Straight Lines

IB Mathematics: Applications and Interpretation SL

Topic 2: Functions | Syllabus SL 2.1 | 50 questions

Learning goal	Use line equations, gradients and intercepts to interpret graphs, solve coordinate problems and build simple linear models.
Syllabus link	AI SL 2.1: $y = mx + c$, $ax + by + d = 0$, $y - y_1 = m(x - x_1)$; gradients, intercepts, parallel and perpendicular lines.
Structure	Questions 1–15: short response; 16–30: extended response; 31–50: original IB exam-style questions with marks.
Technology	A graphic display calculator (GDC) may be used throughout, as in AI SL examinations. Written setup and reasoning remain essential.

Instructions. Work on separate paper. Show the equation, substitution or reasoning that supports each answer. When using a graph or numerical solver, record the equations used and interpret the resulting coordinates. Give exact answers unless an approximation is requested; round money to the nearest cent. Label axes, intercepts and units clearly. Use the supplied grids for sketches.

Section 1: Short response

Core fluency

1.

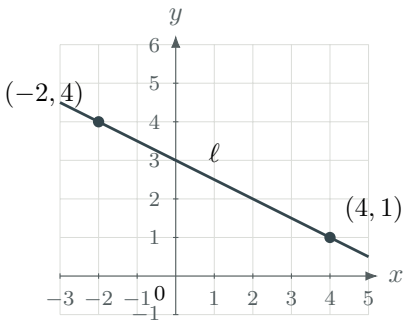
For the line $y = 2.5x + 6$, write down the gradient and the coordinates of the y-intercept.

2.

Find the gradient of the line through $A(1, -2)$ and $B(6, 8)$.

3.

The line ℓ passes through the two labelled points. Find its equation in the form $y = mx + c$.



4.

Write $4x + 5y - 20 = 0$ in the form $y = mx + c$. Hence state its gradient.

5.

A line has gradient -2 and passes through $(3, 1)$. Write its equation in point-gradient form, then in the form $y = mx + c$.

6.
Find the coordinates of both axis intercepts of $3x + 2y = 12$.

7.
State the equation and gradient of the line through each pair of points.
(a) $A(-3, 4)$ and $B(5, 4)$.
(b) $C(2, -1)$ and $D(2, 6)$.

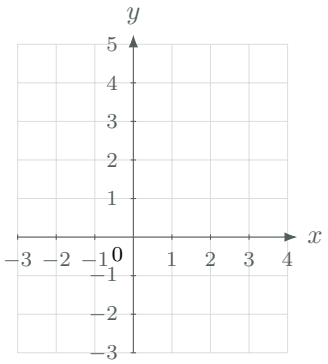
8.
The line joining $P(2, -1)$ and $Q(6, k)$ has gradient 1.5. Find k.

9.
Find the equation of the line through $(2, 5)$ parallel to $y = 0.75x - 1$.

10.
Find the equation of the line through $(0, 3)$ perpendicular to $y = -0.4x + 2$.

11.
Determine which of $P(2, 1)$ and $Q(4, -2)$ lies on $2x + y = 5$. Show your checks.

12.
On the supplied grid, sketch $y = \frac{1}{2}x + 1$ and $y = -2x + 1$.
Label each line and both of its axis intercepts.



13.
Classify each pair as *parallel and distinct*, *perpendicular*, or *coincident* (the same line).

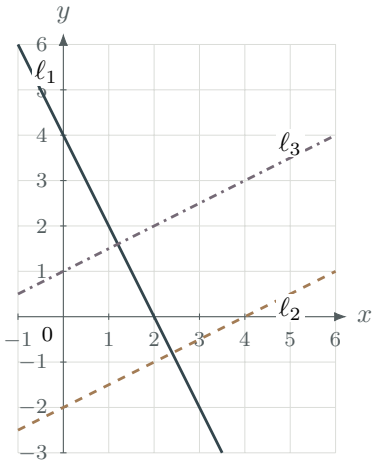
Pair	First line	Second line
(a)	$y = 3x - 2$	$6x - 2y = 10$
(b)	$y = -\frac{1}{2}x + 1$	$y = 2x + 4$
(c)	$3x + 6y = 12$	$x + 2y = 4$

14.
A delivery cost is modelled by $C = 6.50 + 0.85d$, where C is in euros and d is the distance in kilometres. Interpret the numbers 6.50 and 0.85, including units.

15.
A tank's volume is modelled by $V = 28 - 3.5t$, where V is in litres and t is in minutes. A student says, "The gradient is 28 and the vertical intercept is -3.5." Correct the statement and interpret both values.

Section 2: Extended response — Interpretation and applications

16.



The three lines are shown on an equal-scale grid.

(a) Find an equation for each line.

(b) Identify the parallel pair and justify your answer.

(c) Identify every perpendicular pair and justify your answers.

17.
Line ℓ passes through $P(1, 5)$ and $Q(5, 3)$.

(a) Find its gradient and write its equation in point-gradient form.

(b) Write the equation in general form with integer coefficients.

(c) Find the coordinates of both axis intercepts.

18.
A taxi fare consists of a fixed charge plus a constant charge per kilometre. The table gives some fares.

Distance d (km)	2	6	10
Cost C (euros)	7.20	12.00	16.80

(a) Find a model for C in terms of d.

