



# Arithmetic Sequences and Series

IB Mathematics Applications and Interpretation SL

50 questions | Terms, sums, sigma notation and applications

|                      |                                                                                                                                                                                                                                              |
|----------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Learning goal</b> | Recognise arithmetic structure, find terms and sums, and build and interpret suitable models.                                                                                                                                                |
| <b>Syllabus link</b> | SL 1.2: arithmetic sequences and series, sigma notation, applications, simple interest and approximately arithmetic data.                                                                                                                    |
| <b>Structure</b>     | 15 short + 15 extended + 20 IB-style questions.                                                                                                                                                                                              |
| <b>Technology</b>    | GDC permitted throughout. The  icon highlights technology-focused tasks. Give non-exact numerical results to 3 significant figures unless stated otherwise. |

**Instructions.** Show your method; identify the first term and common difference even when using technology. Unless stated otherwise,  $u_n$  is the  $n$ th term,  $S_n = u_1 + \dots + u_n$ , and  $n \in \mathbb{Z}^+$ . Distinguish an individual term from a cumulative total. Check neighbouring integers in threshold questions, state units, and give money to the nearest cent when needed. Work on separate paper.

## Section 1: Short response

1. For 14, 20, 26, ..., state the common difference and write down the next two terms.

2. An arithmetic sequence begins 120, 112, 104, .... Find  $u_9$ .

3. An arithmetic sequence has  $u_n = 8 + 5(n - 1)$ . State  $u_1$  and  $d$ , and find  $u_{18}$ .

4. The table represents an arithmetic sequence. Find  $p$  and  $u_8$ .

| Term $n$ | 1 | 2  | 3   | 4  |
|----------|---|----|-----|----|
| $u_n$    | 7 | 11 | $p$ | 19 |

5. Given  $u_1 = 18$  and  $d = -1.5$ , find the twelfth term.

6. Find the index of the term 73 in 9, 13, 17, ....

7. Determine whether 100 is a term of 6, 13, 20, .... Justify your answer.

8. Find the sum of the first 12 terms of the arithmetic sequence with  $u_1 = 5$  and  $d = 3$ .

9. An arithmetic series has 20 terms. Its first term is 18 and its last is 94. Find the sum.

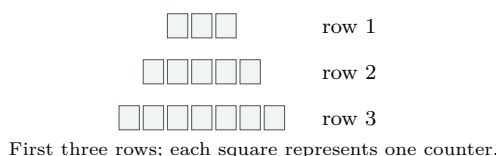
10. Evaluate  $\sum_{k=1}^8 (4k + 3)$ , identifying the first term and common difference.

11. Write  $20 + 17 + 14 + \dots + 2$  using sigma notation with lower limit  $k = 1$ .

12. A technician earns €28 400 in year 1. The annual salary increases by €950 at the start of each later year. Find the salary in year 6.
13. An account earns 3.5% simple interest per year on its initial €2400. Find the interest added each year and the balance after four complete years.
14. A learner claims that 11, 14, 18, 21 are four consecutive terms of an arithmetic sequence. Explain the error.
15. An arithmetic sequence has  $d = 5$  and  $S_8 = 244$ . Find  $u_1$ .

## Section 2: Extended response

16. An arithmetic sequence has  $u_4 = 22$  and  $u_{10} = 49$ .
- (a) Find  $d$  and  $u_1$ .
- (b) Find the sum of its first ten terms.
17. A sequence is defined by  $u_n = 40 - 3(n - 1)$ .
- (a) Find the first negative term and its index.
- (b) Find the sum of all the positive terms.
18. A display is built in rows, starting from the top. Its first three rows are shown. Each new row contains two more counters than the previous row.
- (a) Find a formula for the number of counters in row  $n$ .
- (b) Find the total number of counters in an eight-row display.



19. A charity records the following weekly donations, in euros.

| Week $n$      | 1  | 2  | 3  | 4  | 5  |
|---------------|----|----|----|----|----|
| Donations (€) | 37 | 42 | 46 | 52 | 57 |

- (a) Explain why the data are not exactly arithmetic.
- (b) Use the mean of the four successive increases to form an arithmetic model beginning at €37.
- (c) Predict the donation in week 8 and give one limitation of the prediction.

