



Calculus Practice Worksheet

IB Mathematics: Analysis and Approaches SL & HL

Derivative rules, second derivative, extrema and optimisation - 50 questions

Learning goal	Practise differentiating standard functions, using the chain, product and quotient rules, analysing first and second derivatives, and solving local maximum/minimum and optimisation problems.
Syllabus link	IB Mathematics AA SL/HL: SL 5.6, SL 5.7 and SL 5.8 . These are shared SL content and are suitable for both AA SL and AA HL students.
Structure	Section 1: 15 short-response questions. Section 2: 15 mixed/extended-response questions. Section 3: 20 original IB-style questions.
Calculator / technology guidance	Primarily non-calculator practice. Give exact answers unless a decimal approximation is explicitly requested. Use technology only when a question specifically calls for it.

Instructions

Show clear mathematical working. Angles are in radians. When classifying stationary points, justify the classification using a valid derivative test. In optimisation questions, state the relevant model or constraint, check the feasible domain and interpret the final answer in context.

Section 1: Short response - core fluency

Questions 1–15

1. Differentiate $f(x) = x^7 - 3x^4 + 2x - 9$. [2 marks]
2. Differentiate $y = 5\sqrt{x} - \frac{4}{x^2} + 3x^{-1/3}$, where $x > 0$. [3 marks]
3. Differentiate $f(x) = 4\sin x - 3\cos x + 2e^x$. [2 marks]
4. Differentiate $g(x) = 2\ln x - 7e^x + x^{-2}$, where $x > 0$. [2 marks]
5. Differentiate $y = e^{3x-1}$. [2 marks]
6. Differentiate $y = \sin(5x + 2)$. [2 marks]
7. Differentiate $f(x) = \ln(4x^2 + 1)$. [3 marks]



8. Differentiate $y = (x^2 + 1)^5$. [3 marks]

9. Differentiate $f(x) = x^2 \sin x$. [3 marks]

10. Differentiate $g(x) = (3x - 1)e^x$. [3 marks]

11. Differentiate $h(x) = \frac{x^2 + 1}{x - 2}$, where $x \neq 2$. [3 marks]

12. Differentiate $f(x) = \frac{\ln x}{x}$, where $x > 0$. [3 marks]

13. For $y = e^x \cos x$, find $\frac{d^2 y}{dx^2}$. [3 marks]

14. For $f(x) = x^3 - 6x^2 + 9x + 1$, find the x -coordinates of the stationary points. [3 marks]

15. For the function in Question 14, classify each stationary point as a local maximum or a local minimum. [3 marks]

Section 2: Mixed skills / extended response

Questions 16–30

16. Find and classify the stationary points of $f(x) = x^4 - 4x^2$. [4 marks]

17. The curve $y = x^3 + kx^2 - 3x$ has a stationary point when $x = 1$. Find k . [3 marks]

18. Find the interval on which $f(x) = x^3 - 3x^2 + 5$ is decreasing. [3 marks]

19. For $f(x) = \cos(2x)$, determine whether the curve is concave-up or concave-down at $x = 0$. [2 marks]

20. The area of a rectangle is modelled by $A(x) = x(12 - 2x)$, where $0 < x < 6$. Find the maximum possible area. [3 marks]



21. Find $f'(x)$ for $f(x) = e^x(2x^2 - 1)^4$. Give your answer in factorised form. [5 marks]

22. Find $f'(x)$ for $f(x) = \frac{\ln x}{x^2 + 1}$, where $x > 0$. [5 marks]

23. Let $f(x) = e^{2x} - 3 \ln x$, where $x > 0$. Find the equation of the tangent to the graph of f at $x = 1$. [5 marks]

24. Find the equation of the normal to the curve $y = \sin(2x) + x$ at $x = \frac{\pi}{4}$. [5 marks]

25. The function $f(x) = x^3 + ax^2 + bx + 4$ has a stationary point at $(1, 2)$. Find the values of a and b . [6 marks]

26. The function $f(x) = x^4 - 2kx^2 + 5$ has a local minimum point at $x = 2$. Find the value of k and the minimum value of f . [6 marks]

