



Differentiation: the power rule

IB Mathematics: Analysis and Approaches SL and HL

Integer powers, gradients and algebraic reasoning | 50 shared + 10 HL questions

Learning goal	Differentiate sums of integer powers and apply the derivative.
Syllabus link	SL 5.3: $\frac{d}{dx}(ax^n) = anx^{n-1}$, $n \in \mathbb{Z}$; sums of such terms.
Structure	15 short + 15 extended + 20 IB-style + 10 HL-exclusive; answer key.
Technology	Work without a calculator unless marked GDC (Paper 2 practice). All unmarked exam-style questions are Paper 1 practice.

Instructions. SL students: Questions 1–50. HL students: Questions 1–60. Work on separate paper and show your differentiation steps. Give exact answers unless instructed otherwise. Rewrite fractions using integer powers where useful. Reciprocal expressions have $x \neq 0$ unless a narrower domain is stated.

1 | Short response

1. Differentiate $f(x) = x^7$.
2. Differentiate $y = 6x^4$.
3. Find $f'(x)$ when $f(x) = -3x^5$.
4. Differentiate $y = 9x - 12$.
5. Find $g'(x)$ when $g(x) = 14$.
6. Differentiate $f(x) = \frac{1}{2}x^6$.
7. Differentiate $y = 4x^{-2}$.
8. Differentiate $h(x) = -5x^{-3}$.
9. Write $f(x) = \frac{7}{x}$ using a negative power. Hence find $f'(x)$.
10. Find $\frac{dy}{dx}$ when $y = \frac{3}{x^2} + 2x$.
11. Differentiate $f(x) = 2x^3 - 5x^2 + 4x - 8$.



12. Differentiate $g(x) = \frac{x^4}{4} - \frac{x^2}{2} + 6$.

13. Given $f(x) = 3x^2 - 2x + 1$, find $f'(2)$.

14. A student writes $\frac{d}{dx}(5x^3 + 8) = 15x^2 + 8$. Identify and correct the error.

15. A student writes $\frac{d}{dx}(x^{-2}) = -2x^{-1}$. Identify and correct the error.

2 | Extended response

16. Let $f(x) = 4x^3 - 7x^2 + 2x - 9$.

(a) Find $f'(x)$.

(b) Evaluate $f'(-1)$ and $f'(2)$.

17. Let $g(x) = \frac{6}{x^2} - \frac{4}{x} + 5$, $x \neq 0$.

(a) Rewrite $g(x)$ using negative powers and differentiate.

(b) Find the gradient of $y = g(x)$ at $x = 2$.

18. Let $h(x) = x(x^2 - 3x + 5)$.

(a) Expand $h(x)$ and find $h'(x)$.

(b) Find $h'(3)$.

19. The curve $y = \frac{2x^3 - 5x + 4}{x}$ is defined for $x \neq 0$.

(a) Simplify the expression for y and find $\frac{dy}{dx}$.

(b) Find the gradient at $x = -1$.

20. Let $p(x) = (x - 2)(x + 3)$.

(a) Expand $p(x)$ and differentiate.

(b) Find the value of x for which $p'(x) = 7$.

21. Let $f(x) = ax^3 + 2x$, where a is a constant. Given that $f'(2) = 38$, find a .

22. Let $g(x) = x^3 + bx^2 - 4x$. The gradient at $x = 1$ is 5. Find b .

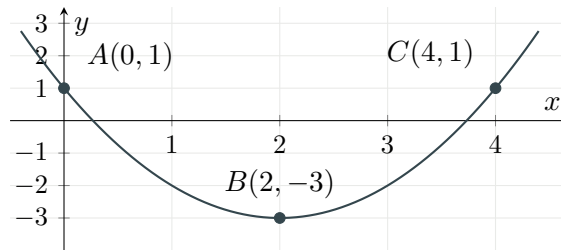


23. Let $f(x) = 2x^2 + k$ and $g(x) = 2x^2 - 7$, where k is a constant.

- (a) Show that $f'(x) = g'(x)$.
(b) Given that $f(3) = 23$, find k .

24. The graph shows $y = f(x) = x^2 - 4x + 1$.

- (a) Find $f'(x)$.
(b) Find the gradient at each of A , B and C .