20.  *GDC* At time  $n = 0$ , €6500 is invested at 2.8% simple interest per year. Let  $B_n$  be the balance after  $n$  complete years.
- (a) Write a formula for  $B_n$ .
- (b) Find the first whole number of years after which the balance exceeds €8000.

- 21.**  *GDC* Noor saves €12 in week 1 and increases the weekly saving by €3 each week.
- (a) Write an expression for the total saved after  $n$  weeks.
- (b) Find the least number of weeks needed to save at least €500.

- 22.** Two jobs have the salary schedules below. In each job, the annual increase is fixed.

| Job           | A       | B       |
|---------------|---------|---------|
| Year-1 salary | €24 000 | €27 000 |

Job A adds €1200 each year; job B adds €750.

- (a) Find the first year in which the annual salary in job A is greater.
- (b) Compare the total earnings over the first eight complete years.

- 23.** A garden mosaic has 6 tiles in row 1 and 4 more tiles in each subsequent row. There are 10 rows. Each tile has area  $0.04 \text{ m}^2$ .
- (a) Find the number of tiles in the final row.
- (b) Calculate the total tiled area.

- 24.** An arithmetic sequence has first term 11 and  $S_{10} = 425$ .
- (a) Find its common difference.
- (b) Find  $u_{10}$  and check the sum using the first and last terms.

- 25.** A recycling centre models weekly collections with an arithmetic sequence. It collects 145 kg in week 2 and 225 kg in week 6.
- (a) Find the weekly increase and the model for week  $n$ .
- (b) Predict the total collected in the first ten weeks.

- 26.** A store of fuel is used at a constant daily rate. At the end of day 3 there are 39 litres left; at the end of day 8 there are 24 litres left.
- (a) Find the daily use and the amount initially stored.
- (b) Find the greatest number of complete days for which the fuel lasts.

- 27.** A contractor charges €180 for day 1 and €15 more for each subsequent day.
- (a) Write the charge for an eight-day project using sigma notation.
- (b) Calculate that total charge.

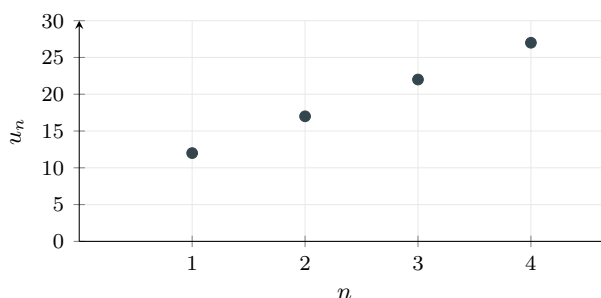
- 28.** A seedling is measured once per week.

| Week $n$    | 1  | 2  | 3  | 4  | 5  |
|-------------|----|----|----|----|----|
| Height (cm) | 24 | 28 | 33 | 37 | 42 |

- (a) Use the first and last measurements to estimate a constant weekly growth.
- (b) Predict the height in week 6. The actual height is 46 cm; state whether the model overestimates or underestimates it.
- (c) Explain why this model is unsuitable for prediction many years ahead.

**29.** The graph shows the first four terms of an arithmetic sequence.

- (a) Find  $u_1$ ,  $d$  and a formula for  $u_n$ .  
 (b) Find the first term that is at least 60.



**30.**  *GDC* A printer produces 250 booklets on day 1 and 35 more each day thereafter. An order requires at least 6000 booklets.

- (a) Write an expression for production over the first  $n$  days.  
 (b) Find the first day by whose end the order can be completed. Justify your integer choice.

