



Introduction to Integration

IB Mathematics: Analysis and Approaches SL / HL | SL 5.5

50 shared questions + 10 AA HL challenges

Learning goal	Find antiderivatives, use a given point to determine a function, and model areas and accumulated quantities.
Syllabus link	SL 5.5: integer powers x^n ($n \neq -1$), boundary conditions, definite integrals using technology, and area above the x -axis.
Structure	1–15 short response; 16–30 extended response; 31–50 IB exam-style. 51–60: AA HL exclusive challenge.
Technology	 GDC required. Other questions: work without a calculator. In Section 3, unmarked questions follow Paper 1 expectations; marked questions follow Paper 2 expectations.

Instructions. Work on separate paper and show your method. Include $+C$ for general antiderivatives. Give exact algebraic answers unless a decimal is requested. For GDC calculations, retain full precision and give non-exact final values to 3 significant figures, unless stated otherwise. Write the definite integral before calculating an area. All diagrams are schematic.

The HL section develops SL 5.5 through harder algebra and connections to earlier content; no additional AHL integration techniques are assumed.

1. Short response

15 questions

1. Find $\int (12x^2 - 5) \, dx$.

2. Find $\int (7x^6 - 3x^2 + 2x) \, dx$.

3. Find $\int \left(\frac{3}{2}x^2 - \frac{4}{5}x^3 \right) \, dx$.

4. Find $\int 5x^{-2} \, dx$, for $x > 0$.

5. Find $\int \left(\frac{8}{x^3} - \frac{6}{x^4} \right) \, dx$, for $x > 0$.

6. Find $\int (x - 3)(x + 2) \, dx$.

7. Find $\int \frac{x^4 + 3x^2 - 2}{x^2} \, dx$, for $x > 0$.

8. Given $f'(x) = 4x^3 - 6x$ and $f(0) = 7$, find $f(x)$.

9. Given $F'(x) = 6x^2 + 4x - 1$ and $F(1) = 8$, find $F(2)$.

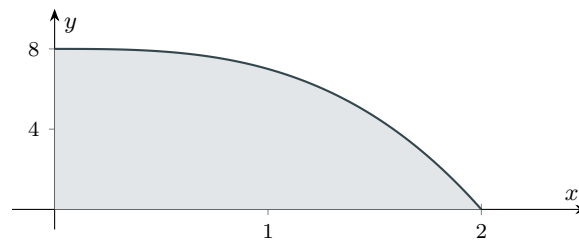
10. Explain why $x^3 - 2x + 1$ and $x^3 - 2x - 7$ have the same derivative.

11. A student claims that $\int x^{-3} dx = -3x^{-4} + C$. Correct the error and verify your result.

12. Complete the table. Take $x > 0$ where needed.

$f(x)$	General antiderivative
ax^4	...
...	$-\frac{b}{3}x^{-3} + C$
$2x - 5$	...

13. The shaded region is bounded by $y = 8 - x^3$, the coordinate axes, and lies in the first quadrant. Write down an integral for its area.



14. Use your GDC to evaluate $\int_{0.5}^{2.5} (x^4 + 2) dx$, giving your answer to three significant figures.

15. For a continuous function with $f(x) > 0$ on $[a, b]$, where $a < b$, explain what $\int_a^b f(x) dx$ represents geometrically.

2. Extended response

15 questions

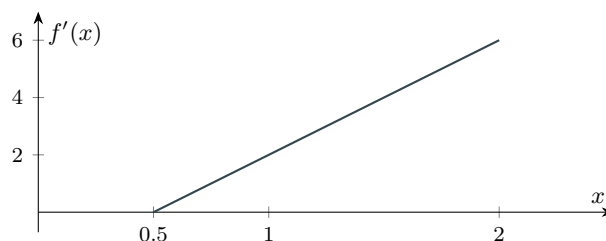
16. Given $f'(x) = 3(x - 2)^2$ and $f(0) = 1$, find $f(x)$ in expanded form and find $f(2)$.