25. Let $f(x) = x^3 - 6x^2 + 11x$. Find all values of x at which the gradient of the curve is 2.

26. The temperature of a liquid is modelled by $T(t) = 80 - 6t + 0.2t^2$ for $0 \leq t \leq 10$, where T is in $^{\circ}\text{C}$ and t is in minutes.

- (a) Find $T'(t)$.
(b) Find and interpret $T'(4)$, including units.

27. Let $f(x) = x^4 - 2x^2$.

- (a) Find $f(2)$ and $f'(2)$.
(b) Explain what these two values represent on the graph of $y = f(x)$.

28. Let $f(x) = 3x^2 + \frac{12}{x}$, $x > 0$.

- (a) Find $f'(x)$.
(b)  GDC Use your GDC to evaluate $f'(1.7)$. Give your answer to three significant figures.

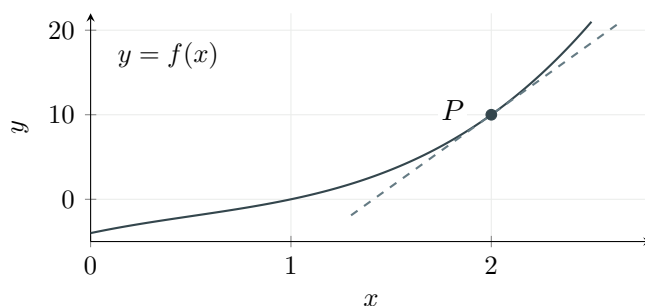
29. A student differentiates $y = \frac{x^3 + 4}{x^2}$ and obtains $\frac{3x^2}{2x}$.

- (a) Explain why this method is invalid.
(b) Rewrite y as a sum of powers and find the correct derivative.

30. Let $f(x) = ax^2 + bx + 4$. Given that $f'(1) = 7$ and $f'(3) = 15$, find a and b .



3 | IB exam-style practice: AA SL and HL

31. [Maximum mark: 4]Consider $f(x) = 2x^3 - 3x^2 + 5x - 4$.The dashed line is the tangent at P , where $x = 2$.

- (a) Find $f'(x)$. [2]
- (b) Find the gradient of the curve $y = f(x)$ at the point P , where $x = 2$. [2]

32. [Maximum mark: 4]Consider $g(x) = \frac{9}{x} + 2x^2$, $x \neq 0$.

- (a) Find $g'(x)$. [3]
- (b) Find $g'(3)$. [1]

33. [Maximum mark: 4]Let $h(x) = \frac{1}{4}x^4 - 2x^2 + 7$.

- (a) Find $h'(x)$. [2]
- (b) Find $h(2)$. [1]
- (c) Show that the gradient at $x = 2$ is zero. [1]

34. [Maximum mark: 4]Let $f(x) = ax^3 - 5x + 2$, where a is a constant.

- (a) Find $f'(x)$ in terms of a . [2]
- (b) Given that $f'(2) = 19$, determine a . [2]

35. [Maximum mark: 5]A curve has equation $y = (x + 1)(x^2 - 2x + 3)$.

- (a) Expand the expression for y . [2]
- (b) Find $\frac{dy}{dx}$. [2]
- (c) Find the gradient at $x = -1$. [1]

**3 | IB exam-style practice: AA SL and HL (continued)****36.** [Maximum mark: 5]

A curve has equation $y = \frac{3x^4 - 2x^2 + 5}{x^2}$, $x \neq 0$.

- (a) Write y as a sum of terms with integer powers of x . [2]
- (b) Find $\frac{dy}{dx}$. [2]
- (c) Find the gradient at $x = 1$. [1]

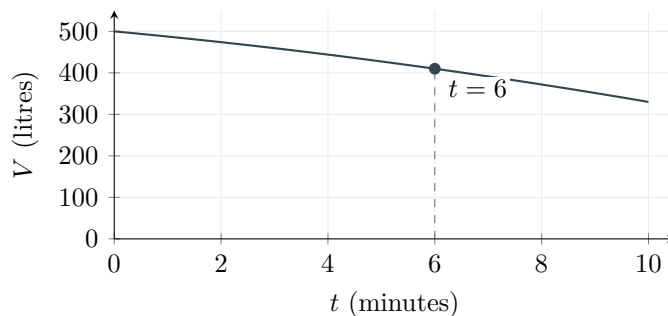
37. [Maximum mark: 4]

The height of a plant is modelled by $H(t) = 12 + 1.5t + 0.2t^2$ for $0 \leq t \leq 8$. Here H is in centimetres and t is in weeks.

