



Geometric Sequences and Series

IB Mathematics Applications and Interpretation SL

50 questions | Terms, finite sums, sigma notation and applications

Learning goal	Recognise multiplicative patterns; calculate terms and finite sums; build and interpret geometric models.
Syllabus link	SL 1.3: geometric sequences and series, n th terms, finite sums, sigma notation and applications. Question 48 revisits SL 1.2.
Structure	15 short-response + 15 extended-response + 20 original IB exam-style questions.
Technology	GDC permitted throughout. The  icon highlights technology-focused questions.

Instructions. Show your method and identify the first term and common ratio, including when using technology. Unless stated otherwise, u_n is the n th term, $S_n = u_1 + \dots + u_n$ and $n \in \mathbb{Z}^+$. Give exact answers for pure number and algebra questions. For numerical predictions in applications, use 3 significant figures unless instructed differently. Give money to the nearest cent. Retain unrounded values until the final answer and check neighbouring integers in threshold questions. Work on separate paper.

Section 1: Short response

- For 6, 12, 24, ..., state the common ratio and find u_8 .
- A geometric sequence begins 243, 162, 108, Find its sixth term.
- A sequence has $u_n = 9(1.4)^{n-1}$. State u_1 and r , and find u_4 .
- The table shows a geometric sequence. Find p and q .

n	1	2	3	4
u_n	12	30	p	q

- Find the next two terms and the seventh term of 16, -8, 4,
- Which term of 5, 15, 45, ... equals 10 935?
- Determine whether 500 occurs in 4, 12, 36, Justify your answer.
- Find the sum of the first five terms when $u_1 = 7$ and $r = 3$.
- Find the sum of the first six terms of 120, 60, 30,
- Evaluate $\sum_{k=1}^8 4(1.5)^{k-1}$, stating the first term and common ratio.
- Write $81 + 27 + 9 + 3 + 1$ in sigma notation with lower limit $k = 1$.



12. A model multiplies a quantity by 1.075 at each step. State the percentage change. State the ratio for a decrease of 18% at each step.

13. A geometric sequence has $r = 1$ and $u_1 = 13$. Find u_{20} and S_{20} .

14. A learner says that 10, 15, 20, 25 is geometric because each term increases by 5. Explain the error.

15. A geometric sequence has $r = 2$ and $S_6 = 441$. Find its first term.

Section 2: Extended response

16. A geometric sequence has positive terms, with $u_3 = 45$ and $u_6 = 1215$.

(a) Find the common ratio and first term.

(b) Find S_8 .

17. A geometric sequence has $u_2 = 18$ and $u_4 = 162$.

(a) Find the two possible common ratios.

(b) For each ratio, find u_1 and u_3 .

18.  *GDC* A wildlife survey records 450 birds in year 1. A model predicts an 8% annual increase.

(a) Write an expression for the predicted population u_n in year n .

(b) Predict the population in year 8, to the nearest bird.

(c) Find the first year in which the predicted population exceeds 900.

19.  *GDC* At time $n = 0$, a sample contains 640 mg of a substance. After each hour, 82% of the previous amount remains.

(a) Write the mass M_n after n complete hours.

(b) Find the mass after six hours.

(c) Find the first whole number of hours at which the mass is below 100 mg.

20.  *GDC* A workshop uses 35 m of fabric in week 1, then 12% more each week.

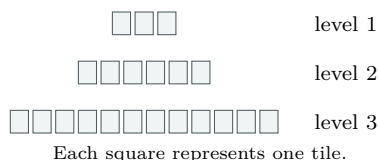
(a) Write a formula for total fabric use over the first n weeks.

(b) Find the least number of weeks for cumulative use to reach at least 800 m.

21. A tile display has 3 tiles at level 1. The number doubles at each new level.

(a) Find the number at level 8.

(b) Find the total number of tiles in an eight-level display.



