Functions, Domain and Range

IB Mathematics: Applications and Interpretation SL

SL 2.2 | 50 questions | Models and inverse functions

Learning goal	Use functions, domains and ranges, interpret models, and understand how an inverse reverses a function.
Syllabus link	AI SL 2.2: functions, notation, models, domain and range; inverse values, one-to-one functions and reflection in $y = x$.
Structure	15 short-response questions; 15 extended-response questions; 20 original IB exam-style questions; answer key and solution outlines.
Technology	A graphic display calculator (GDC) may be used throughout, including the Paper 1-style section.

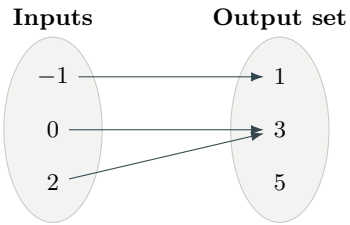
Instructions. Work on separate paper, except where axes are supplied. Show your method and give units where appropriate. Give exact answers unless an approximation is requested. Use the stated domain; otherwise use the largest possible real domain. An open point is excluded; a filled point is included.

1. Short response

1. Let $f(x) = 4x - 7$. Find $f(3)$ and $f(-2)$.

2. A printer has $P(t) = 180 - 12t$ pages left to print after t minutes, for $0 \le t \le 15$. Find $P(5)$ and interpret your answer.

3. The arrows show all the assignments in a mapping.



State whether the mapping is a function. Write down its domain and its range.

4. A relation consists of $\{(1, 4), (2, 5), (1, 7), (3, 6)\}$. Explain why it does not define y as a function of x.

5. The function f has an inverse and $f(4) = 9$. Write down $f^{-1}(9)$.

1. Short response (continued)

6. Let $g(x) = \sqrt{9 - x}$. State its domain and range.

7. Let $h(x) = \dfrac{4}{x + 2}$. State its domain and explain why $h(-2)$ is undefined.

8. The function $p(x) = x^2 + 1$ has domain $\mathbb{R}$.

(a) Write down its range.

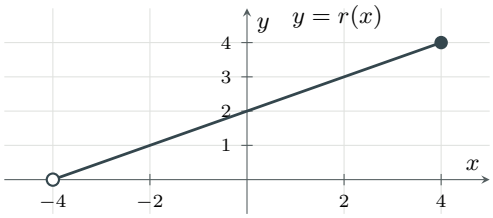
(b) Explain why p does not have an inverse function on this domain.

9. The table gives every value of a one-to-one function g.

x	-2	0	3	5
$g(x)$	1	4	8	11

Write down $g(3)$ and $g^{-1}(4)$.

10. The complete graph of $y = r(x)$ is shown. State its domain and range.



11. A function machine multiplies its input by 3, then subtracts 5. Its output is 16. Find the input and state the two operations that undo the machine, in the correct order.

12. A one-to-one function has points $(-2, 7)$ and $(4, 1)$ on its graph. Write down the corresponding points on the graph of its inverse.

1. Short response (continued)

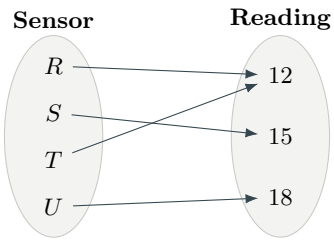
13. The function $f(x) = x^2$ has domain $\{-2, 0, 2\}$. List its range. Is it one-to-one?

14. A walker travels a distance $D(t) = 6t$ kilometres in t hours, for $0 \le t \le 2$. State the domain of D^{-1} and explain the meaning of $D^{-1}(9) = 1.5$.

15. A one-to-one function satisfies $f(2) = 5$ and $f(4) = 2$. Find $f^{-1}(2)$ and $\dfrac{1}{f(2)}$. Explain why these are different operations.

2. Extended response

16. Four sensors produce the readings shown in the mapping.



(a) State the domain and range.

(b) Explain why the mapping is a function, although two arrows end at 12.

(c) Can the reading always be used to identify exactly one sensor? Explain what this means for an inverse function.

17. Let $f(x) = (x - 2)^2 + 1$, with domain $-1 \le x \le 5$.

(a) Complete the table.

x	-1	0	1	2	3	4	5
$f(x)$							

(b) Use a graph or the expression for $f(x)$ to determine its range.

(c) Give two different inputs with the same output. Hence state whether f^{-1} is a function on the whole range.

2. Extended response (continued)

18. The function $g(x) = \sqrt{12 - 3x}$ has domain $-8 \leq x \leq 4$.

(a) Find $g(-8)$, $g(1)$ and $g(4)$.

(b) State the range of g.

(c) Find $g^{-1}(3)$.