- (a) Find $H'(t)$. [2]
- (b) Find and interpret $H'(5)$. [2]

38. [Maximum mark: 4]

For $0 \leq t \leq 10$, the volume of water in a tank is modelled by $V(t) = 500 - 12t - 0.5t^2$, where V is in litres and t is in minutes.



The model for the volume of water remaining in the tank.

- (a) Find $V'(t)$. [2]
- (b) Find $V'(6)$ and explain the meaning of its sign. [2]

39. [Maximum mark: 6]

Let $f(x) = x^3 + bx^2 + cx$. The gradients at $x = 1$ and $x = 2$ are 0 and 5, respectively.

- (a) Find $f'(x)$ in terms of b and c . [2]
- (b) Determine b and c . [4]

40. [Maximum mark: 6]

Let $g(x) = 4x^{-1} + bx + c$, $x > 0$. The gradient at $x = 2$ is 3, and the curve passes through (2, 11).

- (a) Find $g'(x)$. [2]
- (b) Determine b . [2]
- (c) Determine c . [2]

**3 | IB exam-style practice: AA SL and HL (continued)****41.** [Maximum mark: 5]Consider $f(x) = 2x^3 - 9x^2 + 12x + 5$.

- (a) Find $f'(x)$. [2]
- (b) Find the values of x for which the curve has gradient 12. [3]
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42. [Maximum mark: 5]Consider $f(x) = \frac{12}{x^2} + 3x$, $x > 0$.

- (a) Find $f'(x)$. [2]
- (b) Find the positive value of x at which the gradient is zero. [3]
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43. [Maximum mark: 5]The cost, in euros, of producing q items is modelled by $C(q) = 250 + 18q + 0.04q^2$, for $0 \leq q \leq 200$. Treat q as a continuous variable.

- (a) Find $C'(q)$. [2]
- (b) Find $C'(50)$ and state its units. [2]
- (c) Interpret your answer in the context of the model. [1]
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44. [Maximum mark: 5]Let $p(x) = x^3 - 2x$ and $q(x) = 2x^2 + 4x$.

- (a) Find $p'(x)$ and $q'(x)$. [2]
- (b)  GDC Find both values of x at which the two curves have the same gradient. Give your answers to three significant figures. [3]
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45. [Maximum mark: 6]Let $f(x) = ax^{-2} + bx$, $x > 0$. It is given that $f'(1) = -5$ and $f'(2) = 2$.

- (a) Find $f'(x)$. [2]
- (b) Determine a and b . [4]

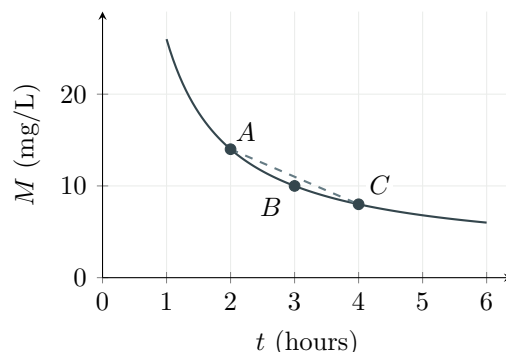


3 | IB exam-style practice: AA SL and HL (continued)

46. [Maximum mark: 6]

For $1 \leq t \leq 6$, the concentration of a substance is modelled by $M(t) = \frac{24}{t} + 2$, where M is in mg/L and t is in hours.

- (a) Find $M'(t)$. [2]
- (b) Find and interpret $M'(3)$. [2]
- (c) Calculate the average rate of change from $t = 2$ to $t = 4$ (the gradient of AC). [2]



A , B and C occur at $t = 2, 3$ and 4 . The dashed segment is AC .

47. [Maximum mark: 5]

GDC Let $f(x) = 0.3x^4 - 1.7x^2 + 2.4x$.

