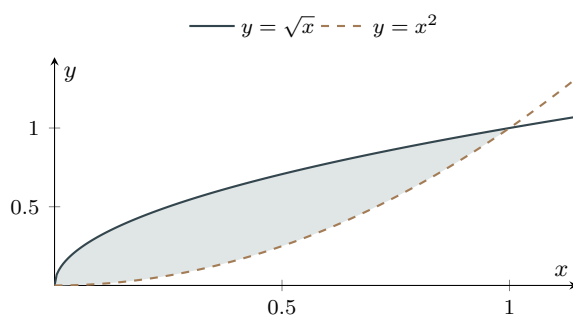
38. The graph shows $f(x) = x^3 - 4x$ on $[-2, 2]$.

- (a) Explain, without integrating, why the signed integral over this interval is zero. [2 marks]
- (b) Write a suitable expression for the total shaded area. [2 marks]
- (c) Find the exact value of that area. [3 marks]



39. Consider $y = \sqrt{x}$ and $y = x^2$, where $x \geq 0$.

- (a) Find all intersection points. [3 marks]
- (b) Determine which curve is upper between these points. [1 mark]
- (c) Find the exact area enclosed by the curves. [4 marks]



40. Consider $I = \int_0^1 \frac{2x}{(x^2 + 1)^2} dx$.

- (a) Using $u = x^2 + 1$, write I as an integral in u , including the limits. [3 marks]
- (b) Evaluate I exactly. [3 marks]
- (c) Interpret I as an area. [1 mark]

41. Let $I = \int_1^e \frac{\ln x}{x} dx$.

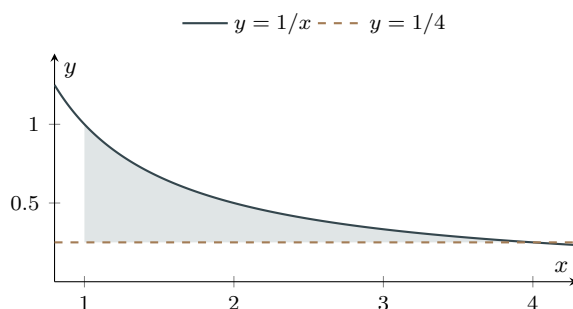
- (a) State a suitable substitution and its transformed limits. [2 marks]
- (b) Find I exactly. [3 marks]
- (c) Explain why the result equals the area under the curve on $[1, e]$. [1 mark]

42. For $k > 0$, let $f(x) = x^2 - k$. It is given that $\int_0^2 f(x) dx = 0$.

- (a) Find k . [3 marks]
- (b) Find the zero of f in $[0, 2]$. [2 marks]
- (c) Find the total area between f and the x -axis on this interval. [5 marks]

43. A region is bounded by $y = 1/x$, $y = 1/4$ and $x = 1$.

- (a) Find the x -coordinate where the two graphs intersect. [1 mark]
- (b) Write an integral for the enclosed area. [2 marks]
- (c) Find the exact area. [3 marks]

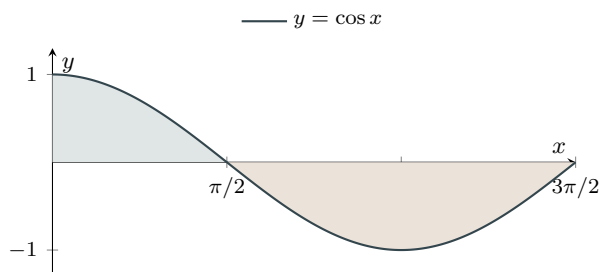


44. The area bounded by $y = e^x$, the x -axis and $x = 0, b$, where $b > 0$, is 5.

- (a) Write an equation involving a definite integral. [2 marks]
- (b) Find b exactly. [3 marks]
- (c) Find the area under the same curve on $[b, 2b]$. [3 marks]

45. The shaded regions lie between $y = \cos x$ and the x -axis for $0 \leq x \leq 3\pi/2$.

- Evaluate the signed integral of $\cos x$ over the interval. [2 marks]
- Write an expression for the total shaded area. [2 marks]
- Find this area exactly. [3 marks]



46. A student evaluates $\int_0^3 (x^2 - 2x) dx = 0$ and concludes that the total area between the curve and the x -axis is zero.