(d) State the domain and range of g^{-1}.

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19. A tank contains $V(t) = 720 - 24t$ litres of water t minutes after draining starts. The model applies until the tank is empty.

(a) State a suitable domain for V.

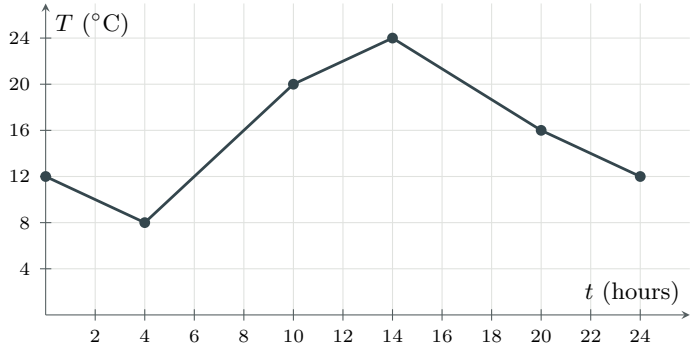
(b) Find $V(12)$, with units.

(c) Find $V^{-1}(240)$ and interpret your answer.

(d) State the domain and range of V^{-1}, with the corresponding units.

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20. The graph shows a model for the temperature $T(t)$, in °C, t hours after midnight. Consecutive plotted points are joined by straight line segments.



(a) Write down $T(4)$.

(b) State the domain and range of T.

(c) Find the two times when $T(t) = 20$.

(d) Explain why this model does not have an inverse function over its whole domain.

2. Extended response (continued)

21. A booking website charges €6 per booking plus €4 for each ticket. A booking must contain between 1 and 12 tickets, inclusive. Let $C(n)$ be the total cost, in euros, of n tickets.

(a) Write down $C(n)$ and its domain.

(b) Find $C(5)$ and $C^{-1}(38)$.

(c) State the domain of C^{-1}. Make clear whether the possible values are discrete or continuous.

(d) Explain why $C^{-1}(27)$ is undefined in this model.

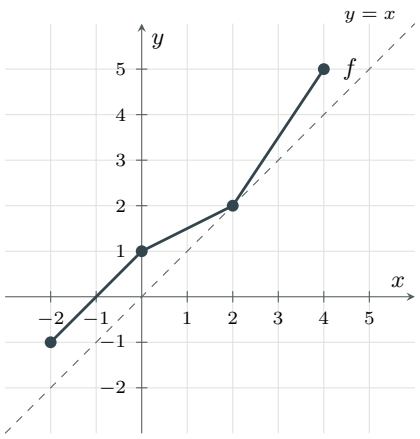
22. A taxi fare is modelled by $F(d) = 4 + 1.8d$, where d is the distance travelled in kilometres and F is in euros. The model is used for $0 \leq d \leq 40$.

(a) State the range of F.

(b) Find $F^{-1}(31)$ and interpret your answer.

(c) A traveller tries to evaluate $F^{-1}(80)$. Explain why this is outside the stated model.

23. The complete graph of f joins the points $(-2, -1)$, $(0, 1)$, $(2, 2)$ and $(4, 5)$ with straight line segments. The dashed line is $y = x$.



(a) Write down $f(0)$ and $f^{-1}(5)$.

(b) Draw the graph of f^{-1} on the same axes.

(c) State the domain and range of f^{-1}.

2. Extended response (continued)

24. The table defines all the values of a function p.

x	-3	-1	2	4
$p(x)$	-4	0	6	10

(a) Make a table giving every value of p^{-1}.

(b) Find $p^{-1}(6)$ and $\dfrac{1}{p(2)}$.

(c) State whether $p^{-1}(5)$ is defined. Give a reason.

25. Let $f(x) = x^3 + 2x + 1$, for $-2 \leq x \leq 3$.

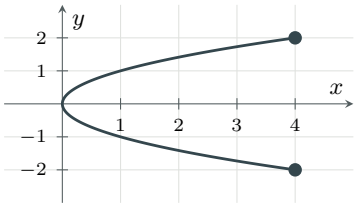
(a) Use your GDC to view the graph. Explain why f has an inverse function.

(b) State the range of f.

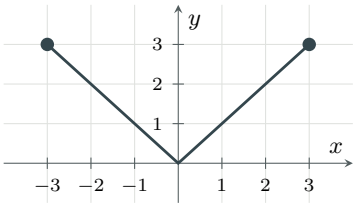
(c) Find $f^{-1}(12)$, correct to three significant figures. Write down the equation you solve.

26. Each diagram shows a complete relation between x and y.

Relation A



Relation B



(a) Which relation defines y as a function of x? Explain your decision for both diagrams.