- (a) Find $f'(x)$ analytically. [2]
- (b) Calculate $f'(1.8)$, giving your answer to three significant figures. [2]
- (c) Use your GDC's numerical derivative feature to check your answer. [1]

48. [Maximum mark: 5]

Two models are $A(t) = 100 + 8t + 0.5t^2$ and $B(t) = 120 + 2t + t^2$, for $0 \leq t \leq 10$. Both give quantities in grams, with t in minutes.

- (a) Find $A'(t)$ and $B'(t)$. [2]
- (b) Find when the two quantities have the same instantaneous rate of change. [2]
- (c) Find that common rate, with units. [1]

49. [Maximum mark: 6]

Let $f(x) = ax^3 + bx^2 + c$. It is given that $f(0) = 7$, $f'(1) = 5$ and $f'(2) = 16$.

- (a) Write down c and find $f'(x)$. [2]
- (b) Determine a and b . [4]

50. [Maximum mark: 8]

Let $f(x) = \frac{k}{x^2} + mx + n$, $x > 0$. The curve passes through $(1, 8)$ and has gradients -6 at $x = 1$ and 1 at $x = 2$.

- (a) Find $f'(x)$ in terms of k and m . [2]
- (b) Determine k and m . [4]
- (c) Hence determine n . [2]

**4 | AA HL-exclusive challenge**

These questions deepen SL 5.3 through algebra, parameters and proof. No additional differentiation rules are required. Work without a calculator.

51. [Maximum mark: 6]

Let $f(x) = \frac{(x^2 - 1)^2}{x^2}$, $x \neq 0$.

- (a) Write $f(x)$ as a sum of integer powers of x . [2]
- (b) Hence find $f'(x)$ without using the quotient rule. [2]
- (c) Find the exact gradient at $x = 2$. [2]

52. [Maximum mark: 7]

A cubic polynomial is $f(x) = ax^3 + bx^2 + cx + d$. It satisfies $f'(0) = 6$, $f'(1) = 0$, $f'(2) = 6$ and $f(1) = 5$.

- (a) Determine a , b , c and d . [7]

53. [Maximum mark: 5]

Let $f(x) = ax^n$, where $a > 0$ and n is an integer greater than 1. It is given that $f'(2) = 12$ and $f'(4) = 96$.

- (a) By considering a ratio, determine n . [3]
- (b) Hence find a . [2]

54. [Maximum mark: 8]

Let $f(x) = x^3 - 3kx^2 + 3x$, where $k \in \mathbb{R}$.

- (a) Find $f'(x)$. [2]
- (b) Find all values of k for which the curve has zero gradient at two distinct real values of x . [3]
- (c) Find all values of k for which the curve has zero gradient at exactly one real value of x . State that x -value in each case. [3]

55. [Maximum mark: 7]

Let $f(x) = \frac{a}{x} + bx^2$, $x > 0$. The gradient is zero at $x = 1$ and is 7 at $x = 2$.

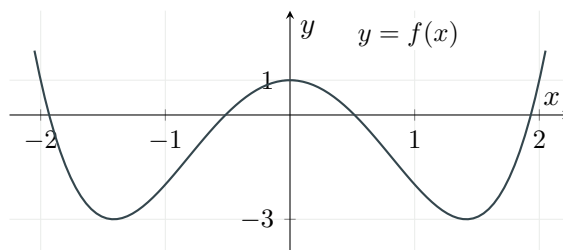
- (a) Determine a and b . [5]
- (b) Find the gradient at $x = \frac{1}{2}$. [2]



4 | AA HL-exclusive challenge (continued)

56. [Maximum mark: 8]

Let $f(x) = x^4 - 4x^2 + 1$. Points P and Q on the curve have x -coordinates a and $-a$, respectively, where $a > 0$.



The curve is symmetric about the y -axis.

- (a) Show that $f'(-a) = -f'(a)$. [3]
- (b) Given that the gradients at P and Q are equal, find a . [3]
- (c) Find the coordinates of P and Q . [2]

57. [Maximum mark: 7]

Let $f(x) = \frac{x^3 - 1}{x}$, $x > 0$.