- 22.**  *GDC* A job pays €28 500 in year 1. At the start of each later year, the annual salary increases by 3.5%.
- (a) Calculate the salary in year 6.
- (b) Calculate the total salary earned over the first six years.

23. Consider $\sum_{k=3}^8 6(2^{k-1})$.

- (a) State the first term, common ratio and number of terms in this sum.
- (b) Evaluate the sum.

24.  *GDC* A model for monthly demand is $u_n = 24(1.25)^{n-1}$ crates.

- (a) Write the predicted total demand from month 5 to month 12 inclusive in sigma notation.
- (b) Calculate this total to the nearest crate.

25. The volumes of a set of containers form an increasing geometric sequence. The third volume is 288 cm^3 and the sixth is 972 cm^3 .

- (a) Find the common ratio and the smallest volume.
- (b) Find the eighth volume.

26. A geometric sequence has $u_1 = 16$ and $S_2 = 28$.

- (a) Find u_2 and the common ratio.
- (b) Find the sum of the first ten terms.

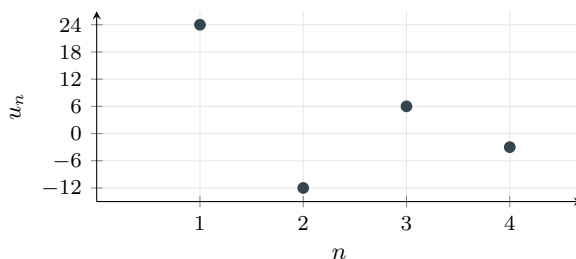
27.  *GDC* A website records the following weekly downloads.

Week n	1	2	3	4	5
Downloads	80	92	107	122	140

- (a) Explain why these data are not exactly geometric.
- (b) Use weeks 1 and 2 to choose a constant ratio and form a model.
- (c) Predict week 6 to the nearest download, and state one limitation.

28. The graph shows the first four terms of a geometric sequence.

- (a) Find u_1 , r and u_6 .
- (b) Explain why a curve joining all the plotted points would not represent the sequence itself.



- 29.**  *GDC* A venue predicts $120(1.2)^{n-1}$ visitors on day n , but can admit only 500 per day.
- Find the first day when predicted demand exceeds capacity.
 - Find the predicted total demand during days 1 to 8, to the nearest visitor.
 - Explain why summing the original model for many more days could overestimate actual admissions.

- 30.**  *GDC* A machine produces 400 items in month 1 and 700 in month 9. Assume a positive geometric model between these months.
- Find the common ratio and the monthly percentage increase.
 - Write a formula for u_n , retaining the exact ratio, and predict month 12 to the nearest item.

Section 3: Original IB exam-style questions

- 31.**  *GDC* A cycle shop forecasts sales of 65 helmets in January 2027. It models a monthly increase of 8%, with January 2027 indexed by $n = 1$.
- State the first term and common ratio of the monthly sales model. [2 marks]
 - Predict sales in December 2027, to the nearest helmet. [2 marks]
 - Predict the totals sold in 2027 and in 2028 separately, each to the nearest helmet. [4 marks]

- 32.**  *GDC* A rower completes 2.4 km in her first practice session. Each later session is 6% longer. She completes three sessions per week for eight weeks.
- Explain why the session distances form a geometric sequence and state its ratio. [2 marks]
 - Calculate the distance in her final session. [2 marks]
 - Calculate the total distance over the eight weeks. [3 marks]
 - Find the first session in which she rows at least 8 km. [3 marks]

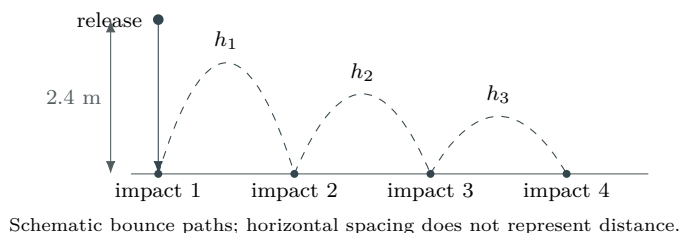
- 33.**  *GDC* A laboratory culture contains 720 bacteria at time $t = 0$. Every 15 minutes, every bacterium divides into two; none die. Let u_n be the population after n such intervals.

