

Kinematics

IB Mathematics AA SL/HL

50 shared questions + 10 HL challenges | Straight-line motion

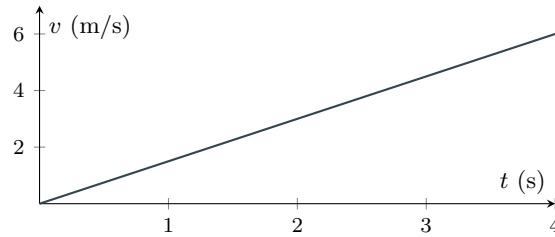
Learning goal	Connect displacement, velocity and acceleration; distinguish total distance from displacement.
Syllabus link	SL 5.9: kinematics, speed, displacement and total distance. HL challenges deepen the same shared content.
Structure	15 short + 15 extended + 20 exam-style questions + 10 HL challenges.
Technology	Non-calculator unless marked  GDC. Unmarked exam questions are Paper 1-style; marked questions are Paper 2-style.

Instructions. Give exact answers unless a GDC is specified; then use 3 significant figures. Use radians. Show integrations and constants, split distance at direction changes, and include units. SL: Q1–50. HL: also Q51–60. Work on separate paper.

1. Short response

- A particle has displacement $s(t) = t^3 - 4t^2 + 5t$ metres. Find $v(t)$ and $a(t)$.
- At an instant a particle has velocity -7 m/s and acceleration 2 m/s². State its speed and whether it is speeding up or slowing down.
- For $v(t) = 4t - 12$ m/s, find when the particle is at rest and state its direction on each side of that time, for $t \geq 0$.
- A particle has acceleration $a(t) = 6t$ m/s² and $v(0) = 4$ m/s. Find $v(t)$.
- A particle has velocity $v(t) = 2t + 1$ m/s and $s(0) = -3$ m. Find $s(t)$.
- A particle moves from $s(0) = -2$ m to $s(5) = 8$ m. Find its displacement and average velocity over these five seconds.
- A particle travels 8 m in the positive direction and then 5 m in the negative direction in 4 seconds. Find its displacement, total distance, average velocity and average speed.
- Given $v(t) = (t - 2)^2$ for $t \geq 0$, does the particle change direction at $t = 2$? Explain.
- Write integrals for the displacement and total distance of a particle between times a and b , where $a < b$ and its velocity is $v(t)$.

10. The graph shows velocity in m/s against time in seconds. Find the displacement from $t = 0$ to $t = 4$.



11. Find the speed and acceleration at $t = 1$ for $s(t) = t^3 - 6t^2 + 9t + 2$ metres.

12. The signed areas under a velocity graph are +12 m, -5 m and +3 m on successive intervals. Find the displacement and total distance.

13. A particle has constant acceleration -3 m/s^2 and initial velocity 8 m/s . Find its velocity at $t = 2 \text{ s}$.

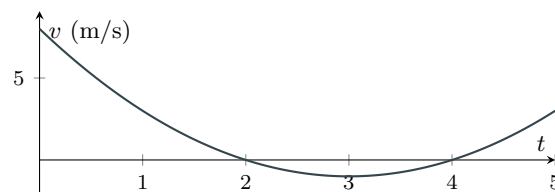
14. A differentiable displacement graph has a local maximum at time T . State its instantaneous velocity and describe its direction immediately before and after T , assuming the graph increases then decreases there.

15. For $v(t) = t^2 - 5t + 6 \text{ m/s}$, find the acceleration and all times when the particle is at rest.

2. Extended response

16. A particle has velocity $v(t) = t^2 - 6t + 8 \text{ m/s}$ for $0 \leq t \leq 5$ and $s(0) = 3 \text{ m}$.

- Find its times of rest and state its direction between them.
- Find its final position and displacement.
- Find the total distance travelled.



17. A particle has velocity $v(t) = 3t^2 - 12t + 9 \text{ m/s}$ for $0 \leq t \leq 4$ and $s(0) = -2 \text{ m}$.

- Find its times of rest and state its direction between them.
- Find its final position and displacement.
- Find the total distance travelled.

18. A particle has velocity $v(t) = 4t - 8 \text{ m/s}$ for $0 \leq t \leq 5$ and $s(0) = 1 \text{ m}$.

- Find its times of rest and state its direction between them.
- Find its final position and displacement.
- Find the total distance travelled.



19. A particle has acceleration $a(t) = 4t - 6$ m/s², $v(0) = 4$ m/s and $s(0) = 2$ m.

- (a) Find $v(t)$ and $s(t)$.
 - (b) Find the times of rest.
 - (c) Find its displacement and total distance from $t = 0$ to $t = 3$.
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20. A particle moves with $v(t) = 3 \cos t$ m/s for $0 \leq t \leq \pi$, with $s(0) = 2$ m.