- (a) Find $f'(x)$ using the power rule. [2]
- (b) Show that $f'(x) = 3$ is equivalent to $(x - 1)^2(2x + 1) = 0$. [3]
- (c) Hence determine every point on the curve at which the gradient is 3. [2]

58. [Maximum mark: 6]

Let $f(x) = x^3 + ax^2 + bx$. The curve has zero gradient at $x = -1$ and $x = 3$.

- (a) Write $f'(x)$ in factorized form and hence determine a and b . [4]
- (b) Find the gradient at the midpoint of the two given x -values. [2]

59. [Maximum mark: 8]

Let $f(x) = ax^{-2} + bx^{-1}$, $x > 0$. It is given that $f'(1) = 2$ and $f'(2) = 0$.

- (a) Determine a and b . [5]
- (b) Show that there is exactly one point on the curve where the gradient is zero, and find its coordinates. [3]

60. [Maximum mark: 9]

Let $f(x) = x^3 + ax^2 + bx + c$. For every real x , $f'(x) = f'(2 - x)$. It is also given that $f'(0) = 5$ and $f(1) = 4$.

- (a) Show that $a = -3$. [4]
- (b) Determine b and c . [2]
- (c) Prove that $f(1 + h) + f(1 - h) = 8$ for every real h . [3]

**Answer key and solution outlines**

Use $\frac{d}{dx}(ax^n) = anx^{n-1}$ term by term. Constants differentiate to zero.

1 | Short response

1. $f'(x) = 7x^6$.
2. $\frac{dy}{dx} = 24x^3$.
3. $f'(x) = -15x^4$.
4. $\frac{dy}{dx} = 9$.
5. $g'(x) = 0$; the derivative of a constant is zero.
6. $f'(x) = 3x^5$.
7. $\frac{dy}{dx} = -8x^{-3} = -\frac{8}{x^3}$.
8. $h'(x) = 15x^{-4} = \frac{15}{x^4}$.
9. $f(x) = 7x^{-1}$, so $f'(x) = -7x^{-2} = -\frac{7}{x^2}$.
10. $\frac{dy}{dx} = -6x^{-3} + 2$.
11. $f'(x) = 6x^2 - 10x + 4$.
12. $g'(x) = x^3 - x$.
13. $f'(x) = 6x - 2$; $f'(2) = 12 - 2 = 10$.
14. The constant 8 differentiates to 0, so the derivative is $15x^2$.
15. Subtract 1 from the exponent: $-2 - 1 = -3$. The derivative is $-2x^{-3}$.



Answer key and solution outlines

2 | Extended response

16. (a) $f'(x) = 12x^2 - 14x + 2$. (b) $f'(-1) = 28$; $f'(2) = 22$.
17. (a) $g = 6x^{-2} - 4x^{-1} + 5$; $g' = -12x^{-3} + 4x^{-2}$. (b) $g'(2) = -\frac{12}{8} + \frac{4}{4} = -\frac{1}{2}$.
18. (a) $h = x^3 - 3x^2 + 5x$; $h' = 3x^2 - 6x + 5$. (b) $h'(3) = 14$.
19. (a) $y = 2x^2 - 5 + 4x^{-1}$; $\frac{dy}{dx} = 4x - 4x^{-2}$. (b) Gradient $= -4 - 4 = -8$.
20. (a) $p = x^2 + x - 6$; $p' = 2x + 1$. (b) $2x + 1 = 7 \Rightarrow x = 3$.
21. $f' = 3ax^2 + 2$. Thus $12a + 2 = 38$, giving $a = 3$.
22. $g' = 3x^2 + 2bx - 4$; $3 + 2b - 4 = 5 \Rightarrow b = 3$.
23. (a) Both derivatives are $4x$ because constants differentiate to zero. (b) $18 + k = 23 \Rightarrow k = 5$.
24. (a) $f' = 2x - 4$. (b) At A : -4 ; at B : 0 ; at C : 4 .
25. $f' = 3x^2 - 12x + 11$. Set $f' = 2$: $3x^2 - 12x + 9 = 0$, so $(x - 1)(x - 3) = 0$. Thus $x = 1, 3$.
26. (a) $T' = -6 + 0.4t$. (b) $T'(4) = -4.4^\circ\text{C/min}$: after 4 minutes the liquid is cooling at 4.4°C/min .
27. (a) $f(2) = 8$; $f' = 4x^3 - 4x$ gives $f'(2) = 24$. (b) The point is $(2, 8)$ and the gradient there is 24 .
28. (a) $f' = 6x - 12x^{-2}$. (b) $f'(1.7) = 10.2 - 12/(1.7)^2 \approx 6.05$.
29. (a) Differentiating numerator and denominator separately and dividing does not give the derivative of the quotient. (b) $y = x + 4x^{-2}$; $\frac{dy}{dx} = 1 - 8x^{-3}$.
30. $f' = 2ax + b$. Hence $2a + b = 7$, $6a + b = 15$. Subtracting gives $4a = 8$, so $a = 2$ and $b = 3$.