(b) State the fixed charge and the charge per kilometre.

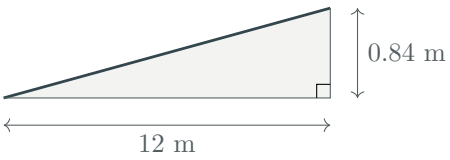
(c) Find the greatest distance that can be travelled for 24 euros under this model.

19.
A ramp rises 0.84 m over a *horizontal* distance of 12 m. For this design, the maximum permitted gradient is $\frac{1}{12}$.

(a) Find the gradient as a decimal.

(b) Determine whether this ramp meets the design limit.

(c) Another ramp must rise 0.75 m. Find its minimum horizontal run under the same limit.



Side profile; not to scale.

20.
A tank drains at a constant rate. Its volume V litres at time t minutes is recorded below.

t (min)	0	5	12
V (L)	240	200	144

(a) Find a linear model for V in terms of t.

(b) Find when 88 litres remain.

(c) Find when the tank is empty. Explain why this model should not be used to predict the volume at $t = 35$.

21.
Two bicycle-hire firms model the cost C, in euros, for $h \geq 0$ hours as

$$C_A = 14 + 2.6h, \qquad C_B = 5 + 4.1h.$$

Hourly charges use the exact hire duration.

(a) Use your GDC, or another method, to find the duration at which the costs are equal and the common cost.

(b) Which firm is cheaper for 4 hours? Find the saving.

(c) Explain which firm is cheaper for durations longer than the value found in part (a).

22.
A straight road is represented by $3x + 4y = 20$ on a coordinate map. A point C has coordinates $(2, -1)$.

(a) Write the road's equation in the form $y = mx + c$.

(b) Find the equation of a line through C parallel to the road.

(c) Find the equation of a line through C perpendicular to the road.

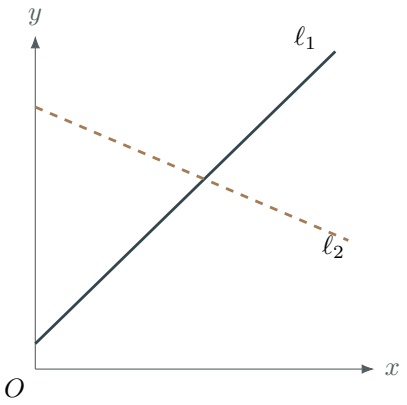
23.

The lines ℓ_1 and ℓ_2 have equations

$$y = 1.6x + 0.8, \qquad y = -0.7x + 8.25.$$

(a) Use your GDC to find their intersection. Give both coordinates to three significant figures.

(b) Determine whether $(3, 5.6)$ lies on both lines. Show a substitution check.



Schematic; scales omitted.

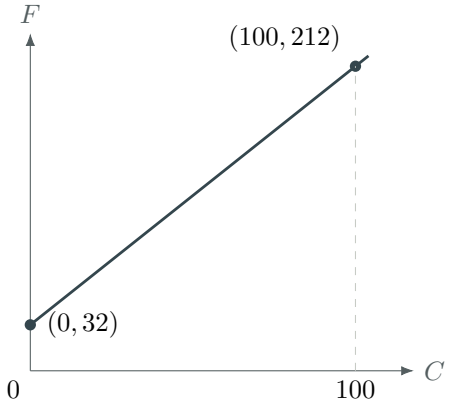
24.

Celsius temperature C and Fahrenheit temperature F are related linearly. The graph shows two known conversions.

(a) Find F in terms of C.

(b) Convert 95°F to degrees Celsius.

(c) Interpret the gradient of this conversion graph.



C in °C; F in °F.

25.

A printing service charges a fixed setup fee and a constant price per poster. Printing 20 posters costs 42 euros, while 50 posters costs 87 euros.

(a) Find a model for the total cost C of printing n posters.

(b) Interpret the gradient and the vertical intercept.

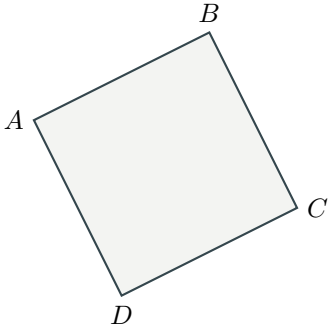
(c) Find the greatest whole number of posters that can be printed for a budget of 100 euros.

26.

A quadrilateral has vertices $A(0, 1)$, $B(4, 3)$, $C(6, -1)$ and $D(2, -3)$.

(a) Use gradients to show that it is a rectangle.

(b) Find an equation of diagonal AC.



27.

A simplified model for a vehicle's resale value is $V = 18500 - 2200t$, where V is in euros and t is the age in years.

(a) Interpret the gradient and vertical intercept.

(b) Use your GDC, or another method, to find when the predicted value is 10000 euros. Give your answer to three significant figures.

(c) Calculate the predicted value at $t = 9$. Explain why this result shows a limitation of the model.

28.

A train travels towards a station at a constant speed. Its distance D from the station is recorded below, where t is the number of minutes since timing began.

t (min)	20	50
D (km)	90	45

(a) Find a linear model for D in terms of t.

(b) Find the train's speed in kilometres per hour.