### Section 3: Original IB exam-style questions

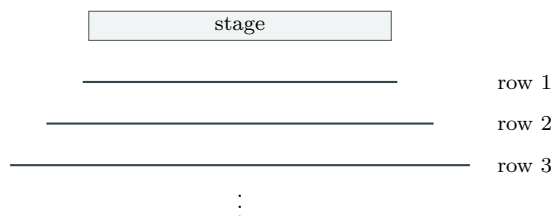
**31.** A theatre reduces the ticket price by the same amount for each row farther from the stage. The first three row prices are shown.

| Row       | 1  | 2    | 3  |
|-----------|----|------|----|
| Price (€) | 42 | 40.5 | 39 |

- (a) State the common difference. [1 mark]  
 (b) Calculate the price of one ticket in row 12. [2 marks]  
 (c) A group buys three tickets in each of rows 1 to 12. Find the total cost. [3 marks]

**32.** A small performance venue has 16 seats in its first row. Each successive row has 3 more seats. There are 18 rows.

- (a) Find the number of seats in row 18. [2 marks]  
 (b) Find the total seating capacity. [2 marks]  
 (c) Every seat is sold for €12. Find the ticket revenue. [2 marks]



Seating rows shown schematically; not to scale.

**33.** Amira cycles 12 km during week 1 of a training plan. Her total weekly distance increases by 2.5 km each week.

- (a) Find her distance in week 10. [2 marks]  
 (b) Find the total distance over the first ten weeks. [2 marks]  
 (c) Find the first week in which her weekly distance is at least 40 km. [3 marks]

**34.**  *GDC* A community fund starts with €500. At the end of year 1 it receives a €350 grant. Each later annual grant is €80 larger. No money is spent and no interest is earned.

- (a) Calculate the grant in year 8. [2 marks]
- (b) Find the total of the first eight grants. [2 marks]
- (c) Find the first year-end at which the fund contains at least €10 000. [4 marks]

**35.**  *GDC* Lina invests €12 500 at 4.2% simple interest per year. Interest is added at the end of each complete year, always calculated on the original investment.

- (a) Find the annual interest. [2 marks]
- (b) Find the balance after seven complete years. [2 marks]
- (c) Find the least number of complete years needed for the balance to exceed €20 000. [3 marks]

**36.** A solar-powered pump transfers 900 litres on day 1, then 45 litres less each day. It stops when the model first gives zero daily transfer. The receiving tank is initially empty and has capacity 10 000 litres.

- (a) Write a formula for the daily transfer  $u_n$ . [2 marks]
- (b) Find the number of days on which a positive volume is transferred. [2 marks]
- (c) Determine whether this pump fills the tank. State the remaining capacity when it stops. [4 marks]

**37.** A club uses an arithmetic model for the number of memberships sold each month. It sells 125 in month 3 and 209 in month 9.

- (a) Find the common difference and the number sold in month 1. [3 marks]
- (b) Find the total sold in the first twelve months. [2 marks]
- (c) Find the mean monthly sales over those twelve months. [2 marks]

**38.**  *GDC* The total amount saved after  $n$  weeks is modelled by  $S_n = 2.5n^2 + 17.5n$  euros, with  $S_0 = 0$ .

- (a) Find  $S_5$  and  $S_6$ . [2 marks]
- (b) Hence find the deposit made in week 6. [2 marks]
- (c) Find the first week by whose end the savings reach at least €1000. [3 marks]

**39.** A literacy project distributes 75 notebooks in month 1 and 18 more in each subsequent month.

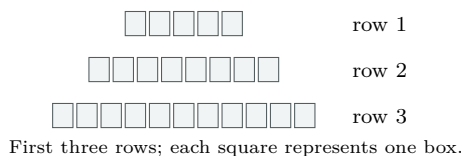
- (a) Find the first month in which at least 200 notebooks are distributed. [3 marks]
- (b) Find the total distributed from month 4 to month 10 inclusive. [3 marks]
- (c) Write the total in part (b) using sigma notation. [2 marks]

40.  *GDC* A shop builds a display with 5 boxes in its top row, 8 in its next row and 11 in its third row. The pattern continues.