- (a) Find its position function and time of direction change.
 - (b) Find its displacement and total distance.
 - (c) Find its average speed.
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21. A particle has velocity $v(t) = e^t - 2$ m/s for $0 \leq t \leq \ln 4$, with $s(0) = 0$.

- (a) Find its position and time of rest.
 - (b) Find its displacement and total distance, giving exact answers.
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22. For $v(t) = t^2 - 4t + 3$ m/s on $0 \leq t \leq 4$,

- (a) find when the speed is increasing;
 - (b) find the greatest speed on the interval;
 - (c) explain why zero acceleration alone does not imply zero velocity.
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23. A particle has displacement $s(t) = t(t - 3)^2$ metres for $0 \leq t \leq 4$.

- (a) Find all times it is at the origin and all times it is at rest.
 - (b) Describe what happens at $t = 3$.
 - (c) Find the total distance travelled.
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24. Particles P and Q move on the same straight line with $s_P = t^2$ and $s_Q = 6t - 5$, in metres, for $t \geq 0$.

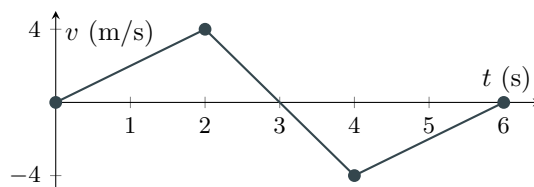
- (a) Find when they meet.
 - (b) Find their velocities at each meeting.
 - (c) Find their separation at $t = 3$.
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25. A particle has constant acceleration a , initial velocity u and initial position 0. It has $s(2) = 10$ m and $s(4) = 28$ m.

- (a) Use integration to express $s(t)$ in terms of u, a .
- (b) Find u, a and the velocity at $t = 3$.

26. The velocity graph is made of straight segments joining $(0, 0)$, $(2, 4)$, $(4, -4)$ and $(6, 0)$, with seconds and m/s on the axes.

- Find the time of direction change.
- Find the displacement and total distance over the six seconds.
- Find the acceleration on $2 < t < 4$.



27.  *GDC* A particle has velocity $v(t) = 2 + \sin(t^2)$ m/s for $0 \leq t \leq 3$ and initial position -1 m. Use radians.

- Explain why the particle never changes direction.
- Find its final position and total distance using numerical integration.

28.  *GDC* A particle has velocity $v(t) = t^3 - 2t - 1$ m/s for $0 \leq t \leq 2$. Use your GDC to find its time of direction change and its total distance.

29. A particle has $s(t) = t^3 - 6t^2 + 12t - 1$ metres, for $t \geq 0$.

- Show that it is at rest at $t = 2$ but does not change direction.
- Find its acceleration at $t = 2$.
- Find the distance travelled from 0 to 4.

30. A particle has $v(t) = 6 - 2t$ m/s for $0 \leq t \leq 5$, with initial position -4 m.

- Find its greatest position coordinate and when it occurs.
- Find all times it passes the origin.
- Find its total distance travelled.

3. IB exam-style practice

31. A particle moves on a straight line with velocity $v(t) = 3t^2 - 18t + 24$ m/s, for $0 \leq t \leq 5$. Its initial position is 2 m.

- Find its acceleration and all times of rest. [3 marks]
- Find its position function. [3 marks]
- Find its displacement and total distance during the first 5 seconds. [5 marks]

32. A particle moves on a straight line with velocity $v(t) = 6t^2 - 24t + 18$ m/s, for $0 \leq t \leq 4$. Its initial position is -3 m.

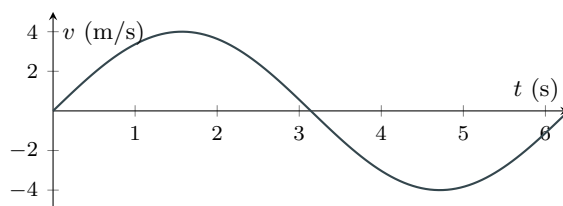
- Find its acceleration and all times of rest. [3 marks]
- Find its position function. [3 marks]
- Find its displacement and total distance during the first 4 seconds. [5 marks]

- 33.** A particle moves on a straight line with velocity $v(t) = t^2 - 8t + 12$ m/s, for $0 \leq t \leq 7$. Its initial position is 4 m.
- Find its acceleration and all times of rest. [3 marks]
 - Find its position function. [3 marks]
 - Find its displacement and total distance during the first 7 seconds. [5 marks]