27. Let $h(x) = x^3 - 3px + 2$. The graph of h has a local maximum point when $x = -2$. Find p and the coordinates of this local maximum point. [6 marks]

28. Consider $f(x) = xe^{-x}$, where $x \in \mathbb{R}$. Find the coordinates of the stationary point and determine its nature. [6 marks]

29. Find the intervals on which $f(x) = x^4 - 8x^2$ is increasing and decreasing. [5 marks]

30. For $f(x) = x^4 - 4x^3$, find the coordinates of any points of inflexion. [5 marks]

Section 3: IB exam-style / Paper 1 practice

Questions 31–50

31. The derivative of a function f is given by

$$f'(x) = (x - 1)^2(x + 2).$$

Determine whether the graph of f has a local maximum point, local minimum point, or neither at each stationary value of x . Justify your answer. [5 marks]

32. A function f satisfies $f'(x) = 3x^2 + 12x - 15$ and $f(-2) = 10$. Find $f(x)$, then find and classify its stationary points. [7 marks]



33. A rectangle has one vertex at the origin, one vertex on the positive x -axis, one vertex on the positive y -axis, and its upper-right vertex on the curve $y = 10 - x^2$. Find the maximum possible area of the rectangle. [6 marks]

34. A rectangle is placed under the curve $y = 8 - 2x^2$ in the first quadrant, with sides parallel to the coordinate axes and one vertex at the origin. Find the maximum possible perimeter of the rectangle. [6 marks]

35. An open box is made from a 30 cm by 20 cm rectangular sheet by cutting out squares of side length x cm from each corner and folding up the sides. Find the value of x that maximizes the volume. [7 marks]

36. A closed cylinder has volume 500 cm^3 . Show that its surface area can be written as

$$S(r) = 2\pi r^2 + \frac{1000}{r},$$

where r is the radius. Hence find the radius that minimizes the surface area.

[7 marks]

37. The profit, in dollars, from selling x items is modelled by

$$P(x) = -0.02x^3 + 3x^2 + 120x - 500, \quad 0 \leq x \leq 120.$$

Find the value of x that maximizes the profit.

[6 marks]

38. For $f(x) = \ln x - \frac{x}{4}$, where $x > 0$, find and classify the stationary point.

[5 marks]

39. A function satisfies $f'(x) = (x - 2)e^{-x}$ and $f(0) = 3$. Find $f(x)$, then find and classify its stationary point.

[6 marks]

40. For $f(x) = x + \frac{4}{x+1}$, where $x > -1$, find and classify the stationary point.

[5 marks]

41. Let $f(x) = e^{2x} \sin x$, where $x \in \mathbb{R}$.

(a) Find $f'(x)$. [3 marks]

(b) Find $f''(x)$. [3 marks]

(c) Find the equation of the tangent to the graph of f at $x = 0$. [2 marks]

(d) State, with a reason, whether the graph is concave-up or concave-down at $x = 0$. [2 marks]

[10 marks]



42. Consider the function $f(x) = x^3 - 3x^2 - 9x + 5$, where $x \in \mathbb{R}$.

- (a) Find $f'(x)$. [2 marks]
 - (b) Find the coordinates of the stationary points. [3 marks]
 - (c) Use the second derivative to classify each stationary point. [3 marks]
 - (d) Find the intervals on which f is increasing. [3 marks]
- [11 marks]

43. The function $f(x) = x^3 + ax^2 + bx + 6$ has stationary points when $x = 1$ and $x = 3$.

- (a) Show that $a = -6$ and find b . [5 marks]
 - (b) Find the coordinates of both stationary points. [3 marks]
 - (c) Classify both stationary points. [4 marks]
- [12 marks]

44. A farmer has 80 metres of fencing and wants to make a rectangular enclosure against a straight wall. The wall forms one side of the rectangle, so only three sides need fencing. Let x be the width perpendicular to the wall and y be the length along the wall.