(b) State the domain and range of the function.

(c) Does that function have an inverse function on its whole range? Explain.

27. A square has side length s metres and area $A(s) = s^2$ square metres, for $0 \leq s \leq 12$.

(a) State the range of A.

(b) Find $A^{-1}(81)$, with units.

(c) Explain why -9 is not another possible value of $A^{-1}(81)$.

2. Extended response (continued)

28. A mapping is defined by the ordered pairs $\{(-2, 4), (0, k), (3, 7)\}$, where k is a real number.

(a) Explain why it is a function for every real value of k.

(b) State the values of k for which it does not have an inverse function.

(c) The pair $(0, 6)$ is added, and the relation must still be a function. Find k, then list the ordered pairs for the inverse.

29. A hiking route is modelled by $H(d) = 100 + 40d - 5d^2$, for $0 \le d \le 8$, where d is distance along the route in kilometres and H is height in metres. The complete graph is shown.



(a) State the range of H.

(b) Find the two distances where $H(d) = 160$.

(c) Explain why $H^{-1}(160)$ cannot be a single inverse-function value for this model.

30. A battery model is $B(t) = 100(0.8)^t$, for $0 \le t \le 10$, where t is time in hours and $B(t)$ is the percentage charge remaining.

(a) Find $B(3)$.

(b) State the range of B, giving the lower endpoint to three significant figures.

(c) Use your GDC to find $B^{-1}(25)$, correct to three significant figures, and interpret your answer.

(d) Explain why an inverse function exists on the stated range.

3. IB exam-style practice

Original questions; GDC permitted. Mark allocations are shown in square brackets.

31. **[Maximum mark: 5]** A function r is defined only by the values in the table.

x	0	1	2	3	4
$r(x)$	12	18	25	33	42

(a) Write down $r(3)$. [1]

(b) Write down $r^{-1}(25)$. [1]

(c) State the domain of r^{-1}. [2]

(d) Explain why $r^{-1}(20)$ is undefined. [1]

32. **[Maximum mark: 6]** The function $F(c) = 1.8c + 32$ converts a temperature of c degrees Celsius to degrees Fahrenheit. Its domain is $-20 \leq c \leq 50$.

(a) Find $F(15)$. [2]

(b) State the range of F. [2]

(c) Find $F^{-1}(86)$ and interpret your answer in this context. [2]

33. **[Maximum mark: 6]** Consider $f(x) = \dfrac{12}{x+1}$, for $1 \leq x \leq 5$.

(a) Find $f(2)$. [1]

(b) State the range of f. [2]

(c) Find $f^{-1}(3)$. [2]

(d) Explain why f has an inverse function on the given domain. [1]

34. **[Maximum mark: 7]** A bicycle rental costs $C(n) = 15 + 8n$ euros for n whole days, where $n \in \{1, 2, \ldots, 14\}$.

(a) Find $C(4)$. [1]

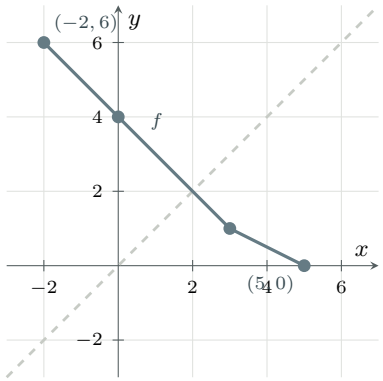
(b) Find $C^{-1}(79)$ and state its unit. [2]

(c) State the domain of C^{-1}, showing the discrete values clearly. [2]

(d) Explain why $C^{-1}(50)$ is undefined for this model. [2]

3. IB exam-style practice (continued)

35. **[Maximum mark: 6]** The graph of f consists of straight-line segments. All endpoints shown are included. The dashed line is $y = x$.



(a) State the range of f. [1]

(b) Write down $f^{-1}(4)$. [1]

(c) Write down the coordinates of the two endpoints of the graph of f^{-1}. [2]

(d) Sketch f^{-1} on the supplied axes. [2]

36. **[Maximum mark: 6]** The volume of a spherical ornament is modelled by $V(r) = \frac{4}{3}\pi r^3$, where r is its radius in centimetres and $0 \leq r \leq 5$.

(a) Find the exact value of $V(3)$. [2]

(b) State the range of V, giving exact endpoints. [2]

(c) Find $V^{-1}(120)$ to three significant figures and interpret your answer. [2]

37. **[Maximum mark: 6]** A one-to-one function g has exactly the input-output pairs

$$(-2, 7), \qquad (0, 4), \qquad (3, 1), \qquad (5, k).$$

(a) State the domain of g. [1]

(b) State the three values that k cannot take. [2]

(c) When $k = -2$, write down all the input-output pairs of g^{-1}. [2]

(d) Write down $g^{-1}(4)$. [1]

38. **[Maximum mark: 6]** A delivery service models the charge, in euros, for a parcel of mass m kilograms by $P(m) = 2.40 + 0.80m$, for $0 < m \leq 15$.