- 34.** A particle has acceleration $a(t) = 12t - 12$ m/s², initial velocity 6 m/s and initial position 1 m, for $0 \leq t \leq 3$.
- Find $v(t)$ and $s(t)$. [5 marks]
 - Find all times it is at rest and state whether it changes direction. [3 marks]
 - Find its total distance travelled. [2 marks]

- 35.** For $0 \leq t \leq 4$, a particle has displacement $s(t) = t^3 - 6t^2 + 9t + 4$ metres.
- Find its times of rest and corresponding positions. [4 marks]
 - Find the intervals on which speed is increasing. [4 marks]
 - Find the greatest speed on the interval. [3 marks]

- 36.** A particle has velocity $v(t) = 4 \sin t$ m/s for $0 \leq t \leq 2\pi$, with $s(0) = 3$ m.
- Find its acceleration and position. [4 marks]
 - State the time of direction change in the interior. [2 marks]
 - Find the total distance and average speed over the interval. [4 marks]



- 37.** A particle has velocity $v(t) = 2 \cos(2t)$ m/s, for $0 \leq t \leq \pi$, with $s(0) = 0$.
- Find all interior rest times. [3 marks]
 - Find the total distance travelled. [4 marks]
 - Find all interior times when speed is greatest, and state that speed. [3 marks]

- 38.** A particle has velocity $v(t) = 3e^{-t} - 1$ m/s for $0 \leq t \leq \ln 9$, with initial position 0.
- Find its position and time of direction change. [4 marks]
 - Find its displacement. [2 marks]
 - Find its total distance. [4 marks]

- 39.** A particle has displacement $s(t) = t^3 + At^2 + Bt + 5$ metres. It is at rest at $t = 1$ and $t = 3$.
- Find A and B . [4 marks]
 - Find its acceleration at each time of rest. [2 marks]
 - Find its total distance from $t = 0$ to $t = 4$. [4 marks]

40. A particle has acceleration $a(t) = 6t - 4 \text{ m/s}^2$ and $v(1) = 0$. Its initial position is 2 m.

- Find $v(t)$ and $s(t)$. [5 marks]
- Find all times of rest for $t \geq 0$. [2 marks]
- Find its total distance from 0 to 2 seconds. [4 marks]

41. Two particles have positions $s_P = t^2 - 2t$ and $s_Q = 4t - 5$ metres, for $t \geq 0$.

- Find the two times when they meet. [3 marks]
- Find their velocities at each meeting. [3 marks]
- Find their greatest separation between the two meetings. [4 marks]

42.  GDC A particle has position $s(t) = t^3 - 4t^2 + 3t$ metres for $0 \leq t \leq 4$. Use technology.

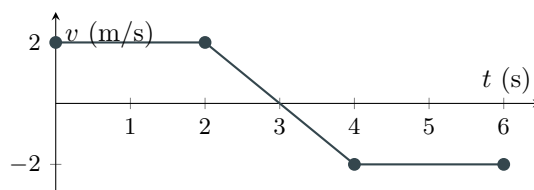
- Find all times it passes the origin. [2 marks]
- Find its interior times of rest. [3 marks]
- Find its total distance travelled. [4 marks]

43.  GDC A particle has velocity $v(t) = 2 \sin t - 0.4t$ m/s for $0 \leq t \leq 4$. Use radians and technology.

- Find the interior time at which it changes direction. [3 marks]
- Find its displacement and total distance. [4 marks]
- Find its acceleration at $t = 2$. [2 marks]

44. A velocity graph joins the points $(0, 2)$, $(2, 2)$, $(4, -2)$ and $(6, -2)$ with straight segments. Time is in seconds and velocity in m/s.

- Find when the particle reverses direction. [2 marks]
- Find the displacement and total distance over the six seconds. [4 marks]
- Find its acceleration on each open segment and explain why no single acceleration is assigned at the two corners. [3 marks]



45. A particle has $v(t) = 12 - 3t^2$ m/s, for $0 \leq t \leq 3$, with $s(0) = 0$.

- Find when it changes direction and its position then. [4 marks]
- Find its displacement, total distance and average speed over the three seconds. [5 marks]
- State when its speed is increasing. [2 marks]

46. A particle has $v(t) = (t - 1)^2(t - 4)$ m/s, for $0 \leq t \leq 5$.

- Find all rest times and identify each direction change. [4 marks]
- Find its acceleration at each rest time. [3 marks]
- Write an expression in definite integrals for its total distance. Evaluation is not required. [3 marks]