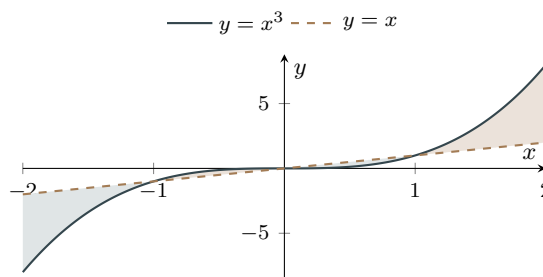
- Verify the value of the definite integral. [3 marks]
- Explain the error in the conclusion. [2 marks]
- Calculate the correct total area. [4 marks]

47.  GDC A flat panel has lower edge $y = 0$ and upper edge $y = \sqrt{1 + x^3}$ for $0 \leq x \leq 3$. Both coordinates are measured in metres.

- Write a definite integral for the area of the panel. [2 marks]
- Use your GDC to find its area. [2 marks]
- A coating costs 12 euros per square metre. Find the cost to the nearest cent. [2 marks]

48. Let $f(x) = x^3$ and $g(x) = x$ for $-2 \leq x \leq 2$.

- Find the intersection x -coordinates in this interval. [2 marks]
- Write a sum of integrals for the total area between the curves over the whole interval. [4 marks]
- Find the exact area. [4 marks]

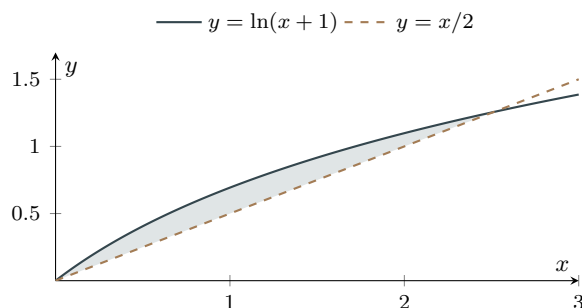


49. For $k > 0$, the curves $y = k - x^2$ and $y = x^2$ enclose a region of area $64/3$.

- Find the intersection x -coordinates in terms of k . [2 marks]
- Show that the enclosed area is $\frac{4}{3}k\sqrt{k/2}$. [4 marks]
- Find k . [3 marks]

50.  *GDC* The graphs $y = \ln(x + 1)$ and $y = x/2$ enclose a region for $x \geq 0$.

- (a) State one intersection and use your GDC to find the other. [3 marks]
- (b) Write a definite integral for the area enclosed by the graphs. [2 marks]
- (c) Use your GDC to calculate the area. [2 marks]



4. AA HL exclusive challenge

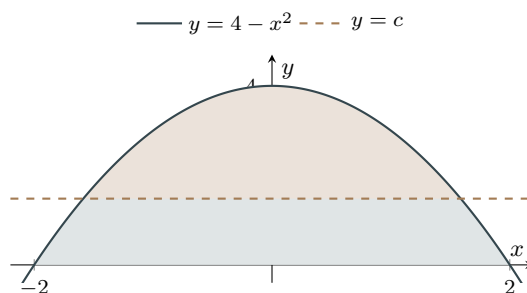
For HL students only. These challenges deepen the shared SL 5.11 content through parameters, symmetry, substitutions and optimization.

51. For $0 < a < 1$, let $A(a)$ be the total area between $y = x^2 - a$ and the x -axis on $0 \leq x \leq 1$.

- (a) Show that $A(a) = \frac{1}{3} - a + \frac{4}{3}a^{3/2}$. [5 marks]
- (b) Find the value of a for which $A(a)$ is least, and find this least area. [5 marks]

52. The horizontal line $y = c$, where $0 < c < 4$, divides the region enclosed by $y = 4 - x^2$ and the x -axis into two equal areas.