(a) Find the number of boxes needed for a twelve-row display. [3 marks]

(b) The shop has 300 boxes. Find the greatest number of complete rows it can build. [3 marks]

(c) Find how many boxes are left over after building this largest display. [1 mark]



41. The table shows selected terms of an arithmetic sequence.

| $n$   | 1  | 4  | 7  |
|-------|----|----|----|
| $u_n$ | 18 | 42 | 66 |

(a) Find  $d$ . [2 marks]

(b) Find  $u_{10}$  and  $S_{10}$ . [3 marks]

(c) Determine whether 105 occurs in the sequence. [2 marks]

42.  *GDC* A visitor centre records the following daily visitor numbers.

| Day $n$  | 1   | 2   | 3   | 4   | 5   |
|----------|-----|-----|-----|-----|-----|
| Visitors | 180 | 201 | 223 | 242 | 264 |

(a) Explain why the observations are not exactly arithmetic. [1 mark]

(b) Estimate a common difference using the first and last days. Form a model  $u_n$ . [3 marks]

(c) Predict the number on day 9. [2 marks]

(d) The centre can admit 360 visitors per day. Find the first day on which the model predicts demand above capacity, and give one reason to treat this prediction cautiously. [3 marks]

43.  *GDC* Two savings plans each begin with €8000. Plan A adds 3.6% simple interest per year. Plan B adds a fixed €310 at each year-end. Neither plan has fees.

(a) Find the annual interest in plan A. [2 marks]

(b) Compare the two balances after five years. [3 marks]

(c) Find the simple annual interest rate that would make plan A add exactly €310 per year. [2 marks]

44. The number of paving stones laid in strip  $n$  of a walkway is  $u_n = 4n + 7$ .

(a) State the first term and common difference. [2 marks]

(b) Find the number of stones in the first 30 strips. [3 marks]

(c) Find the total in strips 11 to 30 inclusive. [3 marks]

**45.** A workshop makes 20 wooden toys on day 1 and 6 more on each subsequent day. Every toy made during the first 12 days is sold for €3.75.

- (a) Find the number made on day 12. [2 marks]
- (b) Calculate the total number sold. [2 marks]
- (c) Calculate the total sales revenue. Give your answer to the nearest cent. [2 marks]

**46.** A museum starts with 2400 brochures and gives out 85 every day. Its stock after  $n$  complete days is  $B_n = 2400 - 85n$ , while this model remains valid.

- (a) State the stock after day 1. [1 mark]
- (b) Find the greatest whole number of days for which the remaining stock is at least 500. [3 marks]
- (c) Find the total number of brochures given out during the first 20 days. [2 marks]

**47.**  *GDC* A ferry operator models monthly ticket sales as an arithmetic sequence. Sales are 1280 in month 1 and 1675 in month 6. The model is used for one year.

- (a) Find the common difference. [2 marks]
- (b) Find the predicted total number of tickets sold in the first twelve months. [3 marks]
- (c) Every ticket costs €1.80. Find the predicted revenue over those twelve months. [2 marks]
- (d) State one reason why the arithmetic model might be unreliable. [1 mark]

**48.**  *GDC* Maya deposits €25 in week 1. For the next 19 weeks, she plans to increase her weekly deposit by €4 each time.

- (a) Calculate her planned total saving over the 20 weeks. [2 marks]
- (b) She changes the weekly increase to a positive integer number of euros,  $d$ , while keeping the first deposit at €25. Find the least  $d$  that gives total savings of at least €1500. [4 marks]

**49.** A studio offers a 24-month rental plan. The first monthly payment is €680 and each later monthly payment is €15 lower. Another plan charges a constant €500 per month.

- (a) Find the final payment under the decreasing plan. [2 marks]
- (b) Compare the total paid under each plan over 24 months. [3 marks]
- (c) Find the first month in which the decreasing payment is strictly below €500. [3 marks]

**50.**  *GDC* A food bank plans to prepare 80 meal boxes in week 1 and 12 more in each subsequent week.

- (a) Find the least number of weeks in which the model predicts a cumulative total of at least 2500 boxes. [3 marks]
- (b) The food bank can actually prepare at most 120 boxes per week. Find the first week in which the original plan exceeds capacity. [2 marks]
- (c) It follows the original plan until the weekly cap is reached, then prepares 120 boxes every week. Find the actual least number of weeks needed to prepare at least 2500 boxes in total. [4 marks]

## Answer key and solution outlines

Equivalent forms and correct alternative methods are accepted. The IB-style questions are original practice items, not official IB examination questions.