47. A particle has $s(t) = t^4/4 - 2t^3 + 4t^2$ metres for $0 \leq t \leq 4$.

- (a) Find and factorize $v(t)$. [3 marks]
- (b) Find its direction of motion on each relevant interval. [3 marks]
- (c) Find its total distance travelled. [3 marks]

48. A particle travels on a straight line with $v(t) = 6/(t+1)^2 - 1$ m/s for $0 \leq t \leq 3$, with $s(0) = 0$.

- (a) Find its position function. [3 marks]
- (b) Find its time of direction change. [2 marks]
- (c) Find its total distance travelled, giving an exact answer. [4 marks]

49. A particle starts from the origin with $v(t) = t^2 - 6t + 8$ m/s for $t \geq 0$.

- (a) Find its position function and two times of rest. [4 marks]
- (b) Show that it does not return to the origin at any positive time. [3 marks]
- (c) Find its smallest position coordinate on $1 \leq t \leq 5$. [3 marks]

50. A particle has velocity $v(t) = k - 2t$ m/s for $0 \leq t \leq 4$, with $k > 0$. Its displacement over the four seconds is 4 m.

- (a) Find k . [3 marks]
- (b) Find when it reverses direction. [2 marks]
- (c) Find the total distance and average speed over the interval. [4 marks]

4. AA HL exclusive challenge

For HL students only. These challenges deepen shared SL 5.9 through parameters and linked reasoning.

51. A particle has velocity $v(t) = t^2 - 2at$ m/s for $0 \leq t \leq 3a$, where $a > 0$.

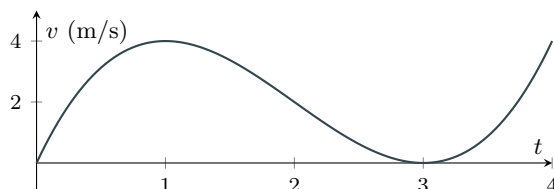
- (a) Find its displacement over this interval in terms of a . [3 marks]
- (b) Find its total distance in terms of a . [4 marks]
- (c) Its total distance is 72 m. Find a . [2 marks]

52. A particle has velocity $v(t) = (t-1)(t-a)$ m/s, for $0 \leq t \leq 3$, where $1 < a < 3$.

- (a) Show that its displacement is $9/2 - 3a/2$. [3 marks]
- (b) Derive an expression for its total distance in terms of a . [5 marks]
- (c) Find a if the total distance exceeds the displacement by $9/4$ m. [3 marks]

53. For $0 \leq t \leq 4$, a particle has $v(t) = t^3 - 6t^2 + 9t$ m/s.

- (a) Find every time at rest and determine whether each interior rest time gives a reversal. [3 marks]
- (b) Find the intervals on which speed is increasing. [4 marks]
- (c) Find its maximum speed and total distance on the interval. [5 marks]



- 54.** A particle has velocity $v(t) = e^t - k$, with $k > 1$, for $0 \leq t \leq \ln(k^2)$.
- (a) Find its time of reversal. [2 marks]
- (b) Find its displacement and total distance in terms of k . [6 marks]
- (c) The total distance is 9 m. Find k . [2 marks]
-
- 55.** Particles P and Q have positions $s_P = t^3 - 6t^2 + 9t$ and $s_Q = t$ metres, for $0 \leq t \leq 4$.
- (a) Find all times when they meet. [3 marks]
- (b) Find their greatest separation between the first and last meeting. Give an exact answer. [6 marks]
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- 56.** A particle has velocity $v(t) = A \cos t + B \sin t$ m/s on $0 \leq t \leq \pi/2$. Its initial velocity is 2 m/s and its displacement over the interval is zero.
- (a) Find A and B . [4 marks]
- (b) Find the time of reversal. [2 marks]
- (c) Find the total distance travelled. [4 marks]
-
- 57.** A particle has acceleration $a(t) = 12t - 6c$, with $c > 0$, and starts from rest at the origin.
- (a) Find $v(t)$ and $s(t)$. [4 marks]
- (b) Find its first positive return time to the origin. [3 marks]
- (c) Find the ratio of total distance to the magnitude of displacement on $0 \leq t \leq 2c$. [4 marks]
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- 58.** A particle has $v(t) = t^2 - 2t + k$ m/s for $0 \leq t \leq 2$, where $k \in \mathbb{R}$.
- (a) Determine the values of k for which it reverses direction in the interior. [4 marks]
- (b) For $0 < k < 1$, find its total distance in terms of k . [6 marks]
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- 59.**  *GDC* A particle starts at the origin and has $v(t) = 3t^2 - 12t + 9$ m/s for $t \geq 0$.
- (a) Find the earliest time at which its accumulated distance is 9 m. [6 marks]
- (b) Find its position and speed at that time. [3 marks]
-
- 60.** A particle has $v(t) = (t - 1)(t - 3)$ m/s and $s(0) = 0$, for $t \geq 0$.
- (a) Find its total distance on $[0, 3]$. [4 marks]
- (b) Find, exactly, the first time $T > 3$ at which its average velocity over $[0, T]$ is 1 m/s. [4 marks]
- (c) Find its average speed over $[0, T]$. [4 marks]

