Graphs of Functions

IB Mathematics: Applications and Interpretation SL

SL 2.3 | 50 questions with worked solution outlines and reference graphs

Learning goal	Produce clear, correctly labelled graphs from equations, given information and contexts; graph sums and differences.
Syllabus link	AI SL **2.3**: graphs and sketches; screen-to-paper transfer; technology for $f+g$ and $f-g$. Current course, first assessment 2021.
Structure	15 short response + 15 extended response + 20 original exam-style questions. Solutions start on page 13.
Technology	A graphical calculator is required. AI SL Paper 1 uses technology. Check angle mode where indicated.

Instructions. Use separate paper or graph paper where no grid is supplied. Show your working and label axes, units, curves and relevant points. Keep exact values unless a decimal is requested; use three significant figures where appropriate. Respect stated domains. A sketch still needs the correct shape and clear labels.

1. Short response

Questions 1–15

1. Draw or sketch?

Two instructions are given: *draw an accurate graph* and *sketch the graph*.

(a) Which instruction requires carefully plotted points and a consistent numerical scale?
(b) For a sketch, state two things that still need to be shown clearly.

2. Coordinates on a function graph

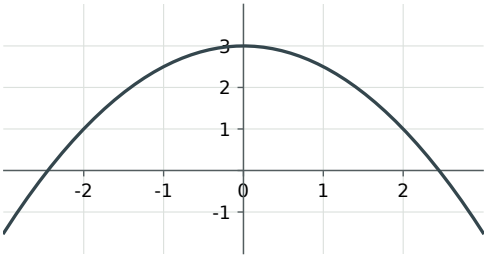
Let $f(x) = 2x - 3$.

(a) Write the coordinates of the points on $y = f(x)$ for $x = -1$, $x = 0$ and $x = 3$.
(b) Determine which of $P(2, 1)$, $Q(1, 2)$ and $R(-2, -7)$ lie on the graph.

3. Finish the graph labels

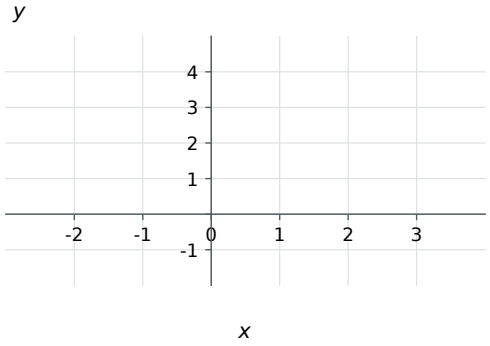
The curve shown is $y = f(x)$. Its highest point is $(0, 3)$.

(a) Label the horizontal axis x, the vertical axis y, and the curve $y = f(x)$.
(b) Mark the point $P(2, 1)$. Write the corresponding statement using function notation.



4. Draw a straight-line segment

Draw $y = x + 1$ for $-2 \leq x \leq 3$ on the grid. Mark the two endpoints and label the line.



5. A table before a curve

For $y = x^2 - 4$:

x	-3	-2	-1	0	1	2	3
y							

(a) Complete the table.
(b) Explain why a smooth curve is needed when these points are used to draw the graph.

6. Choose a useful window

The graph of $y = (x - 12)^2 - 3$, for $8 \leq x \leq 16$, has minimum $(12, -3)$ and endpoints $(8, 13)$ and $(16, 13)$.

Window	$x_{\min}$	$x_{\max}$	$y_{\min}$	$y_{\max}$
A	-10	10	-10	10
B	7	17	-5	15
C	8	16	0	15

Choose the best window below. Explain why the other two are unsuitable.

7. Enter the intended function

The intended function is $f(x) = \dfrac{x+1}{x-2}$.

(a) Choose the correct calculator entry: `x+1/x-2`, `(x+1)/(x-2)`, or `x+1/(x-2)`.
(b) Explain the role of the two pairs of brackets.

8. From notation to points

It is known that $g(-2) = 5$ and $g(1) = -3$.

(a) Write two points that must be on $y = g(x)$.
(b) A student plots $(5, -2)$ instead of the first point. Explain the error.

9. Sum and difference entries

Let $f(x) = 2x + 1$ and $g(x) = x^2$.

(a) Write expressions for $h(x) = f(x) + g(x)$ and $k(x) = f(x) - g(x)$.
(b) Find $h(2)$ and $k(2)$.