17. For $x > 0$, $f'(x) = 6x^{-3} - 4x^{-2} + 3$ and $f(1) = 5$. Find $f(x)$ and $f(2)$.

18. A curve has derivative $6x^2 + 2x - 4$. Its tangent at $x = 1$ passes through $(0, 2)$. Find the equation of the curve.

19. Given $f'(x) = ax^2 + b$, $f(0) = 2$, $f(1) = 6$ and $f(2) = 18$, find a , b and $f(x)$.

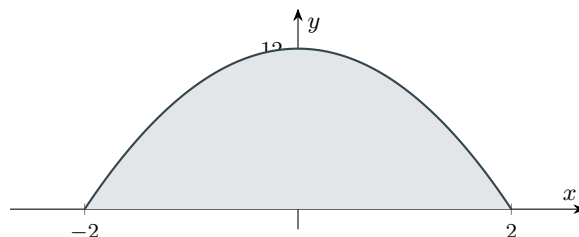
20. The diagram shows $f'(x) = 4x - 2$. Given $f(1) = 3$, find $f(x)$. Explain why f' alone does not determine the y -intercept of f .



21. The derivative of F is $(x + 1)(x - 2)$. Given $F(1) = 0$, find $F(x)$ and all its real zeros.

22. Find a and b if $F(x) = ax^4 + bx^{-2} + 7$ is an antiderivative of $8x^3 + 6x^{-3}$ on $x > 0$. Can this same function satisfy $F(1) = 10$? Explain.

23.  The region shown is bounded by $y = 12 - 3x^2$ and the x -axis. Find its intercepts. Then use technology to calculate its area, first writing the integral.



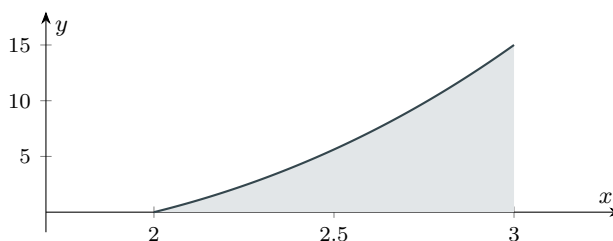
24.  The area under $y = 2x^3 + 1$ from $x = 0$ to $x = k$, with $k > 0$, is 10. Form an integral equation and find k to three significant figures.

25. For $x > 0$, $F'(x) = \frac{(x^2 - 1)^2}{x^2}$ and $F(1) = 0$. Find $F(x)$ and verify your expression by differentiation.

26. A quantity $Q(t)$ satisfies $Q'(t) = 6t^2 - 4t + 5$ for $t \geq 0$, and $Q(1) = 12$. Find $Q(t)$ and the increase in Q between $t = 1$ and $t = 3$.

27. Two functions have the same derivative $3x^2 - 4x + 1$. Given $F(0) = 5$ and $G(2) = 1$, find $F(x) - G(x)$. Explain why your answer is independent of x .

28.  A region is bounded by $y = x^3 - 4x$, the x -axis and $x = 3$, for $2 \leq x \leq 3$. Write and evaluate an integral for its area. Explain why the integral is positive.



29.  The area under $y = a(x^2 + 1)$ from $x = 0$ to $x = 2$ is 21, where $a > 0$. Find a , first writing the appropriate integral equation.

30. The table contains three points on $y = F(x)$.

x	-1	0	1
$F(x)$	1	2	5

It is known that $F'(x) = ax^2 + b$. Determine a and b , or explain why no such function exists.

3. IB exam-style practice

20 questions

31.

[Maximum mark: 5]

Let $f'(x) = 15x^4 - 6x^2 + 2$ and $f(1) = 9$.

- (a) Find $f(x)$. [4]
 (b) Find $f(-1)$. [1]

32.

[Maximum mark: 5]

A curve has derivative $f'(x) = 6x^2 - 8x + 3$. The tangent at $x = 2$ is $y = 11x - 9$.