Answer key and solution outlines

Use unrounded intermediate values. Equivalent exact forms and correctly rounded answers are accepted. All exam-style questions are original practice questions, not official IB examination items.

1. Short response

- $v = s' = 3t^2 - 8t + 5$ m/s; $a = v' = 6t - 8$ m/s².
- Speed 7 m/s. It is slowing down because velocity and acceleration have opposite signs.
- At rest at $t = 3$ s. Negative direction for $0 \leq t < 3$; positive direction for $t > 3$.
- $v = 3t^2 + C$; $C = 4$, so $v = 3t^2 + 4$ m/s.
- $s = t^2 + t + C$; $C = -3$, so $s = t^2 + t - 3$ m.
- Displacement $8 - (-2) = 10$ m; average velocity $10/5 = 2$ m/s.
- Displacement 3 m; distance 13 m; average velocity $3/4$ m/s; average speed $13/4$ m/s.
- No. It is instantaneously at rest at 2, but $v > 0$ on both sides, so it continues in the positive direction.
- Displacement $= \int_a^b v(t) dt$; total distance $= \int_a^b |v(t)| dt$.
- The triangular signed area is $\frac{1}{2}(4)(6) = 12$ m.
- $v = 3t^2 - 12t + 9$, so $v(1) = 0$ and speed is 0 m/s. $a = 6t - 12$ gives $a(1) = -6$ m/s².
- Displacement $12 - 5 + 3 = 10$ m. Total distance $12 + 5 + 3 = 20$ m.
- $v(t) = 8 - 3t$; $v(2) = 2$ m/s.
- $v(T) = 0$. Motion is in the positive direction before T and in the negative direction afterwards.
- $a(t) = 2t - 5$ m/s². $v = (t - 2)(t - 3) = 0$ at $t = 2, 3$ s.

2. Extended response

16. Rest times in the interior: 2, 4 s. The velocity signs on successive intervals are positive, negative, positive. $s(t) = \frac{1}{3}t^3 - 3t^2 + 8t + 3$; $s(5) = \frac{29}{3}$ m, displacement $\frac{20}{3}$ m. Splitting $\int_0^5 |v(t)| dt$ at each rest time gives distance $\frac{28}{3}$ m.

t	0	2	4	5		
$v(t)$		+	0	-	0	+

17. Rest times in the interior: 1, 3 s. The velocity signs on successive intervals are positive, negative, positive. $s(t) = t^3 - 6t^2 + 9t - 2$; $s(4) = 2$ m, displacement 4 m. Splitting $\int_0^4 |v(t)| dt$ at each rest time gives distance 12 m.
18. Rest times in the interior: 2 s. The velocity signs on successive intervals are negative, positive. $s(t) = 2t^2 - 8t + 1$; $s(5) = 11$ m, displacement 10 m. Splitting $\int_0^5 |v(t)| dt$ at each rest time gives distance 26 m.
19. $v = 2t^2 - 6t + 4 = 2(t - 1)(t - 2)$; $s = \frac{2}{3}t^3 - 3t^2 + 4t + 2$. Rest at 1, 2 s; signs +, -, +. Displacement $s(3) - s(0) = 3$ m. Positions $s(0) = 2$, $s(1) = 11/3$, $s(2) = 10/3$, $s(3) = 5$. Distance $5/3 + 1/3 + 5/3 = 11/3$ m.
20. $s = 3 \sin t + 2$. The velocity changes $+$ $\rightarrow$ $-$ at $t = \pi/2$. Displacement 0 m; distance $3 + 3 = 6$ m; average speed $6/\pi$ m/s.
21. $s = e^t - 2t - 1$. Rest at $t = \ln 2$, with signs $- \rightarrow +$. Displacement $3 - 4 \ln 2$ m. At rest $s = 1 - 2 \ln 2$; distance $-(1 - 2 \ln 2) + (2 - 2 \ln 2) = 1$ m.
22. $v = (t - 1)(t - 3)$ and $a = 2t - 4$. Speed increases when $va > 0$: $1 < t < 2$ and $3 < t < 4$. Candidate speeds at $t = 0, 1, 2, 3, 4$ are 3, 0, 1, 0, 3 m/s; greatest 3 m/s. At $t = 2$, acceleration is zero but velocity is -1 m/s.
23. At the origin: $t = 0, 3$. $v = 3(t - 1)(t - 3)$, so rest at 1, 3. At 3 it reaches the origin and reverses from negative to positive motion. Positions at 0, 1, 3, 4 are 0, 4, 0, 4; distance $4 + 4 + 4 = 12$ m.