Interval n	0	1	2	4
Population u_n	720	1440	p	q

- Find p , q and an expression for u_n . [3 marks]
- Find the first recorded time at which the population reaches at least one million. [3 marks]
- Find the total number of individual divisions from $t = 0$ up to that time. Each dividing bacterium contributes one division. [3 marks]

- 34.** An art installation uses 5 cards in row 1, 10 in row 2 and 20 in row 3. Each subsequent row uses twice the previous number. The artist has 1000 cards.
- Find the number of cards in row 7. [2 marks]
 - Write an expression for the total needed for n rows. [2 marks]
 - Find the greatest number of complete rows and the number of unused cards. [4 marks]

- 35.**  *GDC* A ball is released from 2.4 m above the ground. After each impact, its next peak height is 72% of the previous peak height. Let h_n be the peak after impact n , with $h_0 = 2.4$ m.
- Write a formula for h_n and calculate h_6 . [3 marks]
 - Find the first impact after which the peak is below 0.12 m. [3 marks]
 - Calculate the total vertical distance from release up to and including the fifth ground impact. [4 marks]



- 36.**  *GDC* A conservation group receives a grant of €350 in year 1. The grant grows by 4% each year. It saves every grant without earning interest or spending any money.
- Calculate the grant received in year 10. [2 marks]
 - Calculate the total saved after ten grants. [3 marks]
 - Explain why adding the annual grant amounts is appropriate, but adding the successive year-end balances would overcount the money. [2 marks]

- 37.**  *GDC* A coloured tracer has initial concentration 95 units. After each complete hour, the concentration is 87% of its previous value; no tracer is added.
- State the hourly percentage decrease and write the concentration C_n after n hours. [2 marks]
 - Calculate C_{10} . [2 marks]
 - Find the least whole number of hours needed for the concentration to fall below 20 units. [3 marks]
 - State one assumption needed for the model. [1 mark]

- 38.**  *GDC* A designer uses increasing portions of resin for successive ornaments. The second portion is 18 cm^3 and the fifth is 60.75 cm^3 . The portions form a geometric sequence.
- Find the common ratio. [2 marks]
 - Find the first portion. [2 marks]
 - Exactly 591.09375 cm^3 is used. Find the number of complete portions. [3 marks]

- 39.**  *GDC* The distance travelled by a pendulum during its first recorded swing is 26 cm. Each later recorded swing covers 92% of the preceding distance.
- Calculate the distance of the eighth recorded swing. [2 marks]
 - Calculate the total distance in the first twenty recorded swings. [3 marks]
 - The recording begins after one earlier swing. Assuming the same ratio, find the distance of that unrecorded swing. [2 marks]

40.  *GDC* Two online courses model their weekly enrolments geometrically.

Course	A	B
Week-1 enrolments	80	140

Course A grows by 15% per week; course B grows by 5% per week.

- Write the two models a_n and b_n . [2 marks]
- Find the first week in which A predicts more enrolments than B. [3 marks]
- Explain why comparing the first terms alone is insufficient. [1 mark]

41.  *GDC* Two employers offer annual salaries that follow geometric sequences. Job A pays €30 000 in year 1 with annual rises of 5%. Job B pays €34 000 in year 1 with annual rises of 2.5%.

- Find the first year in which job A pays the higher annual salary. [3 marks]
- Calculate total earnings in each job over ten years. [3 marks]
- Which job gives the greater ten-year total, and by how much? [2 marks]

42.  *GDC* In 2027, a science centre records an average of 420 visitors per open day. It predicts that this average will grow by 6% each year. Its daily capacity is 650.