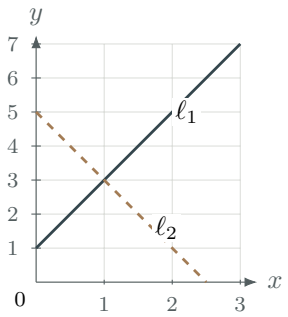
(c) Find when the train reaches the station, measured from when timing began.

29.

A student looks at the display and claims that $\ell_1 : y = 2x+1$ and $\ell_2 : y = -2x + 5$ are perpendicular.

(a) Use gradients to determine whether the claim is correct.

(b) Find the equation of the line through $(0, 5)$ that really is perpendicular to ℓ_1.



Display uses unequal axis scales.

30.

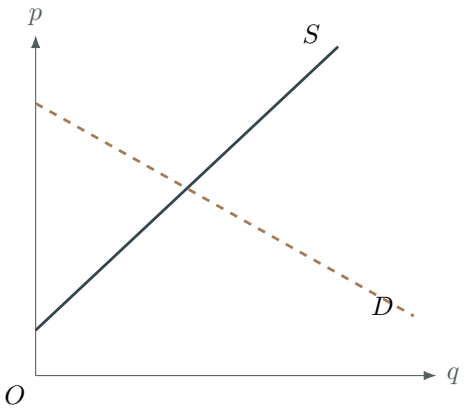
A simplified market model uses

$$p_S = 1.25q + 8, \qquad p_D = -0.75q + 48,$$

where q is quantity in tonnes and p is price in euros per tonne.

(a) Find the equilibrium quantity and price, where $p_S = p_D$.

(b) At a price of 26 euros per tonne, find the quantities supplied and demanded. Find the shortfall to three significant figures.



Supply and demand; scales omitted.

Section 3: IB exam-style practice — GDC permitted throughout

These original questions use IB-style command terms and mark allocations. Show sufficient working even when using your GDC.

31. [Maximum mark: 7]

A tank fills at a constant rate. After 2 minutes it contains 43.0 litres; after 6 minutes it contains 104.2 litres. Let V be its volume in litres at time t minutes after filling begins.

(a) Find the gradient of the volume–time graph and interpret it. [2]

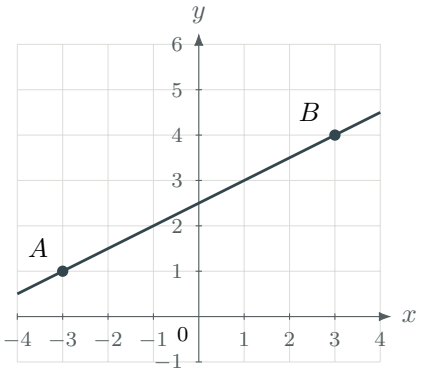
(b) Find a linear model for V in terms of t. [2]

(c) Interpret the vertical intercept of your model. [1]

(d) The tank holds 250 litres. Assuming the rate remains constant, find when it becomes full. Give your answer to 3 significant figures. [2]

32. [Maximum mark: 7]

The diagram shows a line through $A(-3, 1)$ and $B(3, 4)$.



(a) Write down the coordinates of its y-intercept. [1]

(b) Calculate its gradient. [2]

(c) Find its equation in the form $y = mx + c$. [2]

(d) Find an equation of the line through B perpendicular to AB. [2]

33. [Maximum mark: 7]

Two seedlings are studied using the following proposed linear models. Let t be time in days and H be height in centimetres. The models are used only for $0 \le t \le 40$.

Seedling	Height at $t = 0$ (cm)	Increase per day (cm/day)
A	12	0.8
B	18	0.5

(a) Write down a model for each seedling's height. [2]

(b) Find when the two predicted heights are equal and state this height. [3]

(c) According to the models, which seedling is taller after 30 days, and by how much? [2]

34. [Maximum mark: 6]

The line ℓ has equation $4x + 5y - 18 = 0$.

(a) Write the equation in the form $y = mx + c$. [2]

(b) Find the coordinates of both axis intercepts. [2]

(c) Find the equation of the line through $(-2, 1)$ parallel to ℓ. Give your answer in the form $ax + by + d = 0$ with integer coefficients. [2]

35. [Maximum mark: 7]

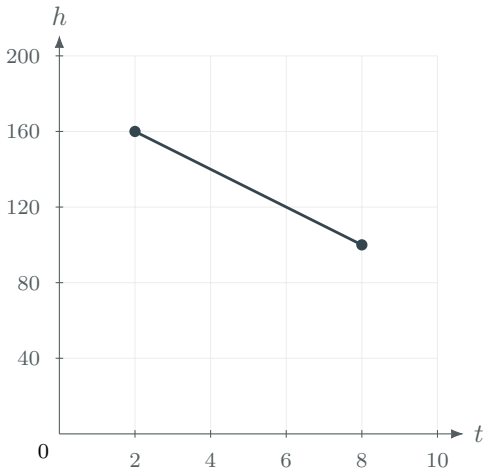
In a laboratory model, the thickness h of an ice block decreases linearly with time t. The two measurements shown are $h = 160$ mm at $t = 2$ minutes and $h = 100$ mm at $t = 8$ minutes.