24. $t^2 = 6t - 5$ gives $t = 1, 5$ s. $v_P = 2t$, $v_Q = 6$: velocities (2, 6) m/s at 1 and (10, 6) m/s at 5. Separation at 3: $|9 - 13| = 4$ m.
25. $v = u + at$ and $s = ut + \frac{1}{2}at^2$. Thus $2u + 2a = 10$ and $4u + 8a = 28$. Solving: $a = 2$ m/s², $u = 3$ m/s; $v(3) = 9$ m/s.
26. Direction changes at $t = 3$. Areas: +4, +2, -2, -4 m. Displacement 0 m; total distance 12 m. Acceleration $(-4 - 4)/(4 - 2) = -4$ m/s².
27. $v \geq 1 > 0$, so motion is always in the positive direction. $\int_0^3 [2 + \sin(t^2)] dt \approx 6.77$ m is both distance and displacement. Final position ≈ 5.77 m.
28. The positive root is $t \approx 1.62$ s. Velocity is initially negative, then positive. Splitting the integral there gives distance ≈ 3.05 m.
29. $v = 3(t - 2)^2 \geq 0$, positive on both sides of 2, so there is no direction change. $a = 6t - 12$ gives $a(2) = 0$. Since motion is nonnegative, distance $s(4) - s(0) = 15 - (-1) = 16$ m.
30. $s = 6t - t^2 - 4$. Maximum position at $t = 3$, where $s = 5$ m. Origin when $t = 3 \pm \sqrt{5}$; only $3 - \sqrt{5}$ lies in $[0, 5]$. Distance $9 + 4 = 13$ m, splitting at $t = 3$.