- (a) Find $f(2)$. [1]
 (b) Find $f(x)$. [4]

33.

[Maximum mark: 6]

For $x > 0$, $f'(x) = 2x + \frac{6}{x^2} - \frac{4}{x^3}$ and $f(1) = 0$.

- (a) Find $f(x)$. [4]
 (b) Find $f(2)$. [2]

34.

[Maximum mark: 5]

For $x > 0$, $F(x) = ax^n + C$ is an antiderivative of $12x^{-5}$, where $a \neq 0$ and n is an integer.

- (a) Find a and n . [3]
 (b) Given $F(1) = 4$, find $F(2)$. [2]

35.

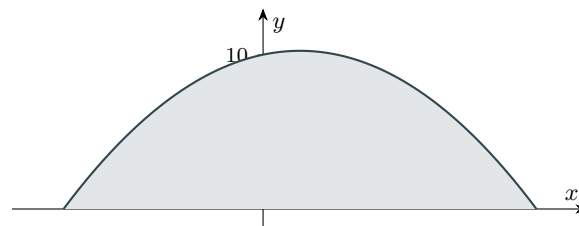
[Maximum mark: 6]

The graph of f' is a line through $(0, -3)$ and $(2, 5)$. The curve $y = f(x)$ passes through $(1, 4)$.

- (a) Find $f'(x)$. [2]
 (b) Find $f(x)$. [3]
 (c) Write down the vertical translation that takes $y = f(x)$ to the antiderivative passing through the origin. [1]

36.

[Maximum mark: 5]

The shaded region is bounded by $y = 10 + x - x^2$ and the x -axis.


- (a) Find the exact x -coordinates of the intercepts. [2]
 (b) Write an integral for the area. [2]
 (c) Find the area to three significant figures. [1]

37.

[Maximum mark: 6]

The curve $y = 6x - x^2$ encloses a region R with the x -axis.

- (a) Find the intercepts and write an integral for the area of R . [3]
 (b) Use your GDC to calculate this area. [1]
 (c) A triangle has vertices $(0, 0)$, $(6, 0)$ and $(a, 12)$. Show that it has the same area as R , for every real a . [2]

38. 

[Maximum mark: 6]

Consider $g(x) = x^2 + \frac{4}{x^2}$, for $x > 0$.

- Find the general antiderivative of g . [2]
- Find the area bounded by $y = g(x)$, the x -axis, $x = 1$ and $x = 3$, first stating the integral. [3]
- Explain why no splitting of the interval is needed. [1]

39. 

[Maximum mark: 6]

The region below $y = x^4 + 1$ and above the x -axis, between $x = 0$ and $x = b$, has area 7.

- Write an integral equation for b . [2]
- Find the positive value of b to three significant figures. [2]
- Find the height of a rectangle with width b and the same area. [2]

40.

[Maximum mark: 6]

A function has derivative $f'(x) = ax^2 + b$. Its graph passes through $(0, 3)$ and $(1, 8)$, and has gradient 9 at $x = 1$.

- Write down two equations in a and b . [3]
- Find a , b and $f(x)$. [3]

41.

[Maximum mark: 4]

A student obtains $F(x) = x^4 - 2x^2 + 5$ as an antiderivative of $4x^3 - 4x$.

- Verify the derivative. [1]
- The required function has $F(2) = 7$. Correct the constant term. [2]
- Explain why the derivative check did not detect the incorrect constant. [1]

42. 

[Maximum mark: 7]

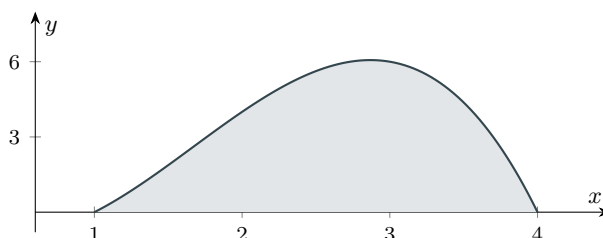
The area under $y = 3x^2 + 2$, above the x -axis from $x = 0$ to $x = 2$, is divided into equal parts by $x = k$.