(a) Calculate and interpret the gradient. [2]

(b) Find h in terms of t. [2]

(c) Assuming this rate continues, find when the thickness reaches 45 mm. [2]

(d) Explain why this model cannot describe the block at $t = 25$ minutes. [1]



t: minutes; h: millimetres.

36. [Maximum mark: 6]

Point A has coordinates $(3.5, -1.2)$, and line ℓ has equation $2x - 3y = 9$.

(a) Write the equations of the horizontal and vertical lines through A. [2]

(b) Find where each of these two lines meets ℓ. Give any non-terminating decimals to 3 significant figures. [3]

(c) State which of the lines in part (a) is perpendicular to $x = 3.5$. [1]

37. [Maximum mark: 7]

The temperatures of two liquids are modelled linearly for $0 \le t \le 20$, where t is measured in minutes. Liquid A cools from 84°C at $t = 0$ to 57°C at $t = 10$. Liquid B has temperature $T_B = 18 + 1.6t$, in degrees Celsius.

(a) Find a model for the temperature T_A of liquid A. [2]

(b) Use your GDC to find when the temperatures are equal and their common temperature. Give both answers to 3 significant figures. [3]

(c) Find the temperature difference between the liquids at $t = 18$, and state which liquid is warmer. [2]

38. [Maximum mark: 7]

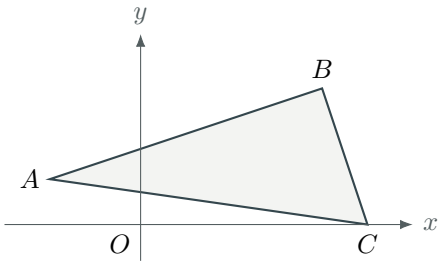
Triangle ABC has vertices $A(-2, 1)$, $B(4, 3)$ and $C(5, 0)$.

(a) Find the gradients of AB and BC. [2]

(b) Hence show that angle ABC is a right angle. [1]

(c) Find the equation of AC. [2]

(d) Find the equation of the line through B parallel to AC. [2]



39. [Maximum mark: 7]

A straight walkway is represented by the line segment joining $A(0, 0.25)$ and $B(7.5, 0.85)$. Coordinates are in metres: x is horizontal distance and y is height above a reference level. The design brief specifies a maximum gradient of 0.09.

(a) Calculate the walkway's gradient and determine whether it meets the design brief. [3]

(b) The walkway is extended along the same straight line. Find its height at $x = 12$. [2]

(c) Find the horizontal distance x at which its height would be 1.45 m. [2]

40. [Maximum mark: 6]

The lines ℓ and n have equations $3x - 2y = 4$ and $y = -0.75x + 5.2$, respectively.

(a) Find the coordinates of their intersection P. [3]

(b) Write down the gradient of ℓ. [1]

(c) Find an equation of the line through P perpendicular to ℓ. Give your answer in point-gradient form. [2]

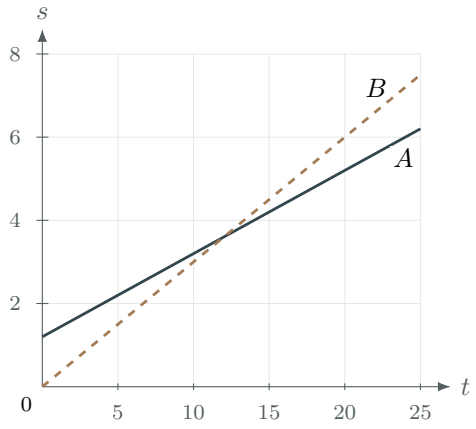
41. [Maximum mark: 8]

Two robots move in the same direction along a straight track. Their positions s metres from a fixed marker are modelled linearly for $0 \le t \le 25$ seconds. Robot A is at 1.2 m when $t = 0$ and at 5.2 m when $t = 20$. Robot B is at 0 m when $t = 0$ and at 6.0 m when $t = 20$.

(a) Find a position model for each robot. [3]

(b) Use your GDC to find when B catches A and their position at that time. [3]

(c) At $t = 15$, which robot is farther along the track, and by how much? [2]



t: seconds; s: metres.

42. [Maximum mark: 7]

The points $A(-1.5, 4.2)$, $B(2.5, 1.4)$ and $C(6.5, -1.4)$ are given.

(a) Use gradients to show that A, B and C are collinear. [3]

(b) Find the equation of their common line. [2]

(c) Find where this line meets $y = x$. Give the coordinates to 3 significant figures. [2]

43. [Maximum mark: 7]

A sensor's output V volts is modelled as a linear function of distance d centimetres. Calibration measurements give $V = 2.35$ when $d = 10$ and $V = 6.10$ when $d = 60$. The sensor has been tested only for $10 \le d \le 60$.

(a) Find the model for V in terms of d. [3]

(b) Use the model to find the distance corresponding to an output of 4.75 V. [2]

(c) Predict the output at $d = 80$ cm and state one reason why this prediction may be less reliable. [2]

44. [Maximum mark: 7]

Consider the four lines in the table.