3. IB exam-style practice

31. $a(t) = 6t - 18$ m/s². Rest at 2, 4 s. $s(t) = t^3 - 9t^2 + 24t + 2$ m. Displacement $\int_0^5 v(t) dt = 20$ m. Split at the rest times and add absolute changes in position: total distance 28 m.
32. $a(t) = 12t - 24$ m/s². Rest at 1, 3 s. $s(t) = 2t^3 - 12t^2 + 18t - 3$ m. Displacement $\int_0^4 v(t) dt = 8$ m. Split at the rest times and add absolute changes in position: total distance 24 m.
33. $a(t) = 2t - 8$ m/s². Rest at 2, 6 s. $s(t) = \frac{1}{3}t^3 - 4t^2 + 12t + 4$ m. Displacement $\int_0^7 v(t) dt = \frac{7}{3}$ m. Split at the rest times and add absolute changes in position: total distance $\frac{71}{3}$ m.
34. $v = 6t^2 - 12t + 6 = 6(t - 1)^2$; $s = 2t^3 - 6t^2 + 6t + 1$. Rest only at 1 s; velocity stays positive on either side, so no change of direction. Distance $s(3) - s(0) = 19 - 1 = 18$ m.
35. $v = 3(t - 1)(t - 3)$ and $a = 6(t - 2)$. Rest at 1, 3 s; positions 8, 4 m. Speed increases when $va > 0$: $1 < t < 2$ and $3 < t < 4$. Candidate speeds at 0, 1, 2, 3, 4 are 9, 0, 3, 0, 9 m/s; maximum speed 9 m/s at 0, 4.
36. $a = 4 \cos t$; $s = 7 - 4 \cos t$. Direction changes at $t = \pi$ (positive to negative). Displacement is 0; distance $8 + 8 = 16$ m; average speed $16/(2\pi) = 8/\pi$ m/s.
37. Rest at $t = \pi/4, 3\pi/4$. $s = \sin(2t)$ takes values 0, 1, -1, 0 at endpoints and rest times, so distance $1 + 2 + 1 = 4$ m. Greatest speed is 2 m/s; in the interior it occurs only at $t = \pi/2$ (also at the two endpoints).
38. $s = 3 - 3e^{-t} - t$. Direction changes at $\ln 3$. Displacement at $\ln 9$ is $8/3 - 2 \ln 3$ m. The turning position is $2 - \ln 3$; distance $2(2 - \ln 3) - (8/3 - 2 \ln 3) = 4/3$ m.
39. $v = 3t^2 + 2At + B = 3(t - 1)(t - 3)$, giving $A = -6$, $B = 9$. $a = 6t - 12$: $a(1) = -6$, $a(3) = 6$ m/s². Positions at 0, 1, 3, 4: 5, 9, 5, 9 m; distance $4 + 4 + 4 = 12$ m.
40. $v = 3t^2 - 4t + C$, with $C = 1$, so $v = (3t - 1)(t - 1)$. $s = t^3 - 2t^2 + t + 2$. Rest at $1/3, 1$ s; signs +, -, +. Positions $2, 58/27, 2, 4$ at 0, $1/3, 1, 2$ give distance $4/27 + 4/27 + 2 = 62/27$ m.
41. Meet when $t^2 - 6t + 5 = 0$, so $t = 1, 5$ s. $v_P = 2t - 2$, $v_Q = 4$; velocities (0, 4) and (8, 4) m/s. On $[1, 5]$, separation $s_Q - s_P = -t^2 + 6t - 5$ is greatest at $t = 3$: 4 m.
42. Origin: $t(t - 1)(t - 3) = 0$, so $t = 0, 1, 3$ s. $v = 3t^2 - 8t + 3 = 0$ gives $t = (4 \pm \sqrt{7})/3 \approx 0.451, 2.22$ s. Split $\int_0^4 |v| dt$ at both times: distance ≈ 17.5 m.
43. Besides $t = 0$, the velocity root is $r \approx 2.6$ s, with signs $+$ $\rightarrow$ $-$. Displacement $\int_0^4 v dt = 2 - 2 \cos 4 - 3.2 \approx 0.107$ m. Splitting at r gives distance ≈ 4.62 m. $a = 2 \cos t - 0.4$; $a(2) \approx -1.23$ m/s².
44. Reversal at $t = 3$. Signed areas 4, +1, -1, -4 m: displacement 0, distance 10 m. Accelerations 0, -2, 0 m/s² on (0, 2), (2, 4), (4, 6). At $t = 2, 4$ the one-sided slopes differ, so the graph has no derivative.
45. Rest at $t = 2$; $s = 12t - t^3$, so turning position 16 m. Final position 9 m: displacement 9 m, distance $16 + 7 = 23$ m, average speed $23/3$ m/s. Since $a = -6t < 0$ for $t > 0$, speed increases where $v < 0$: $2 < t < 3$.

- 46.** Rest at 1, 4 s. The velocity is negative on both sides of 1; it changes $- \rightarrow +$ only at 4. Expanding gives $v = t^3 - 6t^2 + 9t - 4$, so $a = 3t^2 - 12t + 9$; $a(1) = 0$, $a(4) = 9 \text{ m/s}^2$. Total distance $= -\int_0^4 v(t) dt + \int_4^5 v(t) dt$.
- 47.** $v = t^3 - 6t^2 + 8t = t(t-2)(t-4)$. Positive for $0 < t < 2$, negative for $2 < t < 4$. Positions $s(0) = 0$, $s(2) = 4$, $s(4) = 0$; distance 8 m.
- 48.** $s = 6 - 6/(t+1) - t$. The positive turning time is $r = \sqrt{6} - 1$; $s(r) = 7 - 2\sqrt{6}$. Final position $s(3) = 3/2$. Distance $2(7 - 2\sqrt{6}) - 3/2 = 25/2 - 4\sqrt{6} \text{ m}$.
- 49.** $s = t^3/3 - 3t^2 + 8t$; rest at 2, 4 s. For $s = 0$, either $t = 0$ or $t^2 - 9t + 24 = 0$. The quadratic has discriminant $81 - 96 = -15 < 0$, so no positive return. Candidate positions: $s(1) = 16/3$, $s(2) = 20/3$, $s(4) = 16/3$, $s(5) = 20/3$. Minimum coordinate $16/3 \text{ m}$, at $t = 1, 4$.
- 50.** $\int_0^4 (k - 2t) dt = 4k - 16 = 4$, so $k = 5$. Reversal at $t = 5/2$. Positive area $25/4 \text{ m}$; negative area magnitude $9/4 \text{ m}$. Distance $17/2 \text{ m}$; average speed $17/8 \text{ m/s}$.