- Calculate the total area, stating the integral. [3]
- Write an integral equation for k . [2]
- Find k to three significant figures. [2]

43. 

[Maximum mark: 7]

Let $f(x) = x(x-1)(4-x)$. The region R lies between its graph and the x -axis for $1 \leq x \leq 4$.



- Show that $f(x) > 0$ for $1 < x < 4$. [1]
- Write and evaluate an integral for the area of R . [3]
- Find a general antiderivative of f . [3]



44.

[Maximum mark: 6]

A reservoir contains $V(t)$ cubic metres of water, with $V'(t) = 6t^2 + 4t + 3$ for $t \geq 0$. It contains 20 m^3 at $t = 0$.

- (a) Find $V(t)$. [3]
- (b) Find $V(3)$. [1]
- (c) Find the increase in volume between $t = 1$ and $t = 3$. [2]

45.

[Maximum mark: 6]

A curve has derivative $f'(x) = 3x^2 + ax + b$. It passes through $(0, 2)$ and $(1, 5)$, and its tangent at $x = 0$ has gradient 4.

- (a) Find b . [1]
- (b) Find a and the equation of the curve. [4]
- (c) Find its gradient at $x = 2$. [1]

46.

[Maximum mark: 7]

For $x > 0$, $F'(x) = \frac{x^5 + 2x^3 - 6}{x^3}$ and $F(1) = 5$.

- (a) Express $F'(x)$ as a sum of powers of x . [1]
- (b) Find $F(x)$. [4]
- (c) Find $F(2)$. [2]

47.

[Maximum mark: 6]

Functions F and G are antiderivatives of $6x^2 + 2x - 3$ on $\mathbb{R}$. Given $F(1) = 2$ and $G(-1) = 4$:

- (a) find both functions; [4]
- (b) determine whether their graphs intersect, justifying your answer. [2]

48.

[Maximum mark: 7]

A cross-section is bounded by $y = 1.5 + 0.2x^3$, the x -axis, $x = 0$ and $x = 3$. Coordinates are in metres.

- (a) Find its area, first stating the integral. [3]
- (b) A prism with this cross-section has length 5 m. Find its volume. [2]
- (c) Find the height of a rectangle of width 3 m with the same cross-sectional area. [2]

49.

[Maximum mark: 7]

Let $f(x) = x^4 + x^2 + 1$.

- (a) Find a general antiderivative. [2]
- (b) Calculate the area under f between $x = 0$ and $x = 2$, first stating the integral. [3]
- (c) Find $b > 0$ such that the area under f between $x = 0$ and $x = b$ is 5. [2]

50.

[Maximum mark: 9]

A curve has derivative $f'(x) = 6 - 6x^2$ and passes through $(0, 4)$.

- (a) Find $f(x)$. [3]
- (b) Show that $f(x) = 2(2 - x)(x + 1)^2$. [2]
- (c) Write down its distinct x -intercepts. [1]
- (d) Calculate the area between the curve and the x -axis for $-1 \leq x \leq 2$, first stating the integral. [3]



4. AA HL exclusive challenge

10 questions

AA SL students: the shared worksheet ends at Question 50.**51.**

[Maximum mark: 7]

A function satisfies $f'(x) = 3ax^2 + 2bx + c$. It is known that $f(0) = 2$, $f(1) = 5$, $f(-1) = 1$ and $f'(1) = 8$. Determine $f(x)$. [7]

52.

[Maximum mark: 7]

For $x > 0$, $F'(x) = \frac{(x^2 - 1)^2}{x^4}$ and $F(1) = 0$.

- (a) Find $F(x)$ and the exact value of $F(2)$. [5]
(b) Explain why F never decreases on $x > 0$, even though its derivative is zero at one point. [2]

53.