- (a) Write down an equation connecting x and y . [1 marks]
 - (b) Show that the area is $A(x) = 80x - 2x^2$. [3 marks]
 - (c) Find the dimensions that maximize the area. [4 marks]
 - (d) Find the maximum area. [2 marks]
- [10 marks]

45. An open box is made from a 24 cm by 16 cm rectangular sheet by cutting out equal squares of side length x cm from the four corners.

- (a) State the domain of x in this context. [1 marks]
 - (b) Show that the volume is $V(x) = x(24 - 2x)(16 - 2x)$. [2 marks]
 - (c) Find $V'(x)$. [3 marks]
 - (d) Hence find the value of x that maximizes the volume. [4 marks]
 - (e) Find the maximum volume to the nearest cubic centimetre. [2 marks]
- [12 marks]

46. A cylindrical can has no top. Its volume is fixed at 500 cm^3 . The radius is r cm and the height is h cm.

- (a) Show that $h = \frac{500}{\pi r^2}$. [2 marks]
- (b) Show that the surface area is $S(r) = \pi r^2 + \frac{1000}{r}$. [3 marks]
- (c) Find $S'(r)$. [2 marks]
- (d) Find the radius and height that minimize the surface area. [5 marks]



[12 marks]

47. The derivative of a function f is given by

$$f'(x) = 3x^2 + 12x - 15.$$

It is also given that $f(0) = 20$.

- (a) Find $f(x)$. [3 marks]
- (b) Find the coordinates of the stationary points of f . [4 marks]
- (c) Classify each stationary point. [3 marks]
- (d) Find the coordinates of the point of inflexion. [3 marks]
- (e) State the intervals on which the curve is concave-up. [1 marks]

[14 marks]

48. Consider $g(x) = \sin x + \cos x$, for $0 \leq x \leq \frac{\pi}{2}$.

- (a) Find $g'(x)$. [2 marks]
- (b) Find the stationary point in the interval. [3 marks]
- (c) Use the second derivative to show that this stationary point is a maximum. [3 marks]
- (d) Find the maximum value of $g(x)$. [2 marks]

[10 marks]

49. Let $f(x) = \frac{\ln x}{x}$, where $x > 0$.

- (a) Show that $f'(x) = \frac{1 - \ln x}{x^2}$. [3 marks]
- (b) Find the coordinates of the stationary point. [3 marks]
- (c) Show that this stationary point is a local maximum. [3 marks]
- (d) Find the equation of the tangent to the graph of f at $x = 1$. [3 marks]

[12 marks]

50. A window is made from a rectangle with a semicircle on top. The rectangle has width $2r$, height h , and the semicircle has radius r . The total outside perimeter of the window is 20 metres.

- (a) Show that $2h + (\pi + 2)r = 20$. [2 marks]
- (b) Show that the area of the window is

$$A(r) = 20r - \left(2 + \frac{\pi}{2}\right)r^2.$$

[4 marks]

- (c) Find the value of r that maximizes the area. [4 marks]
- (d) Find the corresponding value of h . [3 marks]
- (e) Find the maximum area. [2 marks]

[15 marks]