4. AA HL exclusive challenge

- 51.** $v = t(t - 2a)$: negative on $(0, 2a)$ and positive on $(2a, 3a)$. With $F(t) = t^3/3 - at^2$, displacement $F(3a) - F(0) = 0$. $F(2a) = -4a^3/3$, so distance $8a^3/3$. Hence $8a^3/3 = 72$, $a^3 = 27$ and $a = 3$.
- 52.** Expanding $v = t^2 - (a+1)t + a$ and integrating gives displacement $9/2 - 3a/2$. Only $1 < t < a$ has negative velocity. Its signed integral is $-(a-1)^3/6$, so distance $D = 9/2 - 3a/2 + (a-1)^3/3$. Therefore $(a-1)^3/3 = 9/4$, giving $a = 1 + \sqrt[3]{27/4} \approx 2.89$, which satisfies the domain.
- 53.** $v = t(t-3)^2 \geq 0$: rest at 0, 3, no reversal at 3. $a = 3(t-1)(t-3)$. Speed increases for $0 < t < 1$ and $3 < t < 4$. Speeds at 0, 1, 3, 4 are 0, 4, 0, 4 m/s; maximum 4 m/s at 1, 4. Distance $\int_0^4 v dt = [t^4/4 - 2t^3 + 9t^2/2]_0^4 = 8 \text{ m}$.
- 54.** Reversal at $t = \ln k$. An antiderivative is $F = e^t - kt$. Displacement $k^2 - 1 - 2k \ln k$. Since the velocity is negative then positive, distance $F(0) + F(2 \ln k) - 2F(\ln k) = 1 + k^2 - 2k = (k-1)^2$. Thus $(k-1)^2 = 9$ and $k > 1$ gives $k = 4$.
- 55.** The difference is $d = t(t-2)(t-4)$, so meetings at 0, 2, 4 s. $d' = 3t^2 - 12t + 8 = 0$ gives $t = 2 \pm 2\sqrt{3}/3$. Write $u = t - 2$, so $d = u^3 - 4u$. At these times, $|d| = 16\sqrt{3}/9 \text{ m}$. At meetings the separation is zero, so this is the maximum.
- 56.** $v(0) = A = 2$. The displacement is $A + B = 0$, so $B = -2$. Thus $v = 2(\cos t - \sin t)$ reverses at $t = \pi/4$. With $s(0) = 0$, $s = 2 \sin t + 2 \cos t - 2$; maximum position $2\sqrt{2} - 2$, ending at zero. Distance $4\sqrt{2} - 4 \text{ m}$.
- 57.** $v = 6t^2 - 6ct = 6t(t-c)$ and $s = 2t^3 - 3ct^2 = t^2(2t-3c)$. First positive return is $t = 3c/2$. On $(0, c)$ the particle moves negatively, then positively on $(c, 2c)$. The turning position is $s(c) = -c^3$ and the final position is $s(2c) = 4c^3$. Total distance $= c^3 + 5c^3 = 6c^3$, displacement magnitude $4c^3$: ratio $3/2$.
- 58.** $v = (t-1)^2 + k - 1$. Exactly two interior sign-changing zeros occur for $0 < k < 1$, at $1 \pm \sqrt{1-k}$. For $k \leq 0$, velocity is negative in the interior; for $k > 1$, it is positive; for $k = 1$, it only touches zero. For $0 < k < 1$, set $b = \sqrt{1-k}$. Displacement $= 2k - 4/3$. The negative signed area between $1-b$ and $1+b$ is $-4b^3/3$. Distance $D = 2k - 4/3 + 8(1-k)^{3/2}/3$.

t	0	$1 - \sqrt{1 - k}$	$1 + \sqrt{1 - k}$	2		
$v(t)$		+	0	-	0	+

- 59.** $s = t^3 - 6t^2 + 9t = t(t-3)^2$. Turning times 1, 3 give positions 4, 0 and accumulated distances 4, 8 m. The requested time is after 3, when accumulated distance $= 8 + s(t)$. Solve $t(t-3)^2 = 1$ for $t > 3$: $t \approx 3.53 \text{ s}$. Position 1 m; speed $3t^2 - 12t + 9 \approx 4.04 \text{ m/s}$, using the unrounded root.
- 60.** $s = t^3/3 - 2t^2 + 3t$. Positions at 0, 1, 3: 0, $4/3$, 0, so distance on $[0, 3]$ is $8/3 \text{ m}$. Average velocity $s(T)/T = T^2/3 - 2T + 3 = 1$ gives $T = 3 \pm \sqrt{3}$; take $T = 3 + \sqrt{3}$. Since $s(T) = T$ and motion is positive after 3, distance to T is $8/3 + T$. Average speed $= 1 + 8/[3(3 + \sqrt{3})] = (21 - 4\sqrt{3})/9 \text{ m/s}$.