[Maximum mark: 6]

Let $k > 0$. A function satisfies $f'(x) = 12x^2 - 12kx$, $f(0) = 0$ and $f(k) = -4$. Find the exact value of k and hence find $f(2k)$. [6]

54.

[Maximum mark: 7]

A function has derivative $f'(x) = 3x^2 + 2ax + b$, with $f(1) = f(-1) = 0$ and $f(2) = 15$. Determine a , b and $f(x)$ in factorized form. [7]

55.

[Maximum mark: 8]

A curve satisfies $f'(x) = 4x^3 - 4ax$ and $f(0) = b$. The x -axis is tangent to the curve at $x = 2$.

- (a) Find a and b . [5]
(b) Factorize $f(x)$ and find its other x -intercept. [2]
(c) Explain why the curve never lies below the x -axis. [1]

56.

[Maximum mark: 8]

For $x > 0$, $F'(x) = \frac{ax^2 + b}{x^4}$. Given $F(1) = 0$, $F(2) = 1$ and $F'(1) = 3$, determine a , b and $F(x)$. [8]

57.

[Maximum mark: 7]

A function satisfies $F'(x) = x^3 - ax$ and $F(-1) = F(2) = 0$. Find a and all real zeros of F . [7]

58.

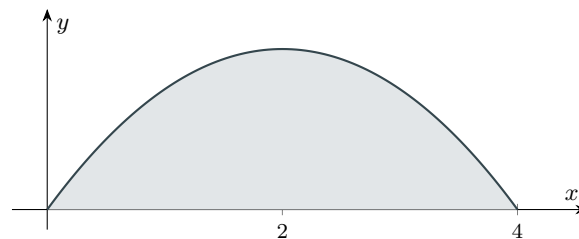
[Maximum mark: 6]

Let $f'(x) = kx^n$, where $k > 0$ and n is a nonnegative integer. Given $f(0) = 0$, $f(1) = 3$ and $f(2) = 96$, find n , k and $f(3)$. [6]

59. 

[Maximum mark: 8]

For $0 \leq x \leq 4$, let $f(x) = kx(4 - x)$ with $k > 0$. The area under the curve is 32.



- Write an integral equation and find k . [3]
- The line $x = c$ divides off one quarter of the total area on its left. Find c to three significant figures. [3]
- Explain why this value of c is unchanged if k is replaced by any other positive value. [2]

60.

[Maximum mark: 8]

A family of curves has derivative $f'(x) = 3x^2 - 2ax$, where a is real. The line $y = 3x + 1$ is tangent to a curve at $x = a$.

- Find the two possible values of a . [2]
- Find the equation of each possible curve. [4]
- Find the exact product of their y -intercepts. [2]



Answer key and solution outlines

AA SL/HL 5.5

Equivalent algebraic forms are accepted. For numerical answers, use unrounded intermediate values.

Section 1: Short response

1. $4x^3 - 5x + C$.
2. $x^7 - x^3 + x^2 + C$.
3. $\frac{1}{2}x^3 - \frac{1}{5}x^4 + C$.
4. $-5x^{-1} + C$.
5. $-4x^{-2} + 2x^{-3} + C$.
6. Expand to $x^2 - x - 6$; integral $= \frac{x^3}{3} - \frac{x^2}{2} - 6x + C$.
7. Integrand $= x^2 + 3 - 2x^{-2}$; integral $= \frac{x^3}{3} + 3x + \frac{2}{x} + C$.
8. $f(x) = x^4 - 3x^2 + 7$.
9. $F(x) = 2x^3 + 2x^2 - x + C$; $C = 5$. Thus $F(2) = 27$.
10. They differ by the constant 8, whose derivative is zero. Both derivatives are $3x^2 - 2$.
11. $\int x^{-3} dx = -\frac{1}{2}x^{-2} + C$. Differentiating gives $(-\frac{1}{2})(-2)x^{-3} = x^{-3}$.
12. $\frac{a}{5}x^5 + C$; bx^{-4} ; $x^2 - 5x + C$.
13. $A = \int_0^2 (8 - x^3) dx$.
14. 23.5 (unrounded 23.525).
15. The area between the graph, the x -axis and the vertical lines $x = a$ and $x = b$.