Answer key and solution outlines

Section 1: Short response - answers 1–15

1. $f'(x) = 7x^6 - 12x^3 + 2$.
2. $\frac{dy}{dx} = \frac{5}{2\sqrt{x}} + \frac{8}{x^3} - x^{-4/3}$.
3. $f'(x) = 4 \cos x + 3 \sin x + 2e^x$.
4. $g'(x) = \frac{2}{x} - 7e^x - 2x^{-3}$.
5. $\frac{dy}{dx} = 3e^{3x-1}$.
6. $\frac{dy}{dx} = 5 \cos(5x + 2)$.
7. $f'(x) = \frac{8x}{4x^2 + 1}$.
8. $\frac{dy}{dx} = 10x(x^2 + 1)^4$.
9. $f'(x) = 2x \sin x + x^2 \cos x$.
10. $g'(x) = (3x + 2)e^x$.
11. $h'(x) = \frac{x^2 - 4x - 1}{(x - 2)^2}$.
12. $f'(x) = \frac{1 - \ln x}{x^2}$.
13. $\frac{d^2y}{dx^2} = -2e^x \sin x$.
14. $f'(x) = 3(x - 1)(x - 3)$, so $x = 1, 3$.
15. $f''(x) = 6x - 12$. At $x = 1$: local maximum, $(1, 5)$. At $x = 3$: local minimum, $(3, 1)$.

Section 2: Mixed / extended response - answers 16–30

16. $f'(x) = 4x(x^2 - 2)$. Stationary points: $x = 0, \pm\sqrt{2}$. Local maximum at $(0, 0)$. Local minima at $(\pm\sqrt{2}, -4)$.
17. $f'(x) = 3x^2 + 2kx - 3$. Since $f'(1) = 0$, $k = 0$.
18. $f'(x) = 3x(x - 2)$. Decreasing on $(0, 2)$.
19. $f''(x) = -4 \cos(2x)$, so $f''(0) = -4 < 0$. Concave-down.
20. $A(x) = 12x - 2x^2$. Maximum when $x = 3$, giving $A = 18$.
21. $f'(x) = e^x(2x^2 - 1)^3(2x^2 + 16x - 1)$.
22. $f'(x) = \frac{(x^2 + 1)/x - 2x \ln x}{(x^2 + 1)^2}$.
23. $f(1) = e^2$, $f'(1) = 2e^2 - 3$. Tangent: $y - e^2 = (2e^2 - 3)(x - 1)$.
24. At $x = \pi/4$, $y = 1 + \pi/4$ and $dy/dx = 1$. Normal: $y - (1 + \pi/4) = -(x - \pi/4)$.
25. From $f(1) = 2$: $a + b = -3$. From $f'(1) = 0$: $2a + b = -3$. Hence $a = 0, b = -3$.
26. $f'(x) = 4x^3 - 4kx$. Using $f'(2) = 0$ gives $k = 4$. Minimum value: $f(2) = -11$.
27. $h'(x) = 3x^2 - 3p$. Since $h'(-2) = 0$, $p = 4$. Local maximum: $(-2, 18)$.
28. $f'(x) = e^{-x}(1 - x)$. Stationary point: $x = 1$, $y = 1/e$. Since $f''(1) < 0$, it is a local maximum.
29. $f'(x) = 4x(x - 2)(x + 2)$. Increasing on $(-2, 0) \cup (2, \infty)$; decreasing on $(-\infty, -2) \cup (0, 2)$.
30. $f''(x) = 12x(x - 2)$. Concavity changes at $x = 0$ and $x = 2$. Points of inflexion: $(0, 0)$ and $(2, -16)$.