Line	Equation
A	$2x + 5y = 10$
B	$4x + 10y = 25$
C	$5x - 2y = 8$
D	$4x + 10y = 20$

(a) Find the gradients of A and B. [2]

(b) Explain why A and B are parallel and distinct. [2]

(c) Identify the line perpendicular to A and justify your answer. [2]

(d) Identify the line coincident with A and justify your answer. [1]

45. [Maximum mark: 8]

During a test, the charge B remaining in a battery is modelled as a linear function of time t hours from the start. After 1.5 hours the charge is 81.5%; after 4.5 hours it is 48.5%. Assume the same discharge rate continues until the battery is empty.

(a) Find and interpret the gradient of the charge–time graph. [2]

(b) Find a model for B in terms of t. [2]

(c) Find when the charge reaches 20%, to 3 significant figures. [2]

(d) Find when the battery is empty and hence state a physically meaningful time interval for the model. [2]

46. [Maximum mark: 7]

A line ℓ has equation $y - 2.5 = 0.6(x - 1.5)$ and passes through $P(4, 4)$.

(a) Write its equation in the form $y = mx + c$. [2]

(b) Write its equation in the form $ax + by + d = 0$ with integer coefficients. [1]

(c) Find the equation of the line through P perpendicular to ℓ. [2]

(d) Find the coordinates of the x-intercept of this perpendicular line. [2]

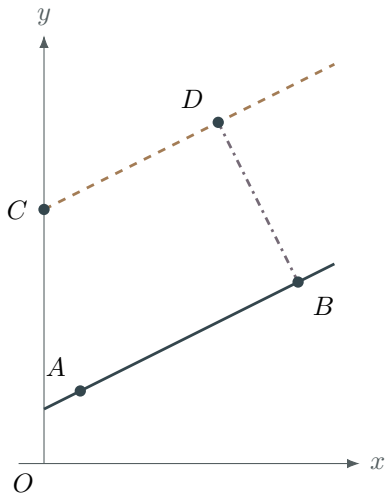
47. [Maximum mark: 7]

On a warehouse plan, a conveyor follows the line through $A(1,2)$ and $B(7,5)$. A second conveyor is parallel to it and passes through $C(0,7)$. Coordinates are in metres. A straight connecting section runs from B perpendicular to both conveyors and meets the second conveyor at D.

(a) Find the equation of the first conveyor's line. [2]

(b) Find the equation of the second conveyor's line. [2]

(c) Find the coordinates of D. [3]



Plan view; equal coordinate scales.

48. [Maximum mark: 6]

A line ℓ meets the axes at $A(7.5,0)$ and $B(0,4.5)$. Point P lies on both ℓ and the line $y=x$.

(a) Find an equation of ℓ. [2]

(b) Find the coordinates of P to 3 significant figures. [2]

(c) Find the equation of the line through P perpendicular to ℓ. Use unrounded values from part (b). [2]

49. [Maximum mark: 7]

In a storage model, every photograph occupies the same amount of space. After 50 photographs have been saved, 1570 MB remain free. After 150 photographs, 1010 MB remain free. Let F be the free space in MB after n photographs are saved; assume no other files change.

(a) Calculate the gradient of the graph of F against n and interpret it. [2]

(b) Find a model for F in terms of n. [2]

(c) Find the maximum total number of photographs that can be saved from the beginning, justifying your integer answer. [3]

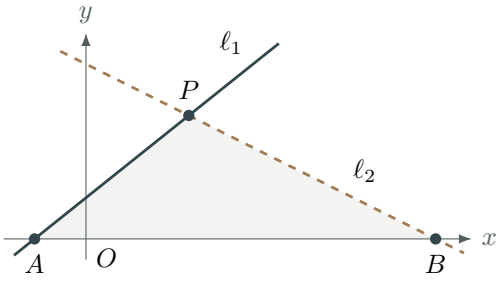
50. [Maximum mark: 7]

The lines $\ell_1 : y = 0.8x + 1.6$ and $\ell_2 : y = -0.5x + 6.8$ intersect at P. They meet the x-axis at A and B, respectively.

(a) Find the coordinates of A and B. [2]

(b) Use your GDC to find the coordinates of P. [3]

(c) Find the area of triangle APB, giving your answer to 3 significant figures. [2]



Answer key and solution outlines — Questions 1–50

Equivalent line equations are accepted. A horizontal and a vertical line are perpendicular; use $m_1 m_2 = -1$ when both gradients exist. GDC results should be supported by the equations used and the appropriate interpretation.

Section 1: Short response | Questions 1–15

1. Gradient 2.5; y-intercept $(0, 6)$.

2. $m = \dfrac{8-(-2)}{6-1} = \dfrac{10}{5} = 2$.

3. $m = \dfrac{1-4}{4-(-2)} = -\frac{1}{2}$. Using $(4, 1)$ gives $1 = -2 + c$, so $c = 3$. Hence $y = -\frac{1}{2}x + 3$.

4. $5y = -4x + 20$, so $y = -\frac{4}{5}x + 4$ and the gradient is $-\frac{4}{5} = -0.8$.

5. $y - 1 = -2(x - 3)$; hence $y = -2x + 7$.

6. When $y = 0$, $x = 4$. When $x = 0$, $y = 6$. The intercepts are $(4, 0)$ and $(0, 6)$.

7. (a) $y = 4$; gradient 0.
(b) $x = 2$; gradient undefined, since the horizontal change is zero.