Section 2: Extended response

16. $f'(x) = 3x^2 - 12x + 12$, so $f(x) = x^3 - 6x^2 + 12x + C$, $C = 1$. Thus $f(2) = 9$.
17. $f(x) = -3x^{-2} + 4x^{-1} + 3x + C$. $5 = 4 + C$ gives $C = 1$. Hence $f(2) = -3/4 + 2 + 6 + 1 = 33/4$.
18. The tangent gradient is $6 + 2 - 4 = 4$. Since its equation is $y = 4x + 2$, the curve contains $(1, 6)$. Integration gives $y = 2x^3 + x^2 - 4x + C$, so $6 = -1 + C$ and $C = 7$.
19. $f(x) = \frac{a}{3}x^3 + bx + 2$. The points give $a/3 + b = 4$ and $8a/3 + 2b = 16$. Thus $a = 4$, $b = 8/3$ and $f(x) = \frac{4}{3}x^3 + \frac{8}{3}x + 2$.
20. $f(x) = 2x^2 - 2x + C$ and $C = 3$. The derivative is unchanged by vertical translations; a boundary condition is needed to fix C .
21. $F(x) = x^3/3 - x^2/2 - 2x + C$ and $C = 13/6$. Then $6F(x) = 2x^3 - 3x^2 - 12x + 13 = (x - 1)(2x^2 - x - 13)$. Zeros: 1 and $\frac{1 \pm \sqrt{105}}{4}$.
22. $F'(x) = 4ax^3 - 2bx^{-3}$, so $a = 2$, $b = -3$. Then $F(1) = 2 - 3 + 7 = 6$, so it cannot satisfy $F(1) = 10$.
23. Intercepts: $(-2, 0)$ and $(2, 0)$. $A = \int_{-2}^2 (12 - 3x^2) dx = 32$ square units.
24. $\int_0^k (2x^3 + 1) dx = 10$. GDC solution $k = 2.00$ (in fact, exactly 2).
25. Expand to $x^2 - 2 + x^{-2}$. $F(x) = x^3/3 - 2x - x^{-1} + C$; $C = 8/3$. Its derivative is $x^2 - 2 + x^{-2} = (x^2 - 1)^2/x^2$.
26. $Q(t) = 2t^3 - 2t^2 + 5t + C$, $C = 7$. $Q(3) = 58$, so the increase is 46.
27. $F = x^3 - 2x^2 + x + 5$. For G , $1 = 8 - 8 + 2 + C_G$ gives $C_G = -1$. Thus $F - G = 6$; their common nonconstant terms cancel.
28. $A = \int_2^3 (x^3 - 4x) dx = 6.25$ square units. On $(2, 3]$, $x^3 - 4x = x(x - 2)(x + 2) > 0$.
29. $\int_0^2 a(x^2 + 1) dx = 21$. GDC gives $a = 4.5$ (exactly $9/2$).
30. $F(x) = \frac{a}{3}x^3 + bx + 2$, so $F(-1) + F(1) = 4$. The table gives $1 + 5 = 6$, a contradiction. No such function exists.