### Section 1: Short response

1.  $d = 20 - 14 = 6$ . The next terms are 32 and 38.
2.  $u_n = u_1 + (n - 1)d$ , with  $d = -8$ . Thus  $u_9 = 120 + 8(-8) = 56$ .
3.  $u_1 = 8$ ,  $d = 5$  and  $u_{18} = 8 + 17(5) = 93$ .
4.  $d = 11 - 7 = 4$ , so  $p = 15$  and  $u_8 = 7 + 7(4) = 35$ .
5.  $u_{12} = 18 + 11(-1.5) = 1.5$ .
6.  $73 = 9 + 4(n - 1)$ , so  $64 = 4(n - 1)$  and  $n = 17$ .
7.  $100 = 6 + 7(n - 1)$  gives  $n = 101/7$ , which is not an integer. Therefore 100 is not a term.
8.  $S_n = \frac{n}{2}[2u_1 + (n - 1)d]$ . Hence  $S_{12} = 6[10 + 11(3)] = 258$ .
9.  $S_n = \frac{n}{2}(u_1 + u_n)$ , so  $S_{20} = 10(18 + 94) = 1120$ .
10. The first term is 7,  $d = 4$ , and the last term is 35. The sum is  $\frac{8}{2}(7 + 35) = 168$ .
11. There are 7 terms; one valid expression is  $\sum_{k=1}^7 [20 - 3(k - 1)]$ .
12. There are five increases:  $28\,400 + 5(950) = \text{€}33\,150$ .
13. Annual interest  $I = Pr = 2400(0.035) = \text{€}84$ . Balance  $= 2400 + 4(84) = \text{€}2736$ .
14. The successive differences are 3, 4, 3. They are not constant, so the listed terms are not arithmetic.
15.  $244 = \frac{8}{2}[2u_1 + 7(5)]$ . Thus  $2u_1 + 35 = 61$ , giving  $u_1 = 13$ .

### Section 2: Extended response

16. (a)  $u_{10} - u_4 = (10 - 4)d$ , so  $d = 27/6 = 4.5$  and  $u_1 = 22 - 3(4.5) = 8.5$ . (b)  $S_{10} = 5(8.5 + 49) = 287.5$ .
17. (a)  $40 - 3(n - 1) < 0$  gives  $n > 14\frac{1}{3}$ . Thus  $u_{15} = -2$  is first negative;  $u_{14} = 1$ . (b) The 14 positive terms sum to  $\frac{14}{2}(40 + 1) = 287$ .
18. (a)  $u_1 = 3$ ,  $d = 2$ , so  $u_n = 3 + 2(n - 1) = 2n + 1$ . (b)  $S_8 = \frac{8}{2}[6 + 7(2)] = 80$  counters.
19. (a) Differences are 5, 4, 6, 5, not constant. (b) Their mean is  $20/4 = 5$ , giving  $u_n = 37 + 5(n - 1)$ . (c)  $u_8 = 72$ : €72. Donations can fluctuate; a constant weekly increase need not continue.
20. (a) Annual interest is  $6500(0.028) = 182$ , so  $B_n = 6500 + 182n$ . (b)  $n > 1500/182 = 8.241\dots$ . Therefore  $n = 9$ ;  $B_8 = 7956$  and  $B_9 = 8138$ .
21. (a)  $S_n = \frac{n}{2}[24 + 3(n - 1)] = 1.5n^2 + 10.5n$ . (b)  $S_{15} = 495 < 500$  and  $S_{16} = 552 \geq 500$ . Since all weekly deposits are positive, the least number is 16.
22. (a)  $24\,000 + 1200(n - 1) > 27\,000 + 750(n - 1)$  gives  $n > 7\frac{2}{3}$ , so year 8. (b)  $S_8^A = 4[48\,000 + 7(1200)] = 225\,600$  and  $S_8^B = 4[54\,000 + 7(750)] = 237\,000$ . Job B pays €11 400 more in total.
23. (a)  $u_{10} = 6 + 9(4) = 42$ . (b)  $S_{10} = 5(6 + 42) = 240$  tiles. Area  $= 240(0.04) = 9.60 \text{ m}^2$ .
24. (a)  $425 = 5[22 + 9d]$ , so  $d = 7$ . (b)  $u_{10} = 11 + 9(7) = 74$  and  $5(11 + 74) = 425$ .
25. (a)  $d = (225 - 145)/(6 - 2) = 20 \text{ kg}$ ;  $u_1 = 125$ , so  $u_n = 125 + 20(n - 1)$ . (b)  $u_{10} = 305$  and  $S_{10} = 5(125 + 305) = 2150 \text{ kg}$ .
26. (a) Daily use  $= (39 - 24)/5 = 3$  litres. Initial amount  $= 39 + 3(3) = 48$  litres. (b)  $48/3 = 16$  complete days. The end-of-day model is  $B_n = 48 - 3n$ , valid until the fuel runs out.
27. (a)  $\sum_{k=1}^8 [180 + 15(k - 1)]$ . (b)  $S_8 = \frac{8}{2}[360 + 7(15)] = \text{€}1860$ .