(a) Find the charge for a parcel of mass 4.5 kg. [2]

(b) State the range of P, showing whether each endpoint is included. [2]

(c) Find $P^{-1}(10.40)$ and state its unit. [2]

3. IB exam-style practice (continued)

39. **[Maximum mark: 6]** Consider $h(x) = 9 - (x-1)^2$, for $-1 \leq x \leq 3$.

(a) Find $h(-1)$ and $h(3)$. [2]

(b) Use your GDC to determine the range of h. [2]

(c) Explain why h does not have an inverse function on this domain. [2]

40. **[Maximum mark: 6]** The table gives a bus's recorded distance from its starting point. In this question, d is defined only at the five observation times shown.

Time t (minutes)	0	10	20	30	40
Distance $d(t)$ (km)	0	8	17	17	24

(a) State the domain of d. [1]

(b) State the range of d. [2]

(c) Explain why d is a function. [1]

(d) Explain why d does not have an inverse function. [2]

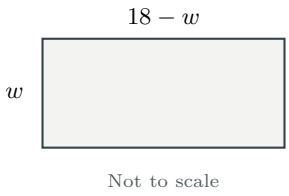
41. **[Maximum mark: 6]** A device's stored charge is modelled by $Q(t) = 80(1 - e^{-0.3t})$, where Q is measured in coulombs and t in seconds. The model is used for $0 \leq t \leq 10$.

(a) Find $Q(4)$ to three significant figures. [2]

(b) Use your GDC to find $Q^{-1}(60)$ to three significant figures. State what this value represents. [2]

(c) State the range of Q. Give an exact upper endpoint or a value correct to three significant figures. [2]

42. **[Maximum mark: 6]** A rectangle has perimeter 36 cm. Its area is modelled by $A(w) = w(18 - w)$, where w is the shorter side (or either side for a square), in centimetres, and $0 < w \leq 9$.



Not to scale

(a) Find $A(4)$. [1]

(b) State the range of A. [2]

(c) Find $A^{-1}(65)$. [2]

(d) Explain why 13 is not a valid answer to part (c). [1]

3. IB exam-style practice (continued)

43. **[Maximum mark: 6]** A telephone service models the monthly charge by $C(t) = 0.06t + 4.50$, where t is the call time in minutes, $0 \leq t \leq 500$, and $C(t)$ is in euros.

(a) Calculate $C(120)$ and interpret this value. [2]

(b) Find $C^{-1}(19.50)$. [2]

(c) State the domain and range of C^{-1}, with units. [2]

44. **[Maximum mark: 6]** The function $f(x) = \sqrt{x+7}$ has domain $-7 \leq x \leq 18$.

(a) Find $f(2)$. [1]

(b) State the domain and range of f^{-1}. [2]

(c) Find $f^{-1}(4)$. [2]

(d) A point (a, b) lies on the graph of f. Write down the coordinates of the corresponding point on the graph of f^{-1}. [1]

45. **[Maximum mark: 6]** A function p is defined only by the table.

x	-1	0	2	3
$p(x)$	5	3	-1	0

(a) Write down $p^{-1}(3)$ and $p(3)$. [2]

(b) Find $p^{-1}(5)$ and $\dfrac{1}{p(-1)}$. [2]

(c) State the range of p^{-1}. [1]

(d) Write down the input x for which $\dfrac{1}{p(x)}$ is undefined. [1]

46. **[Maximum mark: 6]** Use the conversion model $K(m) = 1.60934m$, where m is a distance in miles and $K(m)$ is a distance in kilometres. The model is restricted to $0 \leq m \leq 100$.

(a) Find $K(26.2)$ to three significant figures. [2]

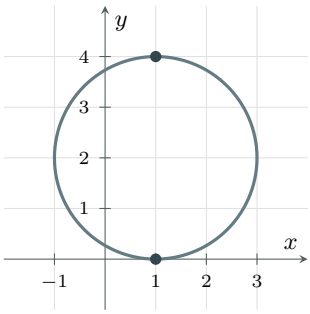
(b) Find $K^{-1}(100)$ to three significant figures and state its unit. [2]

(c) State the domain of K^{-1}. [1]

(d) Explain why $K^{-1}(-5)$ is undefined for this model. [1]

3. IB exam-style practice (continued)

47. [**Maximum mark: 6**] The diagram shows a relation between x and y.