Section 3: IB exam-style practice

31. (a) $f(x) = 3x^5 - 2x^3 + 2x + C$; $9 = 3 + C$, so $C = 6$. (b) $f(-1) = -3 + 2 - 2 + 6 = 3$.
32. (a) $f(2) = 22 - 9 = 13$. (b) $f(x) = 2x^3 - 4x^2 + 3x + C$; $13 = 6 + C$, hence $f(x) = 2x^3 - 4x^2 + 3x + 7$.
33. (a) $f(x) = x^2 - 6/x + 2/x^2 + C$; $C = 3$. (b) $f(2) = 4 - 3 + 1/2 + 3 = 9/2$.
34. (a) $anx^{n-1} = 12x^{-5}$ gives $n = -4$ and $a = -3$. (b) $C = 7$, so $F(2) = 7 - 3/16 = 109/16$.
35. (a) $f'(x) = 4x - 3$. (b) $f(x) = 2x^2 - 3x + C$; $C = 5$. (c) Translation by $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$.
36. (a) $x = (1 \pm \sqrt{41})/2$. (b) $A = \int_{(1-\sqrt{41})/2}^{(1+\sqrt{41})/2} (10 + x - x^2) dx$. (c) $A = 43.8$ square units.
37. (a) $(0, 0)$, $(6, 0)$; $A = \int_0^6 (6x - x^2) dx$. (b) $A = 36$. (c) The base is 6 and perpendicular height is 12, independently of a , so its area is $\frac{1}{2}(6)(12) = 36$.
38. (a) $x^3/3 - 4/x + C$. (b) $A = \int_1^3 (x^2 + 4x^{-2}) dx = 11.3$ square units (3 s.f.). (c) $x^2 + 4/x^2 > 0$ for $x > 0$.
39. (a) $\int_0^b (x^4 + 1) dx = 7$. (b) $b = 1.91$ (3 s.f.). (c) $h = 7/b = 3.66$ units (3 s.f.), using unrounded b .
40. (a) $a/3 + b = 5$ from the points; $a + b = 9$ from the gradient. (b) $a = 6$, $b = 3$; $f(x) = 2x^3 + 3x + 3$.
41. (a) $F' = 4x^3 - 4x$. (b) For $F = x^4 - 2x^2 + C$, $7 = 16 - 8 + C$ gives $C = -1$. (c) Every constant differentiates to zero.
42. (a) $A = \int_0^2 (3x^2 + 2) dx = 12$. (b) $\int_0^k (3x^2 + 2) dx = 6$. (c) $k = 1.46$ (3 s.f.).
43. (a) All three factors are positive. (b) $A = \int_1^4 x(x-1)(4-x) dx = 11.25$. (c) Expand to $-x^3 + 5x^2 - 4x$: antiderivative $= -x^4/4 + 5x^3/3 - 2x^2 + C$.
44. (a) $V(t) = 2t^3 + 2t^2 + 3t + 20$. (b) $V(3) = 101 \text{ m}^3$. (c) $V(1) = 27$, so increase $= 74 \text{ m}^3$.
45. (a) $b = 4$. (b) $f = x^3 + (a/2)x^2 + 4x + 2$; $5 = 7 + a/2$, so $a = -4$ and $f = x^3 - 2x^2 + 4x + 2$. (c) $f'(2) = 12 - 8 + 4 = 8$.
46. (a) $x^2 + 2 - 6x^{-3}$. (b) $F = x^3/3 + 2x + 3x^{-2} + C$; $C = -1/3$. (c) $F(2) = 8/3 + 4 + 3/4 - 1/3 = 85/12$.
47. (a) Both have form $2x^3 + x^2 - 3x + C$. For F , $C = 2$; for G , $4 = 2 + C$, so $C = 2$. (b) $F = G$; the graphs coincide, so every point is common to both.
48. (a) $A = \int_0^3 (1.5 + 0.2x^3) dx = 8.55 \text{ m}^2$. (b) $V = 5A = 42.75 \text{ m}^3$. (c) Height $= 8.55/3 = 2.85 \text{ m}$.
49. (a) $x^5/5 + x^3/3 + x + C$. (b) $A = \int_0^2 f(x) dx = 11.1$ square units (3 s.f.). (c) Solve $\int_0^b f(x) dx = 5$: $b = 1.59$ (3 s.f.).
50. (a) $f(x) = 6x - 2x^3 + 4$. (b) $2(2-x)(x^2 + 2x + 1) = -2x^3 + 6x + 4$. (c) $(-1, 0)$ and $(2, 0)$. (d) $A = \int_{-1}^2 (6x - 2x^3 + 4) dx = 13.5$ square units; the factored form is nonnegative throughout.