- (a) Express the area above $y = c$ in terms of c . [5 marks]
- (b) Find c exactly. [4 marks]



53. Let $b = \sqrt{e - 1}$ and $f(x) = x \ln(1 + x^2)/(1 + x^2)$.

- (a) Find $\int_0^b f(x) dx$ exactly by substitution. [5 marks]
- (b) Find $\int_{-b}^b f(x) dx$. [2 marks]
- (c) Find the total area between the curve and the x -axis on $[-b, b]$. [3 marks]

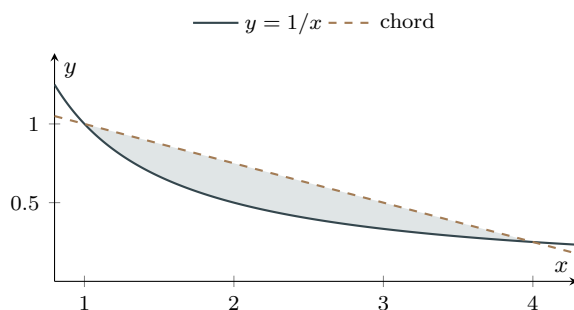
54. For $a > 0$, it is given that $\int_{-a}^a (x^5 - 3x^3 + 2x^2 + 1) dx = 10/3$.

- (a) Use symmetry to obtain an equation in a . [4 marks]
 (b) Find a exactly and prove that there is no other positive solution. [4 marks]

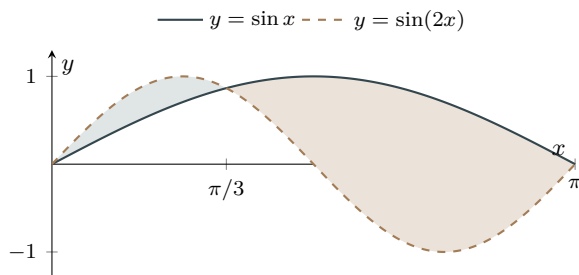
55. For $m > 0$, the curves $y = x^3$ and $y = mx$ enclose two finite regions. Their combined area is 8. Find m exactly, showing the intersections and the area integral.

56. The chord of $y = 1/x$, $x > 0$, joining $(1, 1)$ to $(4, 1/4)$ is shown.

- (a) Find the chord equation and the exact area between the chord and the curve. [6 marks]
 (b) Find the tangent to the curve that is parallel to the chord. [4 marks]
 (c) Find the area between this tangent and the chord on $1 \leq x \leq 4$. [3 marks]



57. Find the total area between $y = \sin x$ and $y = \sin(2x)$ on $0 \leq x \leq \pi$. Give an exact answer and justify every split in your integral.



58. For $k > 0$, let $f(x) = kx(x-1)(x-3)$. The total area enclosed by its graph and the x -axis between $x = 0$ and $x = 3$ is 37.

- (a) Find the ratio of the area on $[0, 1]$ to the area on $[1, 3]$. [5 marks]
 (b) Find k . [3 marks]
 (c) Find $\int_0^3 f(x) dx$. [2 marks]

59. A continuous function f is non-negative on $[1, 3]$ and non-positive on $[3, 5]$, with $\int_1^3 f(x) dx = 4$ and $\int_3^5 f(x) dx = -1$. Define $g(x) = 2f(2x + 1)$ on $[0, 2]$.

(a) Find $\int_0^2 g(x) dx$ and the total area between g and the x -axis. [5 marks]

(b) Find $\int_0^2 [g(x) - 1] dx$. [2 marks]

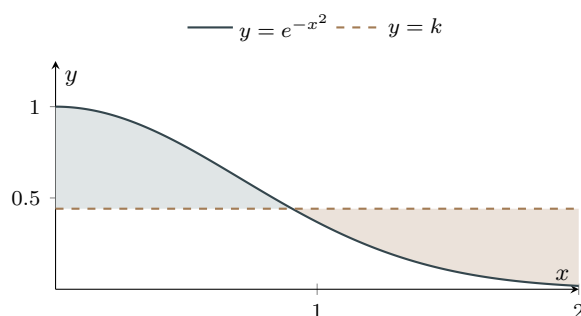
(c) Explain why the total area between $y = g(x) - 1$ and the x -axis cannot be determined from the information given. [2 marks]

60.  *GDC* For $0 \leq x \leq 2$, the region between $y = e^{-x^2}$ and the horizontal line $y = k$, where $e^{-4} < k < 1$, consists of two parts: one above the line and one below it. The two parts have equal area.