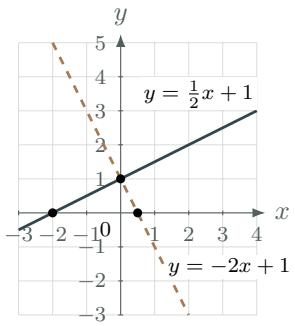
8. $\dfrac{k+1}{6-2} = 1.5$, so $k + 1 = 6$ and $k = 5$.

9. The gradient is 0.75. $y - 5 = 0.75(x - 2)$, so $y = 0.75x + 3.5$.

10. $(-0.4)m = -1$ gives $m = 2.5$. The required line is $y = 2.5x + 3$.

11. For P, $2(2) + 1 = 5$, so P lies on the line. For Q, $2(4) - 2 = 6 \neq 5$, so Q does not.

12. $y = \frac{1}{2}x + 1$ has intercepts $(-2, 0)$ and $(0, 1)$. $y = -2x + 1$ has intercepts $(\frac{1}{2}, 0)$ and $(0, 1)$.



13. (a) Parallel and distinct: $y = 3x - 2$ and $y = 3x - 5$ have the same gradient and different intercepts.
(b) Perpendicular: $(-\frac{1}{2})(2) = -1$.
(c) Coincident: dividing the first equation by 3 gives the second.

14. The fixed charge is 6.50 euros. The cost increases by 0.85 euros per kilometre; this is the gradient of the cost-distance graph.

15. The gradient is -3.5 L/min: the volume decreases by 3.5 litres each minute. The vertical intercept is $(0, 28)$: there are initially 28 litres.

Section 2: Extended response | Questions 16–30

16. (a) $\ell_1 : y = -2x + 4$; $\ell_2 : y = \frac{1}{2}x - 2$; $\ell_3 : y = \frac{1}{2}x + 1$.
(b) $\ell_2 \parallel \ell_3$, since both gradients are $\frac{1}{2}$.
(c) $\ell_1 \perp \ell_2$ and $\ell_1 \perp \ell_3$, since $(-2)(\frac{1}{2}) = -1$.

17. (a) $m = (3-5)/(5-1) = -\frac{1}{2}$; $y - 5 = -\frac{1}{2}(x-1)$.
(b) $y = -\frac{1}{2}x + \frac{11}{2}$, so $x + 2y - 11 = 0$.
(c) $(11, 0)$ and $(0, \frac{11}{2})$.

18. (a) $m = (12 - 7.2)/(6-2) = 1.2$ and $7.2 = 1.2(2) + c$ gives $c = 4.8$. Thus $C = 1.2d + 4.8$.
(b) Fixed charge 4.80 euros; rate 1.20 euros per kilometre.
(c) $24 = 1.2d + 4.8$ gives $d = 16$ km.

19. (a) $m = 0.84/12 = 0.07$.
(b) $0.07 < \frac{1}{12} \approx 0.0833$, so it meets the limit.
(c) For horizontal run r, $0.75/r \le \frac{1}{12}$. Thus $r \ge 9$ m; the minimum is 9 m.

20. (a) $m = (200 - 240)/5 = -8$, so $V = 240 - 8t$.
(b) $88 = 240 - 8t$ gives $t = 19$ min.
(c) $V = 0$ gives $t = 30$ min. At $t = 35$, the formula predicts -40 litres, which is impossible for the volume of water in the tank.

21. (a) Solve $14 + 2.6h = 5 + 4.1h$ (or intersect the two cost graphs). The intersection is $(6, 29.6)$: 6 hours at 29.60 euros.
(b) At $h = 4$, A costs 24.40 euros and B costs 21.40 euros. B saves 3.00 euros.
(c) A is cheaper for $h > 6$, since its cost increases more slowly (2.60 rather than 4.10 euros per hour).

22. (a) $y = -\frac{3}{4}x + 5$.
(b) $y + 1 = -\frac{3}{4}(x-2)$, so $y = -\frac{3}{4}x + \frac{1}{2}$.
(c) The perpendicular gradient is $\frac{4}{3}$. Thus $y + 1 = \frac{4}{3}(x-2)$, or $y = \frac{4}{3}x - \frac{11}{3}$.

23. (a) Graph $y = 1.6x + 0.8$ and $y = -0.7x + 8.25$, then find their intersection. The GDC gives $x = 3.239130\ldots$ and $y = 5.982608\ldots$. Hence the point is $(3.24, 5.98)$ to three significant figures.
(b) At $x = 3$, ℓ_1 gives $y = 5.6$, but ℓ_2 gives $y = 6.15$. The point lies only on ℓ_1.

24. (a) $m = (212 - 32)/(100 - 0) = 1.8$, so $F = 1.8C + 32$.
(b) $95 = 1.8C + 32$ gives $C = 35°$C.
(c) A temperature increase of 1°C corresponds to an increase of 1.8°F.

25. (a) $m = (87 - 42)/(50 - 20) = 1.5$. Then $42 = 1.5(20) + c$ gives $c = 12$, so $C = 1.5n + 12$.
(b) Each poster costs 1.50 euros; the setup fee is 12 euros.
(c) $1.5n + 12 \le 100$ gives $n \le 58.666\ldots$. The maximum is 58 posters, costing 99 euros; 59 would cost 100.50 euros.