(a) Write down the two y-values when $x = 1$. [2]

(b) Explain why the relation does not define y as a function of x. [2]

(c) A student claims: "Two different inputs can never have the same output in a function." State whether the claim is correct and explain your answer. [2]

48. [**Maximum mark: 6**] A continuous one-to-one function g has domain $[-3, 2]$ and range $[-2, 4]$. Its graph passes through $(-3, -2)$, $(0, 1)$ and $(2, 4)$.

(a) Write down $g^{-1}(1)$. [1]

(b) Write down three points on the graph of g^{-1}. [2]

(c) State the domain and range of g^{-1}. [2]

(d) State the equation of the line in which the graph of g is reflected to obtain the graph of g^{-1}. [1]

49. [**Maximum mark: 7**] A ball's height is modelled by $h(t) = 2 + 12t - 3t^2$ metres, where t is the time in seconds and $0 \leq t \leq 4$.

(a) Find $h(1)$. [1]

(b) Find the two times when the height is 11 m. [2]

(c) Use your GDC to determine the range of h. [2]

(d) Explain why an inverse function h^{-1} does not exist on the stated domain. [2]

50. [**Maximum mark: 8**] Consider $f(x) = 2x + \sqrt{x + 1}$, for $0 \leq x \leq 24$. The graph of f is strictly increasing on this domain.

(a) Find $f(3)$. [1]

(b) State the range of f. [2]

(c) Use your GDC to find $f^{-1}(30)$ to three significant figures. [2]

(d) State the domain and range of f^{-1}. [2]

(e) Explain why f has an inverse function. [1]

Answer key and solution outlines

Checking your work. Equivalent interval notation is accepted. For a numerical inverse, solve $f(x) =$ the required output within the stated domain. The symbol f^{-1} means the inverse function; it does not mean the reciprocal $1/f$.

Section 1: Short response

1. $f(3) = 4(3) - 7 = 5$; $f(-2) = 4(-2) - 7 = -15$.

2. $P(5) = 180 - 12(5) = 120$. After 5 minutes, 120 pages remain to be printed.

3. Yes: each input has exactly one output. Domain $= \{-1, 0, 2\}$; range $= \{1, 3\}$. The displayed value 5 is not reached, so it is not in the range.

4. The input $x = 1$ is assigned two different outputs, 4 and 7.

5. $f^{-1}(9) = 4$.

6. The square root requires $9 - x \geq 0$. Domain: $x \leq 9$. Range: $g(x) \geq 0$.

7. Domain: $x \in \mathbb{R}$, $x \neq -2$. At $x = -2$, the denominator is zero; division by zero is undefined.

8.

(a) $p(x) \geq 1$, or $[1, \infty)$.

(b) It is not one-to-one: for example, $p(1) = p(-1) = 2$. The inverse input 2 would have two outputs, 1 and -1.

9. $g(3) = 8$; $g^{-1}(4) = 0$.

10. Domain: $-4 < x \leq 4$, or $(-4, 4]$. Range: $0 < r(x) \leq 4$, or $(0, 4]$. The open endpoint excludes both $x = -4$ and output 0.

11. Undo by adding 5, then dividing by 3. The input is $(16 + 5)/3 = 7$.

12. $(7, -2)$ and $(1, 4)$: the coordinates swap under reflection in $y = x$.

13. Range $= \{0, 4\}$. It is not one-to-one, since $f(-2) = f(2) = 4$.

14. Domain of D^{-1}: $0 \leq d \leq 12$ kilometres. It takes 1.5 hours to walk 9 kilometres.

15. $f^{-1}(2) = 4$, whereas $1/f(2) = 1/5$. The inverse finds the input that produced 2; the reciprocal divides 1 by the output at input 2.

Section 2: Extended response

16.

(a) Domain $= \{R, S, T, U\}$; range $= \{12, 15, 18\}$.

(b) Every sensor has exactly one reading. Different inputs may share an output in a function.

(c) No: the reading 12 could come from R or T. The mapping is not one-to-one, so reversing it does not give an inverse function.

17.

(a) The outputs, in the given order, are 10, 5, 2, 1, 2, 5, 10.

(b) Since $(x - 2)^2 \geq 0$, the smallest output is 1, at $x = 2$. The largest is 10, at the endpoints. The graph is continuous, so the range is $[1, 10]$.

(c) For example, $f(1) = f(3) = 2$. The function is not one-to-one, so it has no inverse function on its whole range.

Section 2: Extended response (continued)

18.

(a) $g(-8) = \sqrt{36} = 6$, $g(1) = \sqrt{9} = 3$, $g(4) = 0$.

(b) $0 \leq g(x) \leq 6$. The function decreases continuously between the endpoint values.