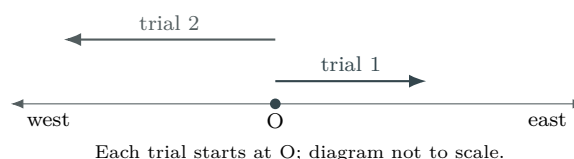
- Write the predicted daily average in year n , with 2027 corresponding to $n = 1$. [2 marks]
- Find the first calendar year in which the prediction exceeds capacity. [3 marks]
- The centre opens on 40 days each year. Predict its total visitors from 2027 to 2031 inclusive, to the nearest visitor. [3 marks]
- Give one limitation of this long-term model. [1 mark]

43.  *GDC* A filtering process starts with 250 g of a pollutant. Each cycle removes the same percentage, $p\%$, of the amount then present. After six cycles, 40% of the original amount remains.

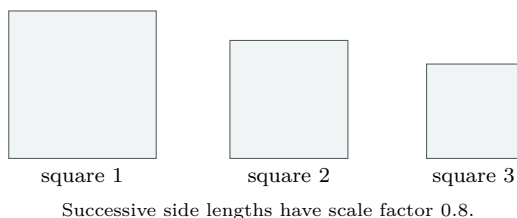
- Find the common ratio for the remaining amounts. [2 marks]
- Find p . [2 marks]
- Predict the mass remaining after three cycles. [2 marks]

44.  *GDC* Each trial of a survey robot starts at O on a straight test track. It travels to an endpoint, then is carried back to O. East is positive. Its endpoint is $x_1 = 3$ m on trial 1 and $x_3 = 5.88$ m on trial 3. Trial 2 ends west of O. The endpoint coordinates form a geometric sequence.

- Find the common ratio. [3 marks]
- Find the endpoint coordinate on trial 6. [2 marks]
- Find the total outward distance travelled under its own power during the first six trials. [3 marks]



45.  *GDC* A design consists of eight separate squares. The first has side length 6 cm. Each subsequent side length is 80% of the preceding one.
- Find the side length of square 5. [2 marks]
 - Show that the square areas form a geometric sequence with common ratio 0.64. [2 marks]
 - Find the total area of all eight squares. [3 marks]



46.  *GDC* The total distance travelled by a machine after n stages is $S_n = 90(1.2^n - 1)$ metres, with $S_0 = 0$.
- Find the distances travelled in stages 1 and 2. [3 marks]
 - Find the distance travelled in stage 7 using two consecutive totals. [2 marks]
 - Write a formula for the distance u_n in stage n . [2 marks]

47.  *GDC* A recovery process collects 75 litres of water in cycle 1. Each following cycle collects 90% of the previous amount. The process can run for at most 20 cycles.
- Write the total collected in the first n cycles using sigma notation. [2 marks]
 - Find the total collected in twenty cycles. [2 marks]
 - Find the least number of cycles needed to collect at least 600 litres. [3 marks]

48.  *GDC* Two walking plans begin at 8 km on day 1. Plan A adds 1.5 km each day. Plan B increases the preceding distance by 12% each day.
- Write a formula for the day- n distance in each plan. [2 marks]
 - Find the first day in each plan on which the distance is at least 25 km. [4 marks]
 - Which plan reaches this daily target sooner? Explain why the comparison concerns terms rather than sums. [2 marks]

49.  *GDC* A recycling project collects 90 kg in week 1. Its forecast multiplies the weekly collection by 1.3. Storage and transport actually limit collection to 300 kg per week.
- Find the first week in which the forecast exceeds the weekly limit. [3 marks]
 - The project follows the forecast until it would exceed the limit, then collects exactly 300 kg each week. Find the actual total over the first ten weeks. [3 marks]
 - Find the first week by whose end the actual cumulative collection reaches at least 2500 kg. [3 marks]

50.  *GDC* A workshop uses 200 kWh of energy in month 1. Its usage increases by 15% per month through month 6. From month 7 onward, the usage in each month is 5% less than in the preceding month.
- Calculate the usage in month 6. [2 marks]
 - Calculate the usage in month 7. [2 marks]
 - Find the total energy used in months 1 to 12 inclusive. [4 marks]
 - Explain why one geometric-series formula cannot be applied across all twelve months. [1 mark]

Answer key and solution outlines

Equivalent forms and correct alternative methods are accepted. These are original practice questions, not official IB examination questions.