(a) Find an integral expression for k and calculate it to 3 significant figures. [4 marks]

(b) Find the x -coordinate where the curve meets the line. [3 marks]

(c) Find the combined area of the two parts. [4 marks]



Answer key and solution outlines

Equivalent exact forms and correct alternative methods are accepted. Area expressions use upper minus lower or split signed contributions. These are original practice questions, not official IB examination items.

1. Short response

1. $[x^3 - x^2 + x]_{-1}^2 = 6 - (-3) = 9$.
2. $[2x^{3/2} - 2/x]_1^4 = \frac{31}{2}$.
3. $[\frac{1}{2}e^{2x}]_0^{\ln 3} = \frac{1}{2}(9 - 1) = 4$.
4. $[2 \ln x]_1^{e^2} = 4$.
5. $[\frac{1}{2} \sin(2x)]_0^{\pi/6} = \frac{\sqrt{3}}{4}$.
6. $[-\cos x]_{\pi/6}^{\pi/2} = \frac{\sqrt{3}}{2}$.
7. $[x^2 + x]_1^0 = -2$. Reversing the limits changes the sign of a definite integral.
8. (a) $g(3) - g(-1) = 7 - 4 = 3$. (b) $g(1) - g(3) = -2 - 7 = -9$.
9. $\int_0^5 f = 5 - 3 = 2$, so the required value is $2(2) + (5 - 0) = 9$.
10. $A = -\int_{-2}^2 (x^2 - 4) dx = \int_{-2}^2 (4 - x^2) dx = \frac{32}{3}$.
11. (a) $[x^2/2 - x]_0^3 = \frac{3}{2}$. (b) $A = \int_0^1 (1 - x) dx + \int_1^3 (x - 1) dx = \frac{1}{2} + 2 = \frac{5}{2}$.
12. $F(4) - F(1) = 2 - 6 = -4$. In $[F(4) + C] - [F(1) + C]$, the constants cancel.
13. $[(x^2 + 1)^3]_0^1 = 8 - 1 = 7$.
14. $A = \int_0^2 (2x - x^2) dx = [x^2 - x^3/3]_0^2 = \frac{4}{3}$.
15. (a) $[-\frac{1}{2} \cos(2x)]_0^\pi = 0$. (b) $A = \int_0^{\pi/2} \sin(2x) dx - \int_{\pi/2}^\pi \sin(2x) dx = 1 + 1 = 2$.

2. Extended response

16. $du = 3 dx$; $x = 0 \Rightarrow u = -1$, $x = 2 \Rightarrow u = 5$. Hence $\frac{1}{3} \int_{-1}^5 u^2 du = [u^3/9]_{-1}^5 = 14$.
17. $u = 2x^2 + 1$, $du = 4x dx$; limits 1, 3. The integral is $\int_1^3 du/u = \ln 3$. Since $2x^2 + 1 \geq 1$, the argument is positive.
18. $[x^2]_1^b = b^2 - 1 = 15$, so $b = 4$. Since $2x > 0$ on $[1, 4]$, the integral equals the area bounded by $y = 2x$, the x -axis and $x = 1, 4$.
19. $[kx^2/2 + 3x]_0^2 = 2k + 6 = 14$, so $k = 4$. Yes: $4x + 3 > 0$ throughout the interval.
20. $\int_0^{\ln 2} -e^x dx = [-e^x]_0^{\ln 2} = -1$. The area is $A = -\int_0^{\ln 2} (-e^x) dx = 1$. The curve is below the axis, so its signed contribution is negative.
21. Zeros: $-1, 0, 1$. With $F = x^4/4 - x^2/2$, the integral is $F(2) - F(-1) = \frac{9}{4}$. The signs are $+, -, +$, so $A = \int_{-1}^0 f dx - \int_0^1 f dx + \int_1^2 f dx = \frac{1}{4} + \frac{1}{4} + \frac{9}{4} = \frac{11}{4}$.
22. $x^2 = x + 2 \Rightarrow (x - 2)(x + 1) = 0$, giving $(-1, 1)$ and $(2, 4)$. The line is upper, so $A = \int_{-1}^2 (x + 2 - x^2) dx = [x^2/2 + 2x - x^3/3]_{-1}^2 = \frac{9}{2}$.
23. Intersections satisfy $x^2 = 4$, so $x = \pm 2$. Thus $A = \int_{-2}^2 [(-x^2 - 1) - (-5)] dx = \int_{-2}^2 (4 - x^2) dx = \frac{32}{3}$. The difference measures a positive vertical separation.
24. $\sin x = \cos x$ at $x = \pi/4$. Thus $A = \int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x) dx = 2(\sqrt{2} - 1) = 2\sqrt{2} - 2$.
25. The odd terms cancel. The result is $2 \int_0^2 (3x^2 + 1) dx = 2[x^3 + x]_0^2 = 20$. Directly, $[x^4/4 + x^3 - x^2/2 + x]_{-2}^2 = 12 - (-8) = 20$.
26. The total area is $\int_0^4 x dx = 8$. Hence $\int_0^c x dx = 4$, so $c^2/2 = 4$ and $c = 2\sqrt{2}$.