Section 4: AA HL exclusive challenge

51. Integrate: $f(x) = ax^3 + bx^2 + cx + d$, with $d = 2$. The points give $a + b + c = 3$ and $-a + b - c = -1$. Hence $b = 1$ and $a + c = 2$. The gradient gives $3a + 2 + c = 8$, so $a = 2$, $c = 0$. Therefore $f(x) = 2x^3 + x^2 + 2$.

52. (a) $F' = 1 - 2x^{-2} + x^{-4}$, so $F = x + 2x^{-1} - \frac{1}{3}x^{-3} + C$. $C = -8/3$; $F(2) = 2 + 1 - 1/24 - 8/3 = 7/24$. (b) The numerator is a square and the denominator is positive, so $F' \geq 0$; it vanishes only at $x = 1$.

53. $f(x) = 4x^3 - 6kx^2 + C$ and $C = 0$. Thus $f(k) = -2k^3 = -4$, so $k = \sqrt[3]{2}$. Then $f(2k) = 32k^3 - 24k^3 = 8k^3 = 16$.

54. $f = x^3 + ax^2 + bx + C$. Adding and subtracting the equations at ± 1 gives $C = -a$ and $b = -1$. Hence $f = (x^2 - 1)(x + a)$. Since $f(2) = 3(2 + a) = 15$, $a = 3$. Thus $b = -1$ and $f = (x - 1)(x + 1)(x + 3)$.

55. (a) Tangency requires $f'(2) = 0$, so $32 - 8a = 0$ and $a = 4$. Integration gives $f = x^4 - 2ax^2 + b$; $f(2) = 0$ gives $16 - 32 + b = 0$, so $b = 16$. (b) $f = (x^2 - 4)^2 = (x - 2)^2(x + 2)^2$; other intercept $(-2, 0)$. (c) A square is nonnegative.

56. $F = -a/x - b/(3x^3) + C$. From $F(1) = 0$, $C = a + b/3$. Then $F(2) = 1$ gives $a/2 + 7b/24 = 1$, or $12a + 7b = 24$. Also $a + b = 3$. Subtracting $7(a + b) = 21$ gives $a = 3/5$; $b = 12/5$, $C = 7/5$. Thus $F = \frac{7}{5} - \frac{3}{5x} - \frac{4}{5x^3}$.

57. $F = x^4/4 - ax^2/2 + C$. Subtracting the conditions gives $15/4 - 3a/2 = 0$, so $a = 5/2$. Then $C = 1$ and $F = \frac{1}{4}(x^4 - 5x^2 + 4) = \frac{1}{4}(x^2 - 1)(x^2 - 4)$. Zeros: $-2, -1, 1, 2$.

58. $f(x) = \frac{k}{n+1}x^{n+1} + C$, with $C = 0$. From $f(1) = 3$, $k/(n+1) = 3$. Hence $f(2) = 3 \cdot 2^{n+1} = 96$, so $2^{n+1} = 32$, $n = 4$ and $k = 15$. Thus $f(3) = 3 \cdot 3^5 = 729$.

59. (a) $\int_0^4 kx(4-x) dx = 32$; $k = 3$. (b) Solve $\int_0^c 3x(4-x) dx = 8$: $c = 1.31$ (3 s.f.). (c) The equation $k \int_0^c x(4-x) dx = \frac{k}{4} \int_0^4 x(4-x) dx$ has the common factor k , which cancels.

60. (a) At tangency, $f'(a) = a^2 = 3$, so $a = \pm\sqrt{3}$. (b) $f = x^3 - ax^2 + C$. At $x = a$, $f(a) = C = 3a + 1$. Thus $f = x^3 - \sqrt{3}x^2 + 1 + 3\sqrt{3}$ or $f = x^3 + \sqrt{3}x^2 + 1 - 3\sqrt{3}$. (c) $(1 + 3\sqrt{3})(1 - 3\sqrt{3}) = 1 - 27 = -26$.