Section 1: Short response

1. $r = 12/6 = 2$. Using $u_n = u_1 r^{n-1}$, $u_8 = 6(2^7) = 768$.
2. $r = 162/243 = 2/3$, so $u_6 = 243(2/3)^5 = 32$.
3. $u_1 = 9$, $r = 1.4$ and $u_4 = 9(1.4)^3 = 24.696$ (exact).
4. $r = 30/12 = 2.5$, so $p = 30(2.5) = 75$ and $q = 75(2.5) = 187.5$.
5. $r = -1/2$. The next terms are -2 and 1 ; $u_7 = 16(-1/2)^6 = 1/4$.
6. $5(3^{n-1}) = 10935$, so $3^{n-1} = 2187 = 3^7$. Thus $n = 8$.
7. $u_5 = 4(3^4) = 324$ and $u_6 = 4(3^5) = 972$. The sequence is increasing, and $324 < 500 < 972$, so 500 is not a term.
8. Using $S_n = u_1(r^n - 1)/(r - 1)$, $S_5 = 7(3^5 - 1)/(3 - 1) = 847$.
9. $r = 1/2$. Thus $S_6 = 120[1 - (1/2)^6]/(1 - 1/2) = 236.25$.
10. $u_1 = 4$ and $r = 1.5$. The sum is $4(1.5^8 - 1)/(1.5 - 1) = 197.03125$ (exact).
11. One valid expression is $\sum_{k=1}^5 81(1/3)^{k-1}$.
12. $1.075 - 1 = 0.075$, so the increase is 7.5%. A decrease of 18% gives ratio $1 - 0.18 = 0.82$.
13. Every term is 13, so $u_{20} = 13$ and $S_{20} = 20(13) = 260$. The sum formula with denominator $r - 1$ is not used when $r = 1$.
14. A geometric sequence requires a constant ratio. Here the ratios are 1.5, $4/3$ and 1.25, so they are not equal. A constant difference describes an arithmetic sequence.
15. $S_6 = u_1(2^6 - 1)/(2 - 1) = 63u_1$. Therefore $u_1 = 441/63 = 7$.

Section 2: Extended response

16. (a) $u_6/u_3 = r^3 = 1215/45 = 27$, hence $r = 3$. Since $u_3 = u_1 r^2$, $u_1 = 45/9 = 5$. (b) $S_8 = 5(3^8 - 1)/(3 - 1) = 16400$.
17. (a) $r^2 = u_4/u_2 = 9$, so $r = 3$ or $r = -3$. (b) Since $u_1 = u_2/r$ and $u_3 = u_2 r$: for $r = 3$, $u_1 = 6$ and $u_3 = 54$; for $r = -3$, $u_1 = -6$ and $u_3 = -54$.
18. (a) $u_1 = 450$, $r = 1.08$, so $u_n = 450(1.08)^{n-1}$. (b) $u_8 = 771.2209\dots$, or 771 birds. (c) A table gives $u_{10} = 899.552\dots < 900$ and $u_{11} = 971.516\dots > 900$. The first year is 11.
19. (a) $M_n = 640(0.82)^n$; the starting mass is indexed by 0. (b) $M_6 = 194.564\dots \approx 195$ mg. (c) $M_9 = 107.276\dots$ and $M_{10} = 87.9667\dots$, so 10 hours.
20. (a) $u_1 = 35$, $r = 1.12$, so $S_n = 35(1.12^n - 1)/0.12$. (b) $S_{11} = 722.910\dots$ and $S_{12} = 844.659\dots$. Since all terms are positive, the least number is 12 weeks.
21. (a) $u_n = 3(2^{n-1})$, so $u_8 = 384$. (b) $S_8 = 3(2^8 - 1)/(2 - 1) = 765$ tiles.
22. (a) $u_1 = 28500$, $r = 1.035$, so $u_6 = 28500(1.035)^5 = €33849.06$. (b) $S_6 = 28500(1.035^6 - 1)/0.035 = €186679.34$.
23. (a) First term $= 6(2^2) = 24$, ratio $r = 2$, number of terms $= 8 - 3 + 1 = 6$. (b) The sum is $24(2^6 - 1)/(2 - 1) = 1512$.
24. (a) $\sum_{k=5}^{12} 24(1.25)^{k-1}$. (b) The block begins with $u_5 = 24(1.25)^4$ and has 8 terms. Its sum is $24(1.25)^4(1.25^8 - 1)/0.25 = 1162.6088\dots$, or 1163 crates.