26. (a) $m_{AB} = m_{CD} = \frac{1}{2}$ and $m_{BC} = m_{DA} = -2$. Opposite sides are parallel and adjacent sides are perpendicular, so the quadrilateral is a rectangle.
(b) $m_{AC} = (-1 - 1)/(6 - 0) = -\frac{1}{3}$. Since $A = (0, 1)$, $y = -\frac{1}{3}x + 1$.

27. (a) Initial value 18500 euros; the predicted value decreases by 2200 euros per year.
(b) Solve $18500 - 2200t = 10000$: $t = 3.863636\ldots \approx 3.86$ years.
(c) The model gives $V = -1300$ euros. A negative resale value is not meaningful here, so this linear rule should not be extended to that age.

28. (a) $m = (45 - 90)/(50 - 20) = -1.5$ km/min. Using $(20, 90)$ gives $D = 120 - 1.5t$.
(b) Speed $= 1.5 \times 60 = 90$ km/h.
(c) $D = 0$ gives $t = 120/1.5 = 80$ minutes after timing began.

29. (a) The gradients are 2 and -2, whose product is -4, not -1. The lines are not perpendicular; unequal display scales distort their apparent angle.
(b) The required gradient is $-\frac{1}{2}$, so $y = -\frac{1}{2}x + 5$.

30. (a) Solve $1.25q + 8 = -0.75q + 48$: $q = 20$ tonnes and $p = 33$ euros per tonne.
(b) Supply: $26 = 1.25q + 8$ gives $q_S = 14.4$ tonnes. Demand: $26 = -0.75q + 48$ gives $q_D = \frac{88}{3} = 29.333\ldots$ tonnes. Shortfall $= q_D - q_S = 14.933\ldots \approx 14.9$ tonnes.

Section 3: IB exam-style practice | Questions 31–50

31. (a) $m = (104.2 - 43.0)/(6 - 2) = 15.3$ L/min: the volume increases by 15.3 litres each minute.
(b) $43 = 15.3(2) + c$, so $c = 12.4$ and $V = 15.3t + 12.4$.
(c) The tank initially contains 12.4 litres.
(d) $250 = 15.3t + 12.4$ gives $t = 15.5294\ldots$, so the tank is full after 15.5 minutes (3 s.f.).

32. (a) $(0, 2.5)$.
(b) $m = (4 - 1)/(3 - (-3)) = 0.5$.
(c) $y = 0.5x + 2.5$.
(d) The perpendicular gradient is -2. Thus $y - 4 = -2(x - 3)$, or $y = -2x + 10$.

33. (a) $H_A = 12 + 0.8t$ and $H_B = 18 + 0.5t$.
(b) $12 + 0.8t = 18 + 0.5t$ gives $t = 20$ days. The common height is 28 cm.
(c) At $t = 30$, $H_A = 36$ cm and $H_B = 33$ cm. Seedling A is 3 cm taller.

34. (a) $y = -0.8x + 3.6$.
(b) Set $y = 0$ to obtain $(4.5, 0)$; set $x = 0$ to obtain $(0, 3.6)$.
(c) The gradient is -0.8, so $y - 1 = -0.8(x + 2)$. Hence $4x + 5y + 3 = 0$.

35. (a) $m = (100 - 160)/(8 - 2) = -10$ mm/min. The thickness decreases by 10 mm per minute.
(b) $160 = -10(2) + c$ gives $c = 180$, so $h = 180 - 10t$.
(c) $45 = 180 - 10t$ gives $t = 13.5$ minutes.
(d) At $t = 25$ the model gives $h = -70$ mm, which is impossible for a thickness. The model reaches zero at $t = 18$ minutes.

36. (a) Horizontal: $y = -1.2$; vertical: $x = 3.5$.
(b) With $y = -1.2$, $2x + 3.6 = 9$, so the point is $(2.7, -1.2)$. With $x = 3.5$, $7 - 3y = 9$, giving $(3.5, -\frac{2}{3}) \approx (3.5, -0.667)$.
(c) The horizontal line $y = -1.2$ is perpendicular to $x = 3.5$.

37. (a) The gradient is $(57 - 84)/10 = -2.7$, so $T_A = 84 - 2.7t$.
(b) Solve $84 - 2.7t = 18 + 1.6t$. The GDC intersection is $(15.3488\ldots, 42.5581\ldots)$: $t = 15.3$ minutes and temperature 42.6°C (3 s.f.).
(c) At $t = 18$, $T_A = 35.4$°C and $T_B = 46.8$°C. Liquid B is 11.4°C warmer.

38. (a) $m_{AB} = (3 - 1)/(4 - (-2)) = \frac{1}{3}$; $m_{BC} = (0 - 3)/(5 - 4) = -3$.
(b) $(\frac{1}{3})(-3) = -1$, so $AB \perp BC$ and angle $ABC = 90°$.
(c) $m_{AC} = -\frac{1}{7}$. Hence $y - 1 = -\frac{1}{7}(x + 2)$, or $x + 7y - 5 = 0$.
(d) $y - 3 = -\frac{1}{7}(x - 4)$, or $x + 7y - 25 = 0$.

39. (a) $m = (0.85 - 0.25)/7.5 = 0.08$. Since $0.08 < 0.09$, the walkway meets the stated design brief.
(b) The equation is $y = 0.08x + 0.25$. At $x = 12$, $y = 1.21$ m.
(c) $1.45 = 0.08x + 0.25$ gives $x = 15$ m.