Answer key and solution outlines

3 | IB exam-style practice: AA SL and HL

31. (a) $f' = 6x^2 - 6x + 5$. (b) $f'(2) = 24 - 12 + 5 = 17$.
32. (a) $g = 9x^{-1} + 2x^2 \Rightarrow g' = -9x^{-2} + 4x$. (b) $g'(3) = -1 + 12 = 11$.
33. (a) $h' = x^3 - 4x$. (b) $h(2) = 4 - 8 + 7 = 3$. (c) $h'(2) = 8 - 8 = 0$.
34. (a) $f' = 3ax^2 - 5$. (b) $12a - 5 = 19 \Rightarrow a = 2$.
35. (a) $y = x^3 - x^2 + x + 3$. (b) $\frac{dy}{dx} = 3x^2 - 2x + 1$. (c) Gradient = 6.
36. (a) $y = 3x^2 - 2 + 5x^{-2}$. (b) $\frac{dy}{dx} = 6x - 10x^{-3}$. (c) Gradient = -4.
37. (a) $H' = 1.5 + 0.4t$. (b) $H'(5) = 3.5$ cm/week: at week 5 the plant's height is increasing at this rate.
38. (a) $V' = -12 - t$. (b) $V'(6) = -18$ L/min. The negative sign means the volume is decreasing (at 18 L/min).
39. (a) $f' = 3x^2 + 2bx + c$. (b) $3 + 2b + c = 0$ and $12 + 4b + c = 5$. Subtract: $9 + 2b = 5$, so $b = -2$; then $c = 1$.
40. (a) $g' = -4x^{-2} + b$. (b) $-1 + b = 3 \Rightarrow b = 4$. (c) $2 + 8 + c = 11 \Rightarrow c = 1$.
41. (a) $f' = 6x^2 - 18x + 12$. (b) $6x^2 - 18x + 12 = 12 \Rightarrow 6x(x - 3) = 0$, so $x = 0, 3$.
42. (a) $f' = -24x^{-3} + 3$. (b) $-24/x^3 + 3 = 0 \Rightarrow x^3 = 8$, so $x = 2$.
43. (a) $C' = 18 + 0.08q$. (b) $C'(50) = 22$ euros/item. (c) Near 50 items, each additional item increases the cost by approximately €22.
44. (a) $p' = 3x^2 - 2$; $q' = 4x + 4$. (b) $3x^2 - 4x - 6 = 0$. Thus $x = \frac{2 \pm \sqrt{22}}{3} \approx -0.897, 2.23$.
45. (a) $f' = -2ax^{-3} + b$. (b) $-2a + b = -5$; $-a/4 + b = 2$. Subtract: $7a/4 = 7$, giving $a = 4$, $b = 3$.
46. (a) $M' = -24t^{-2}$. (b) $M'(3) = -8/3$ mg/L per hour: concentration is falling at 8/3 mg/L per hour at hour 3. (c) $[M(4) - M(2)]/(4 - 2) = (8 - 14)/2 = -3$ mg/L per hour.
47. (a) $f' = 1.2x^3 - 3.4x + 2.4$. (b) $f'(1.8) = 3.2784 \approx 3.28$. (c) Numerical derivative of the original f at 1.8 gives 3.2784, agreeing with (b).
48. (a) $A' = 8 + t$; $B' = 2 + 2t$. (b) $8 + t = 2 + 2t \Rightarrow t = 6$ min. (c) Common rate = 14 g/min.
49. (a) $c = 7$; $f' = 3ax^2 + 2bx$. (b) $3a + 2b = 5$, $12a + 4b = 16$. Subtract twice the first equation: $6a = 6$, so $a = 1$; then $b = 1$.
50. (a) $f' = -2kx^{-3} + m$. (b) $-2k + m = -6$; $-k/4 + m = 1$. Subtract: $7k/4 = 7$, so $k = 4$ and $m = 2$. (c) $4 + 2 + n = 8 \Rightarrow n = 2$.