27. The roots are 0, 3, and $kx(3-x) > 0$ between them. Thus $18 = \int_0^3 kx(3-x) dx = k[3x^2/2 - x^3/3]_0^3 = 9k/2$, so $k = 4$.
28. The integrals are $6 - 2 = 4$ and $6 - 5 = 1$. The table determines net changes, but not any additional sign changes or cancellation within the subintervals. Therefore it does not determine $\int_{-1}^4 |F'(x)| dx$.
29. $A = \int_0^2 e^{-x^2} dx \approx 0.882$. Since $e^{-x^2} > 0$, the curve never crosses the axis.
30. The intersections have $x \approx \pm 0.824$. If r is the positive root, then $A = \int_{-r}^r (\cos x - x^2) dx = 2 \int_0^r (\cos x - x^2) dx \approx 1.09$. Retain the unrounded root in the calculation.

3. IB exam-style practice

31. $f = 2(x-1)^2 + 1$, with minimum 1. The integral is $[2x^3/3 - 2x^2 + 3x]_{-1}^2 = 10/3 - (-17/3) = 9$. Since $f > 0$ throughout, signed and total area coincide.
32. $f = 4 - 3x^{-1/2}$. Then $[4x - 6\sqrt{x}]_1^9 = 18 - (-2) = 20$. On $[1, 9]$, $f \geq 1 > 0$, so this is the area above the axis.
33. An antiderivative is $F = \frac{1}{2}e^{2x} - x$. Since $f \geq 0$, $A = \int_0^{\ln 2} (e^{2x} - 1) dx = [\frac{1}{2}e^{2x} - x]_0^{\ln 2} = \frac{3}{2} - \ln 2$.
34. $F = -\cos(2x) + \frac{1}{2}\sin(2x)$. Hence $\int_0^{\pi/4} f(x) dx = \frac{1}{2} - (-1) = \frac{3}{2}$. Both trigonometric terms are non-negative on this interval, so $f \geq 0$.
35. The intersections are $(-3, 0), (3, 0)$. Since the curve is below the axis, $A = \int_{-3}^3 (9 - x^2) dx = [9x - x^3/3]_{-3}^3 = 36$.
36. The intercept is $x = 3/2$. The signed integral is $[x^2 - 3x]_0^4 = 4$. The total area is $A = \int_0^{3/2} (3 - 2x) dx + \int_{3/2}^4 (2x - 3) dx = \frac{9}{4} + \frac{25}{4} = \frac{17}{2}$.
37. $8x - 2x^2 = 2x \Rightarrow 2x(3-x) = 0$, giving $(0, 0), (3, 6)$. The area is $A = \int_0^3 (6x - 2x^2) dx = [3x^2 - 2x^3/3]_0^3 = 9$. For the tangent, $f' = 8 - 4x = 2$ gives $x = 3/2$, $y = 15/2$. Thus $y = 2x + 9/2$.
38. f is odd, so contributions on a symmetric interval cancel. The total area is $A = \int_{-2}^0 (x^3 - 4x) dx + \int_0^2 (4x - x^3) dx = 2 \int_0^2 (4x - x^3) dx = 8$.
39. $\sqrt{x} = x^2$ gives $x = 0$ or $x = 1$, so $(0, 0), (1, 1)$. On $0 < x < 1$, $\sqrt{x} > x^2$. Thus $A = \int_0^1 (x^{1/2} - x^2) dx = [\frac{2}{3}x^{3/2} - \frac{1}{3}x^3]_0^1 = \frac{1}{3}$.