- 25.** (a) $r^3 = 972/288 = 27/8$, so $r = 3/2$. The first volume is $288/(3/2)^2 = 128 \text{ cm}^3$. (b) $u_8 = 128(3/2)^7 = 2187 \text{ cm}^3$.
- 26.** (a) $u_2 = S_2 - u_1 = 12$, giving $r = 12/16 = 3/4$. (b) $S_{10} = 16[1 - (3/4)^{10}]/(1 - 3/4) = 60.395935 \dots \approx 60.4$.
- 27.** (a) Ratios such as $92/80 = 1.15$ and $107/92 \approx 1.163$ differ. (b) Choose $r = 1.15$, so $u_n = 80(1.15)^{n-1}$. (c) $u_6 = 160.908575$, giving 161 downloads. Interest, advertising or competition can change the growth rate.
- 28.** (a) $u_1 = 24$, $r = -12/24 = -1/2$ and $u_6 = 24(-1/2)^5 = -3/4$. (b) The term index takes positive integer values only; intermediate points on a continuous curve are not terms.
- 29.** (a) $u_8 = 429.982 \dots < 500$ and $u_9 = 515.978 \dots > 500$, so day 9. (b) $S_8 = 120(1.2^8 - 1)/0.2 = 1979.890176$, or 1980 visitors. (c) Once demand exceeds 500, actual daily admissions are capped at 500; the geometric prediction ignores this.
- 30.** (a) $r^8 = 700/400 = 1.75$, so $r = 1.75^{1/8} = 1.07245667 \dots$. Increase $= 100(r - 1) = 7.25\%$ (3 s.f.). (b) $u_n = 400(1.75)^{(n-1)/8}$. Thus $u_{12} = 863.450 \dots$, or 863 items.