(c) $g^{-1}(3) = 1$, since $g(1) = 3$.

(d) Domain of g^{-1}: $[0, 6]$. Range of g^{-1}: $[-8, 4]$.

19.

(a) The tank is empty when $720 - 24t = 0$, giving $t = 30$. Domain: $0 \leq t \leq 30$ minutes.

(b) $V(12) = 720 - 288 = 432$ litres.

(c) Solve $720 - 24t = 240$: $24t = 480$, so $t = 20$. Hence $V^{-1}(240) = 20$: after 20 minutes, 240 litres remain.

(d) Inverse domain: volumes from 0 to 720 litres inclusive. Inverse range: times from 0 to 30 minutes inclusive.

20.

(a) $T(4) = 8\,^{\circ}\text{C}$.

(b) Domain: $0 \leq t \leq 24$ hours. Range: $8 \leq T \leq 24\,^{\circ}\text{C}$.

(c) $t = 10$ hours and $t = 17$ hours. On the falling segment, 20 is halfway between 24 and 16, so the time is halfway between 14 and 20.

(d) The same temperature occurs at different times; for example, $T(10) = T(17) = 20$. Thus the function is not one-to-one.

21.

(a) $C(n) = 6 + 4n$, with domain $n \in \{1, 2, \ldots, 12\}$.

(b) $C(5) = €26$. Solving $6 + 4n = 38$ gives $n = 8$, so $C^{-1}(38) = 8$ tickets.

(c) Inverse domain: $\{10, 14, 18, 22, 26, 30, 34, 38, 42, 46, 50, 54\}$ euros. These values are discrete.

(d) $6 + 4n = 27$ gives $n = 5.25$, which is not an allowed number of tickets. Equivalently, 27 is not in the range of C.

22.

(a) $F(0) = 4$ and $F(40) = 76$. The range is $4 \leq F \leq 76$ euros.

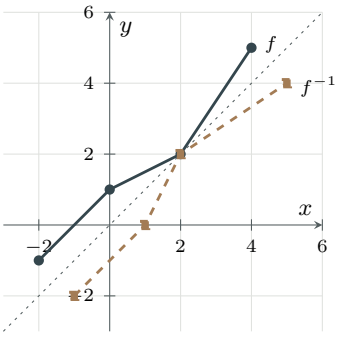
(b) $4 + 1.8d = 31$ gives $d = 27/1.8 = 15$. Thus $F^{-1}(31) = 15$: a fare of €31 corresponds to a journey of 15 km.

(c) €80 is outside the range $[4, 76]$. The required distance would exceed 40 km.

23.

(a) $f(0) = 1$; $f^{-1}(5) = 4$.

(b) Reflect each point in $y = x$. Join $(-1, -2)$, $(1, 0)$, $(2, 2)$ and $(5, 4)$ with straight segments, as shown.



(c) Domain of f^{-1}: $[-1, 5]$. Range of f^{-1}: $[-2, 4]$.

Section 2: Extended response (continued)

24.

(a) Reverse each input–output pair:

x	-4	0	6	10
$p^{-1}(x)$	-3	-1	2	4

(b) $p^{-1}(6) = 2$; $1/p(2) = 1/6$.

(c) It is undefined because 5 is not an output of p, so 5 is not in the domain of p^{-1}.

25.

(a) The graph is strictly increasing throughout the stated domain; every output occurs once. Thus f is one-to-one.

(b) $f(-2) = -11$ and $f(3) = 34$. The continuous increasing graph gives range $[-11, 34]$.

(c) Solve $x^3 + 2x + 1 = 12$, for $-2 \leq x \leq 3$. The GDC gives $x = 1.926270\ldots$, so $f^{-1}(12) = 1.93$ to three significant figures.

26.

(a) B is a function: every x has one y. A is not: for example, $x = 1$ has outputs $y = 1$ and $y = -1$.

(b) For B, domain $= [-3, 3]$ and range $= [0, 3]$.

(c) No. For example, the output 2 comes from both $x = -2$ and $x = 2$, so B is not one-to-one.

27.

(a) $0 \leq A \leq 144\,\text{m}^2$.

(b) $s^2 = 81$ with $s \geq 0$ gives $A^{-1}(81) = 9$ metres.

(c) A negative side length is not in the stated domain. Although $(-9)^2 = 81$, $s = -9$ is not allowed by this model.

28.

(a) Each input $-2, 0, 3$ appears once and has one assigned output, whatever the value of k.

(b) $k = 4$ or $k = 7$ gives a repeated output and prevents an inverse function.

(c) The existing output at input 0 must also be 6, so $k = 6$. The inverse pairs are $\{(4, -2), (6, 0), (7, 3)\}$.