40. $du = 2x dx$ and the limits are 1, 2, so $I = \int_1^2 u^{-2} du = [-u^{-1}]_1^2 = \frac{1}{2}$. The integrand is non-negative, so this is the area under $y = 2x/(x^2 + 1)^2$ above the axis on $[0, 1]$.
41. $u = \ln x$, $du = dx/x$; limits 0, 1. Thus $I = \int_0^1 u du = \frac{1}{2}$. On $[1, e]$, $\ln x \geq 0$ and $x > 0$, so the integrand is non-negative.
42. $8/3 - 2k = 0$, so $k = 4/3$. The zero is $a = 2/\sqrt{3}$. The area is $A = \int_0^a (4/3 - x^2) dx + \int_a^2 (x^2 - 4/3) dx$. The signed integral is zero, so $A = 2 \int_0^a (4/3 - x^2) dx = \frac{32\sqrt{3}}{27}$.
43. The graphs meet at $x = 4$. Hence $A = \int_1^4 (1/x - 1/4) dx = [\ln x - x/4]_1^4 = \ln 4 - \frac{3}{4}$.
44. $\int_0^b e^x dx = 5$, so $e^b - 1 = 5$ and $b = \ln 6$. Then $A = \int_b^{2b} e^x dx = e^{2b} - e^b = 36 - 6 = 30$.
45. $\int_0^{3\pi/2} \cos x dx = [\sin x]_0^{3\pi/2} = -1$. The area is $A = \int_0^{\pi/2} \cos x dx - \int_{\pi/2}^{3\pi/2} \cos x dx = 1 - (-2) = 3$.
46. $[x^3/3 - x^2]_0^3 = 0$. The integrand changes sign at $x = 2$, so positive and negative contributions cancel. The area is $A = \int_0^2 (2x - x^2) dx + \int_2^3 (x^2 - 2x) dx = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$.
47. $A = \int_0^3 \sqrt{1+x^3} dx \approx 7.34 \text{ m}^2$. Using the unrounded area, the cost is $12A \approx 88.10$ euros.
48. Intersections: $x = -1, 0, 1$. For $d = x^3 - x$, the signs on the four intervals are $-, +, -, +$. Hence $A = -\int_{-2}^{-1} d dx + \int_{-1}^0 d dx - \int_0^1 d dx + \int_1^2 d dx = \frac{9}{4} + \frac{1}{4} + \frac{1}{4} + \frac{9}{4} = 5$.
49. Let $a = \sqrt{k/2}$; intersections occur at $x = \pm a$. Then $A = \int_{-a}^a (k - 2x^2) dx = 2[kx - 2x^3/3]_0^a = \frac{4}{3}ka$. Setting this equal to $64/3$ gives $k\sqrt{k/2} = 16$, hence $k^3 = 512$ and $k = 8$.
50. One intersection is $(0, 0)$; the other has $x = b \approx 2.51$, $y \approx 1.26$. The logarithmic curve is upper, so $A = \int_0^b [\ln(x+1) - x/2] dx \approx 0.322$. Use the unrounded value of b .

4. AA HL exclusive challenge

51. The zero is $\sqrt{a}$, so $A = \int_0^{\sqrt{a}} (a - x^2) dx + \int_{\sqrt{a}}^1 (x^2 - a) dx = \frac{1}{3} - a + \frac{4}{3}a^{3/2}$. Then $A'(a) = -1 + 2\sqrt{a} = 0$ gives $a = 1/4$. Since $A''(a) = 1/\sqrt{a} > 0$, this is the minimum. Its value is $A(1/4) = 1/4$.