Section 3: Original IB exam-style questions

- 31.** (a) $u_1 = 65$, $r = 1.08$. (b) $u_{12} = 65(1.08)^{11} = 151.5565 \dots$, giving 152 helmets. (c) $S_{12} = 65(1.08^{12} - 1)/0.08 = 1233.5132 \dots$, giving 1234. For 2028, use $S_{24} - S_{12} = 65(1.08)^{12}(1.08^{12} - 1)/0.08 = 3106.1961 \dots$, giving 3106.
- 32.** (a) Each distance is multiplied by the fixed factor 1.06, so $r = 1.06$. (b) There are $3(8) = 24$ sessions; $u_{24} = 2.4(1.06)^{23} = 9.17 \text{ km}$ (3 s.f.). (c) $S_{24} = 2.4(1.06^{24} - 1)/0.06 = 122 \text{ km}$ (3 s.f.). (d) $u_{21} = 7.697 \dots < 8$ and $u_{22} = 8.158 \dots \geq 8$, so session 22.
- 33.** (a) $u_n = 720(2^n)$, so $p = 2880$ and $q = 11\,520$. (b) $u_{10} = 737\,280 < 10^6$ and $u_{11} = 1\,474\,560 \geq 10^6$. Thus 11 intervals, or 165 minutes. (c) Divisions in the successive intervals are 720, 1440, $\dots$, $720(2^{10})$. Their sum is $720(2^{11} - 1)/(2 - 1) = 1\,473\,840$. This also equals the population increase.
- 34.** (a) $u_7 = 5(2^6) = 320$. (b) $S_n = 5(2^n - 1)/(2 - 1) = 5(2^n - 1)$. (c) $S_7 = 635 \leq 1000$ but $S_8 = 1275 > 1000$. Therefore 7 complete rows can be made, leaving $1000 - 635 = 365$ cards.
- 35.** (a) $h_n = 2.4(0.72)^n$, so $h_6 = 0.334 \text{ m}$ (3 s.f.). (b) $h_9 = 0.124796 \dots$ and $h_{10} = 0.0898537 \dots$, so impact 10. (c) Before the fifth impact there is the initial fall and four complete up-and-down bounces. Distance $= 2.4 + 2 \sum_{k=1}^4 2.4(0.72)^k = 2.4 + 2[1.728(1 - 0.72^4)/0.28] = 11.4 \text{ m}$ (3 s.f.).
- 36.** (a) $u_1 = 350$, $r = 1.04$, so $u_{10} = 350(1.04)^9 = \text{€}498.16$. (b) $S_{10} = 350(1.04^{10} - 1)/0.04 = \text{€}4202.14$. (c) Each grant is new money, counted once. Later balances already include earlier grants, so summing balances would count the same grants repeatedly.
- 37.** (a) Decrease $= 13\%$ and $C_n = 95(0.87)^n$. (b) $C_{10} = 23.6$ units (3 s.f.). (c) $C_{11} = 20.5322 \dots$ and $C_{12} = 17.8630 \dots$, so 12 hours. (d) The fractional reduction per hour remains constant, for example because conditions remain unchanged.
- 38.** (a) $r^3 = 60.75/18 = 3.375$, so $r = 1.5$. (b) $u_1 = 18/1.5 = 12 \text{ cm}^3$. (c) $12(1.5^n - 1)/0.5 = 591.09375$ gives $1.5^n = 25.62890625 = 1.5^8$, hence $n = 8$.
- 39.** (a) $u_1 = 26$, $r = 0.92$, so $u_8 = 26(0.92)^7 = 14.5 \text{ cm}$ (3 s.f.). (b) $S_{20} = 26(1 - 0.92^{20})/0.08 = 264 \text{ cm}$ (3 s.f.). (c) If the earlier distance is x , then $0.92x = 26$, so $x = 26/0.92 = 28.3 \text{ cm}$ (3 s.f.).
- 40.** (a) $a_n = 80(1.15)^{n-1}$ and $b_n = 140(1.05)^{n-1}$. (b) A table gives $a_7 = 185.044 \dots < b_7 = 187.613 \dots$, but $a_8 = 212.801 \dots > b_8 = 196.994 \dots$. The first week is 8; $a_n/b_n = (4/7)(1.15/1.05)^{n-1}$ increases. (c) A starts lower but has the larger growth factor.
- 41.** (a) $a_n = 30\,000(1.05)^{n-1}$ and $b_n = 34\,000(1.025)^{n-1}$. Comparing successive years gives $a_6 < b_6$ but $a_7 > b_7$, so year 7; a_n/b_n increases. (b) $S_{10}^A = 30\,000(1.05^{10} - 1)/0.05 = \text{€}377\,336.78$; $S_{10}^B = 34\,000(1.025^{10} - 1)/0.025 = \text{€}380\,914.98$. (c) Using unrounded totals, B pays $\text{€}3578.20$ more.
- 42.** (a) $u_n = 420(1.06)^{n-1}$. (b) $u_8 = 631.524 \dots < 650$ and $u_9 = 669.416 \dots > 650$. Thus $n = 9$, the year $2027 + 8 = 2035$. (c) There are five years; total $= 40S_5 = 40[420(1.06^5 - 1)/0.06] = 94\,703.161728$, giving 94 703 visitors. (d) Demand may not grow by a constant percentage; admissions also cannot exceed capacity.
- 43.** (a) If the remaining fraction per cycle is r , then $250r^6 = 0.4(250)$, so $r = 0.4^{1/6} = 0.858374 \dots$. (b) $p = 100(1 - r) = 14.2\%$ (3 s.f.). (c) $M_3 = 250r^3 = 250\sqrt{0.4} = 158 \text{ g}$ (3 s.f.).