29.

(a) $100 \leq H \leq 180$ metres.

(b) From the graph, $d = 2$ km and $d = 6$ km. Check: $H(2) = 100 + 80 - 20 = 160$ and $H(6) = 100 + 240 - 180 = 160$.

(c) The height 160 m corresponds to two distances. Reversing the function would not give a unique output at input 160.

30.

(a) $B(3) = 100(0.8)^3 = 51.2\%$.

(b) The exact range is $[100(0.8)^{10}, 100]$. The lower endpoint is $10.737418\ldots\%$, or 10.7% to three significant figures; the upper endpoint is 100%.

(c) Solve $100(0.8)^t = 25$ on $0 \leq t \leq 10$. The GDC gives $t = 6.212567\ldots$, so $B^{-1}(25) = 6.21$ hours. The charge reaches 25% after approximately 6.21 hours.

(d) The function is strictly decreasing on the stated domain, so each permitted percentage corresponds to exactly one time.

Section 3: IB exam-style practice

31.

(a) 33.

(b) 2, since $r(2) = 25$.

(c) $\{12, 18, 25, 33, 42\}$.

(d) 20 is not in the range of r, so it is not an allowed input to r^{-1}.

32.

(a) $F(15) = 1.8(15) + 32 = 59\,°\text{F}$.

(b) $F(-20) = -4$, $F(50) = 122$; the range is $[-4, 122]$ in degrees Fahrenheit.

(c) Solve $1.8c + 32 = 86$: $c = 30$. Thus $F^{-1}(86) = 30$; $86\,°\text{F}$ corresponds to $30\,°\text{C}$.

33.

(a) $f(2) = 12/3 = 4$.

(b) $f(1) = 6$ and $f(5) = 2$; since f decreases continuously, its range is $[2, 6]$.

(c) $12/(x + 1) = 3$ gives $x + 1 = 4$, so $f^{-1}(3) = 3$.

(d) f is one-to-one: each output in $[2, 6]$ occurs for exactly one input.

34.

(a) $C(4) = 15 + 32 = €47$.

(b) $15 + 8n = 79$ gives $n = 8$, so $C^{-1}(79) = 8$ days.

(c) $\{23, 31, 39, \ldots, 127\}$ euros, in steps of 8.

(d) $15 + 8n = 50$ gives $n = 35/8 = 4.375$, which is not an allowed whole number of days; 50 is not in the range of C.

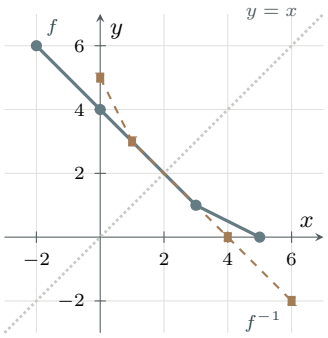
35.

(a) $[0, 6]$.

(b) 0.

(c) The inverse endpoints are $(6, -2)$ and $(0, 5)$.

(d) Join $(0, 5)$, $(1, 3)$, $(4, 0)$ and $(6, -2)$ in this order by straight-line segments; use filled endpoints. The result is the reflection of the supplied graph in $y = x$.



36.

(a) $V(3) = \frac{4}{3}\pi(3^3) = 36\pi$ cm^3.

(b) $\left[0, \frac{500\pi}{3}\right]$ in cm^3.

(c) Solve $\frac{4}{3}\pi r^3 = 120$ for $0 \le r \le 5$: $r = 3.059831\ldots$. Hence $V^{-1}(120) = 3.06$ cm (3 s.f.); this is the radius of an ornament with volume 120 cm^3.

Section 3: IB exam-style practice (continued)

37.

(a) $\{-2, 0, 3, 5\}$.

(b) $k \neq 7, 4, 1$, because outputs must not repeat.

(c) $(7, -2)$, $(4, 0)$, $(1, 3)$, $(-2, 5)$.

(d) 0.

38.

(a) $P(4.5) = 2.40 + 0.80(4.5) = €6.00$.

(b) $2.40 < P(m) \leq 14.40$, or $(2.40, 14.40]$ euros. The lower endpoint is excluded because $m = 0$ is excluded.

(c) $2.40 + 0.80m = 10.40$ gives $m = 10$, so $P^{-1}(10.40) = 10$ kg.

39.

(a) $h(-1) = 9 - 4 = 5$ and $h(3) = 9 - 4 = 5$.

(b) The graph reaches 9 at $x = 1$ and has lowest value 5 on the stated domain, so the range is $[5, 9]$.

(c) It is not one-to-one: for example, the output 5 comes from both $x = -1$ and $x = 3$. Reversing these pairs would assign two outputs to the input 5.