- 28.** (a)  $d = (42 - 24)/4 = 4.5$  cm/week. (b)  $u_6 = 24 + 5(4.5) = 46.5$  cm, an overestimate of 0.5 cm. (c) Growth will not remain constant indefinitely; a plant has biological size and growth limits.
- 29.** (a)  $u_1 = 12$ ,  $d = 5$ , so  $u_n = 12 + 5(n - 1)$ . (b)  $12 + 5(n - 1) \geq 60$  gives  $n \geq 10.6$ . Thus  $u_{11} = 62$  is first;  $u_{10} = 57$ .
- 30.** (a)  $S_n = \frac{n}{2}[500 + 35(n - 1)]$ . (b)  $S_{13} = 5980 < 6000$  and  $S_{14} = 6685 \geq 6000$ , so day 14. Daily production is positive throughout this period.

### Section 3: Original IB exam-style questions

- 31.** (a)  $d = -1.50$ . (b)  $u_{12} = 42 + 11(-1.5) = €25.50$ . (c) One ticket in each row costs  $S_{12} = 6(42 + 25.5) = 405$ . The group pays  $3(405) = €1215$ .
- 32.** (a)  $u_{18} = 16 + 17(3) = 67$ . (b)  $S_{18} = 9(16 + 67) = 747$  seats. (c) Revenue =  $747(12) = €8964$ .
- 33.** (a)  $u_{10} = 12 + 9(2.5) = 34.5$  km. (b)  $S_{10} = 5(12 + 34.5) = 232.5$  km. (c)  $12 + 2.5(n - 1) \geq 40$  gives  $n \geq 12.2$ , so week 13;  $u_{12} = 39.5$  and  $u_{13} = 42$ .
- 34.** (a)  $u_8 = 350 + 7(80) = 910$ . (b)  $S_8 = 4(350 + 910) = 5040$ . (c) Balance  $B_n = 500 + \frac{n}{2}[700 + 80(n - 1)]$ .  $B_{12} = 9980$  and  $B_{13} = 11\,290$ , so the first such year-end is 13.
- 35.** (a)  $I = Pr = 12\,500(0.042) = €525$ . (b)  $B_7 = 12\,500 + 7(525) = €16\,175$ . (c)  $12\,500 + 525n > 20\,000$  gives  $n > 14.2857\dots$ , hence 15 years.  $B_{14} = 19\,850$  and  $B_{15} = 20\,375$ .
- 36.** (a)  $u_n = 900 - 45(n - 1)$ . (b)  $u_{21} = 0$  and  $u_{20} = 45$ , so 20 days have positive transfer. (c)  $S_{20} = 10(900 + 45) = 9450$  litres. It does not fill the tank;  $10\,000 - 9450 = 550$  litres of capacity remain.
- 37.** (a)  $d = (209 - 125)/6 = 14$  and  $u_1 = 125 - 2(14) = 97$ . (b)  $S_{12} = 6[194 + 11(14)] = 2088$ . (c) Mean =  $2088/12 = 174$  memberships per month.
- 38.** (a)  $S_5 = 150$ ,  $S_6 = 195$ . (b)  $u_6 = S_6 - S_5 = €45$ . (c)  $S_{16} = 920$  and  $S_{17} = 1020$ , so week 17. The deposits  $u_n = 5n + 15$  are positive.
- 39.** (a)  $75 + 18(n - 1) \geq 200$  gives  $n \geq 7.944\dots$ , so month 8, with 201 notebooks. (b)  $S_{10} - S_3 = 1560 - 279 = 1281$ .  
(c)  $\sum_{k=4}^{10} [75 + 18(k - 1)]$ .