Answer key and solution outlines

4 | AA HL-exclusive challenge

51. (a) $f = x^2 - 2 + x^{-2}$. (b) $f' = 2x - 2x^{-3}$. (c) $f'(2) = 4 - \frac{1}{4} = \frac{15}{4}$.
52. $f' = 3ax^2 + 2bx + c$. At $x = 0$, $c = 6$. The other gradients give $3a + 2b = -6$ and $12a + 4b = 0$. Subtract twice the first equation: $6a = 12$, so $a = 2$, $b = -6$. Finally $2 - 6 + 6 + d = 5$, so $d = 3$.
53. $f' = anx^{n-1}$. Thus $\frac{f'(4)}{f'(2)} = \left(\frac{4}{2}\right)^{n-1} = 8$, giving $n - 1 = 3$ and $n = 4$. Then $4a(2^3) = 12$, so $a = \frac{3}{8}$.
54. (a) $f' = 3x^2 - 6kx + 3$. (b) Set $f' = 0$: $x^2 - 2kx + 1 = 0$. Its discriminant is $4k^2 - 4$. Two distinct real roots require $k^2 > 1$: $k < -1$ or $k > 1$. (c) A repeated root requires $k^2 = 1$. For $k = 1$, $x = 1$; for $k = -1$, $x = -1$.
55. $f' = -ax^{-2} + 2bx$. Hence $-a + 2b = 0$ and $-a/4 + 4b = 7$. With $a = 2b$, $7b/2 = 7$, so $b = 2$ and $a = 4$. Then $f'(\frac{1}{2}) = -16 + 2 = -14$.
56. (a) $f' = 4x^3 - 8x$, so $f'(-a) = -4a^3 + 8a = -f'(a)$. (b) Equality gives $f'(a) = 0$, hence $4a(a^2 - 2) = 0$. Since $a > 0$, $a = \sqrt{2}$. (c) $f(\pm\sqrt{2}) = 4 - 8 + 1 = -3$: $P(\sqrt{2}, -3)$ and $Q(-\sqrt{2}, -3)$.
57. (a) $f = x^2 - x^{-1}$, so $f' = 2x + x^{-2}$. (b) $2x + x^{-2} = 3 \Rightarrow 2x^3 - 3x^2 + 1 = 0$. Expanding $(x - 1)^2(2x + 1)$ gives this cubic. (c) The roots are 1 and $-\frac{1}{2}$; the domain excludes $-\frac{1}{2}$. Since $f(1) = 0$, the only point is $(1, 0)$.
58. (a) $f' = 3x^2 + 2ax + b$. Its roots and leading coefficient give $f' = 3(x + 1)(x - 3) = 3x^2 - 6x - 9$. Comparing coefficients gives $a = -3$, $b = -9$. (b) The midpoint is $x = 1$, where $f'(1) = 3(2)(-2) = -12$.
59. (a) $f' = -2ax^{-3} - bx^{-2}$. Thus $-2a - b = 2$ and $-a/4 - b/4 = 0$. The second gives $b = -a$; hence $a = -2$, $b = 2$. (b) $f' = (4 - 2x)/x^3$. Since $x > 0$, this is zero only at $x = 2$. As $f(2) = -\frac{1}{2} + 1 = \frac{1}{2}$, the point is $(2, \frac{1}{2})$.
60. (a) $f' = 3x^2 + 2ax + b$ and $f'(2 - x) = 3x^2 + (-12 - 2a)x + 12 + 4a + b$. Comparing coefficients of x gives $2a = -12 - 2a$, so $a = -3$; the constants then agree. (b) $f'(0) = b = 5$ and $1 - 3 + 5 + c = 4$ give $c = 1$. (c) Expanding gives $f(1 + h) = 4 + 2h + h^3$ and $f(1 - h) = 4 - 2h - h^3$. Their sum is 8.