40.

(a) $\{0, 10, 20, 30, 40\}$ minutes.

(b) $\{0, 8, 17, 24\}$ km; repeated outputs appear only once in a set.

(c) Every allowed time has exactly one recorded distance.

(d) $d(20) = d(30) = 17$, so d is not one-to-one; reversing the table would assign both 20 and 30 to the input 17.

41.

(a) $Q(4) = 80(1 - e^{-1.2}) = 55.9044\ldots \approx 55.9$ C.

(b) Solve $80(1 - e^{-0.3t}) = 60$ using the GDC: $t = 4.620981\ldots$. Hence $Q^{-1}(60) = 4.62$ s (3 s.f.), the time required for the stored charge to reach 60 C.

(c) Q increases from $Q(0) = 0$ to $Q(10) = 80(1 - e^{-3}) = 76.0170\ldots$. Its range is $[0, 80(1 - e^{-3})]$ C, approximately $[0, 76.0]$ C.

42.

(a) $A(4) = 4(18 - 4) = 56$ cm^2.

(b) $0 < A(w) \leq 81$, or $(0, 81]$ in cm^2. Zero is excluded because $w > 0$; $A(9) = 81$.

(c) Solving $w(18 - w) = 65$ gives $w = 5$ or $w = 13$. Only 5 lies in the domain, so $A^{-1}(65) = 5$ cm.

(d) $13 \notin (0, 9]$ and cannot be the shorter side.

43.

(a) $C(120) = 0.06(120) + 4.50 = €11.70$; 120 minutes of calls gives a monthly charge of €11.70.

(b) $0.06t + 4.50 = 19.50$ gives $t = 250$, so $C^{-1}(19.50) = 250$ minutes.

(c) Domain of C^{-1}: $[4.50, 34.50]$ euros. Range of C^{-1}: $[0, 500]$ minutes.

44.

(a) $f(2) = \sqrt{9} = 3$.

(b) Domain of f^{-1}: $[0, 5]$; range of f^{-1}: $[-7, 18]$.

(c) $\sqrt{x + 7} = 4$ gives $x + 7 = 16$, so $f^{-1}(4) = 9$.

(d) (b, a).

Section 3: IB exam-style practice (continued)

45.

(a) $p^{-1}(3) = 0$ and $p(3) = 0$.

(b) $p^{-1}(5) = -1$, whereas $1/p(-1) = 1/5$. The inverse value is not a reciprocal.

(c) $\{-1, 0, 2, 3\}$.

(d) $x = 3$, since $p(3) = 0$ and division by zero is undefined.

46.

(a) $K(26.2) = 42.164708 \approx 42.2$ km (3 s.f.).

(b) Solve $1.60934m = 100$: $m = 62.137273\ldots$, so $K^{-1}(100) = 62.1$ miles (3 s.f.).

(c) $[0, 160.934]$ km.

(d) -5 is outside this range of K, so it is not an allowed input to K^{-1}.

47.

(a) $y = 0$ and $y = 4$.

(b) The same input $x = 1$ has two outputs, 0 and 4; a function requires exactly one output for each allowed input.

(c) The claim is false. Different inputs may share an output (a many-to-one function); what is forbidden is a single input having more than one output.

48.

(a) $g^{-1}(1) = 0$.

(b) $(-2, -3)$, $(1, 0)$ and $(4, 2)$.

(c) Domain: $[-2, 4]$; range: $[-3, 2]$.

(d) $y = x$.

49.

(a) $h(1) = 2 + 12 - 3 = 11$ m.

(b) Solve $2 + 12t - 3t^2 = 11$: $t = 1$ s or $t = 3$ s; both are in the domain.

(c) The GDC graph shows $2 \leq h(t) \leq 14$; the range is $[2, 14]$ metres.

(d) h is not one-to-one: for example, the height 11 m occurs at both 1 s and 3 s. Thus a height does not determine one unique time.

50.

(a) $f(3) = 6 + 2 = 8$.

(b) Since f increases continuously, its range runs from $f(0) = 1$ to $f(24) = 48 + 5 = 53$, so it is $[1, 53]$.

(c) Solve $2x + \sqrt{x + 1} = 30$ using the GDC on $[0, 24]$: $x = 13.121097\ldots$, so $f^{-1}(30) = 13.1$ (3 s.f.).

(d) Domain of f^{-1}: $[1, 53]$; range of f^{-1}: $[0, 24]$.

(e) It is strictly increasing and therefore one-to-one; every output comes from exactly one input.

Curriculum reference: IB Mathematics: Applications and Interpretation guide, SL 2.2 (first assessment 2021; August 2025 update). All questions are original Radford Mathematics practice.