10. Combine corresponding outputs

Complete the two blank rows.

x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4
$g(x)$	-1	0	1	2	3
$f(x) + g(x)$					
$f(x) - g(x)$					

11. A point on a combined graph

The point $(3, 7)$ lies on $y = f(x)$ and $(3, 2)$ lies on $y = g(x)$.

(a) Write the corresponding point on $y = f(x) + g(x)$.
(b) Write the corresponding point on $y = f(x) - g(x)$.

12. Subtracting a negative output

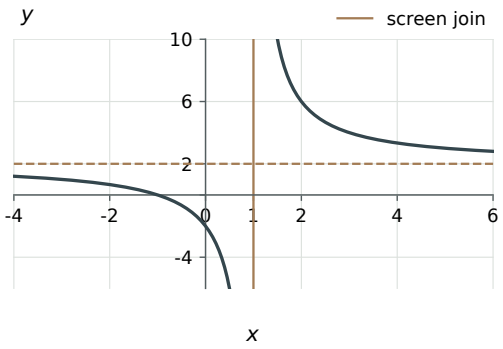
At $x = 2$, $f(2) = 3$ and $g(2) = -5$.

(a) Find $(f - g)(2)$ and $(g - f)(2)$.
(b) A student says that the graphs of $f - g$ and $g - f$ must be identical. Use your answers to explain why this is false.

13. A misleading screen join

The function $y = \dfrac{4}{x-1} + 2$ is undefined at $x = 1$. Its asymptotes are $x = 1$ and $y = 2$. A crude screen display appears to join the two branches with the vertical segment shown.

(a) Explain why that segment must not appear as part of the curve in a paper sketch.
(b) How should the line $x = 1$ be shown instead?



14. Axes in context

The temperature T of a drink, in °C, is measured t minutes after it is poured. The values are $T(0) = 80$, $T(5) = 62$ and $T(10) = 48$.

(a) Write suitable labels, including units, for both axes.
(b) Write the coordinates of the point representing the temperature after five minutes.

15. Window versus domain

A calculator displays $y = (x-2)^2 + 1$ in the window $0 \le x \le 4$, $0 \le y \le 6$. No domain restriction was given for the function.

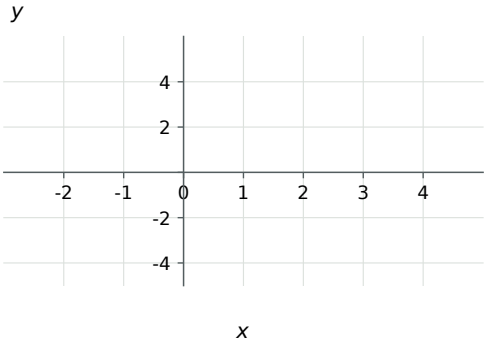
(a) Explain why the displayed horizontal interval is not the full domain.
(b) State the full domain.

2. Extended response — Questions 16–30

16. Transfer a quadratic graph

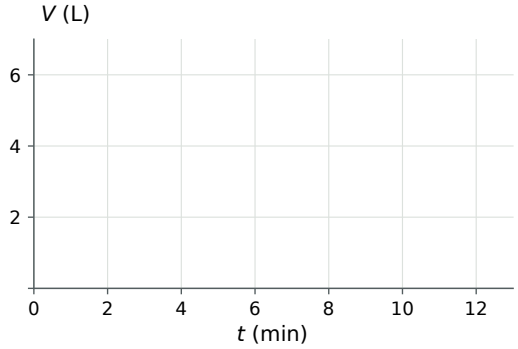
Use technology to graph $f(x) = (x-1)^2 - 4$ for $-2 \le x \le 4$. The vertex is $(1, -4)$; the intercepts are $(-1, 0)$, $(3, 0)$ and $(0, -3)$; the endpoints are $(-2, 5)$ and $(4, 5)$.

(a) Sketch the graph on the grid, labelling the curve and all the supplied points.
(b) Explain why the curve should not continue beyond the endpoints.



17. Sketch from a filling story

An empty tank fills at a constant rate of 1 litre per minute for six minutes. Its volume then stays constant for two minutes. It then empties at a constant rate of 1.5 litres per minute until empty.