Section 3: IB exam-style - answers 31–50

31. Stationary values: $x = -2, 1$. Since $(x - 1)^2 \geq 0$, the sign of f' changes from negative to positive at $x = -2$, so there is a local minimum. At $x = 1$, the sign does not change, so neither.
32. $f(x) = x^3 + 6x^2 - 15x - 36$. Stationary points: $(-5, 64)$, local maximum; $(1, -44)$, local minimum.
33. $A = x(10 - x^2)$. $A' = 10 - 3x^2$, so $x = \sqrt{10/3}$. Maximum area $= \frac{20}{3} \sqrt{\frac{10}{3}}$.
34. $P = 2x + 2(8 - 2x^2) = 16 + 2x - 4x^2$. Maximum when $x = \frac{1}{4}$. Maximum perimeter $= \frac{65}{4}$.
35. $V = x(30 - 2x)(20 - 2x)$. $V' = 12x^2 - 200x + 600$. Valid maximum at $x = \frac{25 - 5\sqrt{7}}{3} \approx 3.92$ cm.
36. Using $\pi r^2 h = 500$, $h = 500/(\pi r^2)$, so $S = 2\pi r^2 + 1000/r$. $S' = 4\pi r - 1000/r^2$. Minimum when $r = \sqrt[3]{250/\pi}$.
37. $P'(x) = -0.06x^2 + 6x + 120$. Valid critical point: $x = 50 + 30\sqrt{5} \approx 117.1$. This gives the maximum profit on the given interval.
38. $f'(x) = 1/x - 1/4$, so $x = 4$. Since $f''(x) = -1/x^2 < 0$, local maximum at $(4, \ln 4 - 1)$.
39. $f(x) = (1 - x)e^{-x} + 2$. Stationary point at $x = 2$, $f(2) = 2 - e^{-2}$. Since $f''(2) > 0$, local minimum.
40. $f'(x) = 1 - \frac{4}{(x+1)^2}$. Since $x > -1$, stationary point at $x = 1$, $f(1) = 3$. Since $f''(1) > 0$, local minimum.
41. $f'(x) = e^{2x}(2\sin x + \cos x)$. $f''(x) = e^{2x}(3\sin x + 4\cos x)$. At $x = 0$, $f(0) = 0$, $f'(0) = 1$, so tangent $y = x$. Since $f''(0) = 4 > 0$, concave-up.
42. $f'(x) = 3(x - 3)(x + 1)$. Stationary points: $(-1, 10)$, local maximum; $(3, -22)$, local minimum. Increasing on $(-\infty, -1) \cup (3, \infty)$.
43. $f'(x) = 3x^2 + 2ax + b$. Conditions give $3 + 2a + b = 0$ and $27 + 6a + b = 0$, so $a = -6$, $b = 9$. Stationary points: $(1, 10)$, local maximum; $(3, 6)$, local minimum.
44. $2x + y = 80$. $A = xy = x(80 - 2x) = 80x - 2x^2$. $A' = 80 - 4x$, so $x = 20$, $y = 40$. Maximum area 800 m^2 .
45. $0 < x < 8$. $V = x(24 - 2x)(16 - 2x)$, $V' = 12x^2 - 160x + 384$. Valid maximum at $x = \frac{20 - 4\sqrt{7}}{3} \approx 3.14$. Maximum volume $\approx 541 \text{ cm}^3$.
46. $h = 500/(\pi r^2)$. $S = \pi r^2 + 1000/r$, $S' = 2\pi r - 1000/r^2$. Minimum when $r = \sqrt[3]{500/\pi}$. Then $h = r$.
47. $f(x) = x^3 + 6x^2 - 15x + 20$. Stationary points: $(-5, 120)$, local maximum; $(1, 12)$, local minimum. $f'' = 6x + 12$, so point of inflexion at $(-2, 66)$. Concave-up for $x > -2$.
48. $g'(x) = \cos x - \sin x$. Stationary point at $x = \pi/4$. Since $g''(x) = -\sin x - \cos x$, $g''(\pi/4) < 0$, so maximum. Maximum value $\sqrt{2}$.
49. $f'(x) = \frac{1 - \ln x}{x^2}$. Stationary point: $(e, 1/e)$. $f''(x) = \frac{2 \ln x - 3}{x^3}$, so $f''(e) < 0$: local maximum. Tangent at $x = 1$: $y = x - 1$.
50. Perimeter: $2h + 2r + \pi r = 20$. Area: $A = 2rh + \frac{1}{2}\pi r^2 = 20r - (2 + \frac{\pi}{2})r^2$. $A' = 20 - (4 + \pi)r$, so $r = \frac{20}{\pi + 4}$, and $h = \frac{20}{\pi + 4}$. Maximum area $= \frac{200}{\pi + 4} \text{ m}^2$.