- 40.** (a)  $S_{12} = 6[10 + 11(3)] = 258$ . (b)  $S_{13} = 299 \leq 300$  but  $S_{14} = 343 > 300$ , so 13 complete rows. (c)  $300 - 299 = 1$  box remains.
- 41.** (a)  $d = (42 - 18)/3 = 8$ . (b)  $u_{10} = 18 + 9(8) = 90$ ;  $S_{10} = 5(18 + 90) = 540$ . (c)  $105 = 18 + 8(n - 1)$  gives  $n = 11.875$ , not an integer. It is not a term.
- 42.** (a) Differences 21, 22, 19, 22 are unequal. (b)  $d = (264 - 180)/4 = 21$ ;  $u_n = 180 + 21(n - 1)$ . (c)  $u_9 = 348$ .  
(d)  $u_9 = 348 \leq 360$ ,  $u_{10} = 369 > 360$ , so day 10. Demand may change with weather or day of the week; the trend may not continue.
- 43.** (a)  $8000(0.036) = 288$  euros. (b)  $B_5^A = 8000 + 5(288) = 9440$ ;  $B_5^B = 8000 + 5(310) = 9550$ . Plan B has €110 more. (c)  $8000r = 310$ , so  $r = 0.03875$ , or 3.875% (about 3.88%).
- 44.** (a)  $u_1 = 11$ ,  $d = 4$ . (b)  $u_{30} = 127$  and  $S_{30} = 15(11 + 127) = 2070$ . (c) There are 20 strips, from  $u_{11} = 51$  to  $u_{30} = 127$ , so the total is  $10(51 + 127) = 1780$ .
- 45.** (a)  $u_{12} = 20 + 11(6) = 86$ . (b)  $S_{12} = 6(20 + 86) = 636$ . (c) Revenue =  $636(3.75) = €2385.00$ .
- 46.** (a)  $B_1 = 2315$ . (b)  $2400 - 85n \geq 500$  gives  $n \leq 22.3529\dots$ , so 22 days:  $B_{22} = 530$  and  $B_{23} = 445$ . (c)  $85(20) = 1700$  brochures; summing the remaining stocks would not measure the number given out.
- 47.** (a)  $d = (1675 - 1280)/5 = 79$ . (b)  $S_{12} = 6[2560 + 11(79)] = 20\,574$  tickets. (c)  $20\,574(1.80) = €37\,033.20$ .  
(d) Seasonal demand or operating capacity may prevent a constant increase.
- 48.** (a)  $S_{20} = 10[50 + 19(4)] = €1260$ . (b)  $10(50 + 19d) \geq 1500$ , so  $d \geq 100/19 = 5.263\dots$ . The least integer is 6;  $d = 5$  gives €1450 and  $d = 6$  gives €1640.
- 49.** (a)  $u_{24} = 680 - 23(15) = €335$ . (b)  $S_{24} = 12(680 + 335) = €12\,180$ ; the constant plan costs  $24(500) = €12\,000$ , saving €180. (c)  $680 - 15(n - 1) < 500$  gives  $n > 13$ , so month 14. The payment in month 13 is exactly €500.
- 50.** (a)  $S_n = \frac{n}{2}[160 + 12(n - 1)]$ ;  $S_{15} = 2460$  and  $S_{16} = 2720$ , so 16 weeks in the uncapped model. (b)  $u_4 = 116$  and  $u_5 = 128$ , so week 5. (c) The first four weeks give  $80 + 92 + 104 + 116 = 392$ . For  $n \geq 4$ , the actual total is  $392 + 120(n - 4)$ . This reaches 2500 when  $n \geq 21.566\dots$ , so 22 weeks: totals 2432 at week 21 and 2552 at week 22.