40. (a) Write ℓ as $y = 1.5x - 2$ and solve $1.5x - 2 = -0.75x + 5.2$. This gives $P = (3.2, 2.8)$.
(b) The gradient is 1.5.
(c) The perpendicular gradient is $-\frac{2}{3}$, so $y - 2.8 = -\frac{2}{3}(x - 3.2)$.

41. (a) For A, $m_A = (5.2 - 1.2)/20 = 0.2$, so $s_A = 1.2 + 0.2t$. For B, $m_B = 6/20 = 0.3$, so $s_B = 0.3t$.
(b) The GDC intersection of s_A and s_B is $(12, 3.6)$. B catches A after 12 seconds, at 3.6 m from the marker.
(c) At $t = 15$, $s_A = 4.2$ m and $s_B = 4.5$ m. B is 0.3 m farther along the track.

42. (a) $m_{AB} = (1.4 - 4.2)/(2.5 + 1.5) = -0.7$ and $m_{BC} = (-1.4 - 1.4)/(6.5 - 2.5) = -0.7$. The segments share B and have equal gradients, so the points are collinear.
(b) $1.4 = -0.7(2.5) + c$ gives $c = 3.15$. Thus $y = -0.7x + 3.15$.
(c) Solve $x = -0.7x + 3.15$ to obtain $x = y = 1.85294\ldots$. The intersection is $(1.85, 1.85)$ (3 s.f.).

43. (a) $m = (6.10 - 2.35)/(60 - 10) = 0.075$ V/cm. Substituting $(10, 2.35)$ gives $c = 1.60$, so $V = 0.075d + 1.60$.
(b) $4.75 = 0.075d + 1.60$ gives $d = 42$ cm.
(c) At $d = 80$, $V = 7.60$ V. This is extrapolation outside the tested interval, where a linear relationship has not been established.

44. (a) $m_A = m_B = -0.4$.
(b) The gradients are equal, but the y-intercepts are 2 and 2.5, so the lines are distinct and parallel.
(c) C has gradient 2.5, and $(-0.4)(2.5) = -1$, so C is perpendicular to A.
(d) D is coincident with A: dividing D's equation by 2 gives $2x + 5y = 10$.

45. (a) $m = (48.5 - 81.5)/(4.5 - 1.5) = -11$. The charge decreases by 11 percentage points per hour.
(b) $81.5 = -11(1.5) + c$ gives $c = 98$, so $B = 98 - 11t$.
(c) $20 = 98 - 11t$ gives $t = \frac{78}{11} \approx 7.09$ hours (3 s.f.).
(d) $B = 0$ gives $t = 98/11 \approx 8.91$ hours. The meaningful interval is $0 \le t \le 98/11$ hours.

46. (a) $y = 0.6x + 1.6$.
(b) $3x - 5y + 8 = 0$.
(c) The perpendicular gradient is $-5/3$, so $y - 4 = -\frac{5}{3}(x - 4)$.
(d) Set $y = 0$: $-4 = -\frac{5}{3}(x - 4)$ gives $x = 6.4$. The intercept is $(6.4, 0)$.

47. (a) $m = (5 - 2)/(7 - 1) = 0.5$, so $y - 2 = 0.5(x - 1)$ and $y = 0.5x + 1.5$.
(b) The same gradient and $C(0, 7)$ give $y = 0.5x + 7$.
(c) The connector has gradient -2, so $y - 5 = -2(x - 7)$, or $y = -2x + 19$. Solving $0.5x + 7 = -2x + 19$ gives $D = (4.8, 9.4)$.

48. (a) $m = (0 - 4.5)/(7.5 - 0) = -0.6$, so $y = -0.6x + 4.5$.
(b) Solve $x = -0.6x + 4.5$ to obtain $P = (2.8125, 2.8125)$, or $(2.81, 2.81)$ to 3 s.f.
(c) The perpendicular gradient is $5/3$. Thus $y - 2.8125 = \frac{5}{3}(x - 2.8125)$, equivalently $y = \frac{5}{3}x - 1.875$.

49. (a) $m = (1010 - 1570)/(150 - 50) = -5.6$ MB per photograph. Each additional photograph uses 5.6 MB.
(b) $1570 = -5.6(50) + c$ gives $c = 1850$, so $F = 1850 - 5.6n$.
(c) Require $F \ge 0$: $n \le 1850/5.6 = 330.357\ldots$. The maximum is 330 photographs; $F(330) = 2$ MB, whereas 331 photographs would require more space than is available.

50. (a) For ℓ_1, $0 = 0.8x + 1.6$ gives $A = (-2, 0)$. For ℓ_2, $0 = -0.5x + 6.8$ gives $B = (13.6, 0)$.
(b) The GDC intersection of the two lines is $P = (4, 4.8)$.
(c) The base is $13.6 - (-2) = 15.6$ and the perpendicular height is 4.8. Area $= \frac{1}{2}(15.6)(4.8) = 37.44 \approx 37.4$ square units (3 s.f.).

Syllabus scope checked against the supplied IB Mathematics: Applications and Interpretation guide, SL 2.1, p. 35. Original practice questions and illustrations by Radford Mathematics.