- 44.** (a) $r^2 = x_3/x_1 = 5.88/3 = 1.96$. Since trial 2 ends west, $r = -1.4$. (b) $x_6 = 3(-1.4)^5 = -16.13472 \approx -16.1$ m. (c) Distances are positive: 3, 4.2, 5.88, ..., with ratio 1.4. Total = $3(1.4^6 - 1)/0.4 = 48.97152 \approx 49.0$ m. Summing signed coordinates would not give distance.
- 45.** (a) Side length $l_5 = 6(0.8)^4 = 2.4576$ cm (exact). (b) $A_{n+1}/A_n = (0.8l_n)^2/l_n^2 = 0.8^2 = 0.64$. The first area is 36 cm^2 . (c) Total area = $36(1 - 0.64^8)/(1 - 0.64) = 97.2 \text{ cm}^2$ (3 s.f.).
- 46.** (a) $u_1 = S_1 = 90(1.2 - 1) = 18$ m. $S_2 = 39.6$, so $u_2 = S_2 - S_1 = 21.6$ m. (b) $u_7 = S_7 - S_6 = 232.486272 - 178.73856 = 53.747712$ m. (c) $u_n = S_n - S_{n-1} = 90(1.2^n - 1.2^{n-1}) = 18(1.2)^{n-1}$.
- 47.** (a) $S_n = \sum_{k=1}^n 75(0.9)^{k-1}$; first term 75, ratio 0.9. (b) $S_{20} = 75(1 - 0.9^{20})/0.1 = 659$ litres (3 s.f.). (c) $S_{15} = 595.581 \dots < 600$ and $S_{16} = 611.023 \dots \geq 600$, so 16 cycles, within the twenty-cycle limit.
- 48.** (a) $a_n = 8 + 1.5(n - 1)$ and $b_n = 8(1.12)^{n-1}$. (b) For A, $a_{12} = 24.5 < 25$ and $a_{13} = 26 \geq 25$, so day 13. For B, $b_{11} = 24.8468 \dots < 25$ and $b_{12} = 27.8284 \dots \geq 25$, so day 12. (c) B reaches the target one day sooner. The target is the distance on one day, not the cumulative distance across days.
- 49.** (a) $u_n = 90(1.3)^{n-1}$. $u_5 = 257.049 < 300$ but $u_6 = 334.1637 > 300$, so week 6. (b) The first five weeks give $S_5 = 90(1.3^5 - 1)/0.3 = 813.879$ kg. Weeks 6–10 add $5(300)$, for $2313.879 \approx 2310$ kg (3 s.f.). (c) The total for $n \geq 5$ is $813.879 + 300(n - 5)$. At $n = 10$ it is 2313.879, and at $n = 11$ it is 2613.879. Therefore week 11.
- 50.** (a) $u_6 = 200(1.15)^5 = 402$ kWh (3 s.f.). (b) $u_7 = 200(1.15)^5(0.95) = 382$ kWh (3 s.f.). (c) Split at the change in ratio, retaining full precision: first block = $200(1.15^6 - 1)/0.15 = 1750.7476875$; second block = $[200(1.15)^5(0.95)](1 - 0.95^6)/0.05 = 2024.734353 \dots$. Total = $3775.482040 \dots \approx 3780$ kWh (3 s.f.). (d) The common ratio changes from 1.15 to 0.95, so the complete twelve-term sequence is not geometric.