52. The intersections with the line are $x = \pm\sqrt{4-c}$. The upper area is $\int_{-\sqrt{4-c}}^{\sqrt{4-c}} (4 - c - x^2) dx = \frac{4}{3}(4 - c)^{3/2}$. The whole region has area $\int_{-2}^2 (4 - x^2) dx = 32/3$. Thus $(4 - c)^{3/2} = 4$, giving $c = 4 - \sqrt[3]{16}$.

53. With $u = \ln(1 + x^2)$, $du = 2x dx/(1 + x^2)$; the limits become 0, 1. Thus $\int_0^b f = \frac{1}{2} \int_0^1 u du = 1/4$. Since f is odd, $\int_{-b}^b f = 0$. It is non-negative on $[0, b]$, so the total area is $2 \int_0^b f = 1/2$.

54. Odd terms cancel, so $2 \int_0^a (2x^2 + 1) dx = 4a^3/3 + 2a = 10/3$. Thus $2a^3 + 3a - 5 = (a - 1)(2a^2 + 2a + 5) = 0$. The quadratic has discriminant $4 - 40 < 0$, so the unique real solution is $a = 1$.

55. Intersections satisfy $x(x^2 - m) = 0$, so $x = -\sqrt{m}, 0, \sqrt{m}$. By symmetry, $A = 2 \int_0^{\sqrt{m}} (mx - x^3) dx = 2[mx^2/2 - x^4/4]_0^{\sqrt{m}} = m^2/2$. Hence $m^2 = 16$ and $m = 4$.

56. The chord has slope $-1/4$, so $y = (5 - x)/4$. It lies above the curve since $\frac{5-x}{4} - \frac{1}{x} = \frac{(x-1)(4-x)}{4x} \geq 0$ on $[1, 4]$. Thus $A = \int_1^4 [(5 - x)/4 - 1/x] dx = 15/8 - \ln 4$. The tangent has $-1/x^2 = -1/4$, hence $x = 2$ and $y = 1 - x/4$. Its vertical separation from the chord is $1/4$, so the area is $\int_1^4 (1/4) dx = 3/4$.

57. $\sin x = 2 \sin x \cos x$ gives $x = 0, \pi/3, \pi$. On $(0, \pi/3)$, $\sin(2x) > \sin x$; on $(\pi/3, \pi)$ the order reverses. Thus $A = \int_0^{\pi/3} (\sin 2x - \sin x) dx + \int_{\pi/3}^{\pi} (\sin x - \sin 2x) dx = 1/4 + 9/4 = 5/2$.

58. An antiderivative is $F = k(x^4/4 - 4x^3/3 + 3x^2/2)$. The first area is $\int_0^1 f = 5k/12$; the second is $-\int_1^3 f = 8k/3$. The ratio is $5 : 32$. Thus $37k/12 = 37$, giving $k = 12$. The signed integral is $F(3) - F(0) = -9k/4 = -27$.

59. With $u = 2x + 1$, $du = 2dx$, $\int_0^2 g = \int_1^5 f = 4 - 1 = 3$. The sign changes at $x = 1$, so the area is $\int_0^1 g - \int_1^2 g = 4 + 1 = 5$. Subtracting 1 gives signed integral $3 - 2 = 1$. The data do not locate crossings of $g = 1$ on $[0, 1]$ or determine the areas on either side of that level; the shifted total area is not fixed.

60. Equal areas give $\int_0^2 (e^{-x^2} - k) dx = 0$, hence $k = \frac{1}{2} \int_0^2 e^{-x^2} dx \approx 0.441$. The intersection is $r = \sqrt{-\ln k} \approx 0.905$. The combined area is $A = \int_0^r (e^{-x^2} - k) dx + \int_r^2 (k - e^{-x^2}) dx = 2 \int_0^r (e^{-x^2} - k) dx \approx 0.619$. Use unrounded k, r .