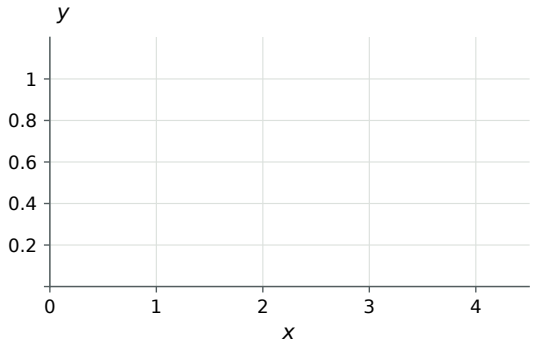
(a) Sketch volume V (litres) against time t (minutes), from the start until the tank is empty. Label every point where the behaviour changes.

(b) Explain what the horizontal part of your graph means.

18. An accurate curved graph

Let $f(x) = \dfrac{1}{x+1}$ for $0 \le x \le 4$.

x	0	1	2	3	4
$f(x)$					



(a) Complete the table, giving exact values.

(b) Draw an accurate graph on the grid, using the points in the table and a smooth curve.

19. Preserve the shape of a cubic

Use technology to graph $y = x^3 - 3x$ on $-2.5 \le x \le 2.5$. You are given the turning points $(-1, 2)$ and $(1, -2)$ and the three intercepts $(-\sqrt{3}, 0)$, $(0, 0)$ and $(\sqrt{3}, 0)$.

(a) Sketch the graph on separate axes, labelling the five supplied points.

(b) State a suitable vertical window that shows the entire specified graph.

20. Choose a window, then sketch

Let $f(x) = \sqrt{x+2}$ for $-2 \le x \le 7$. Four points are $(-2, 0)$, $(-1, 1)$, $(2, 2)$ and $(7, 3)$.

(a) Compare window A: $[-10, 10] \times [-10, 10]$ with window B: $[-3, 8] \times [-0.5, 3.5]$. Which makes the shape easier to see? Explain.

(b) Use technology, then sketch and label the four supplied points.

21. Distance from home

A cyclist leaves home and reaches a point 3 km away after 6 minutes, travelling at constant speed. The cyclist rests there until $t = 10$, then returns home at constant speed, arriving at $t = 16$.

(a) Sketch distance from home d (km) against t (minutes). Label the points where the behaviour changes.
(b) Explain why the final segment slopes down even though the cyclist is moving.

22. Graph a sum and a difference

Let $f(x) = x^2$ and $g(x) = 2x + 1$, for $-2 \le x \le 3$.

(a) Write calculator-ready expressions for $h = f + g$ and $k = f - g$.
(b) Graph both on the same axes. Sketch and label each curve, using the supplied points $(-1, 0)$, $(0, 1)$, $(3, 16)$ on h and $(1, -2)$, $(0, -1)$, $(3, 2)$ on k.

23. Remove a background signal

A measured signal is $f(t) = 8(0.7)^t + 2$ and a constant background is $g(t) = 2$, for $0 \le t \le 8$. Define the corrected signal by $s(t) = f(t) - g(t)$.

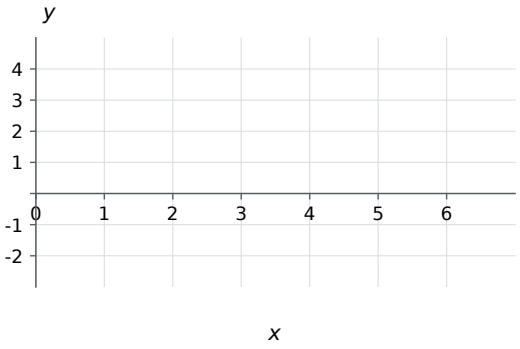
(a) Write an expression for $s(t)$. Use technology to find $s(0)$, $s(2)$, $s(4)$ and $s(8)$.
(b) Sketch s against t, labelling the first and last points. Explain what has been removed.

24. Combine piecewise linear graphs

The values below define f and g for $0 \le x \le 6$. Each function is linear between consecutive listed points.

x	0	2	4	6
$f(x)$	1	5	5	1
$g(x)$	3	3	1	1
$h(x)$				
$k(x)$				

(a) Complete rows for $h = f + g$ and $k = f - g$.
(b) Draw the graph of k on the grid, joining consecutive points with straight segments.



25. Use the common domain

The functions are $f(x) = \sqrt{x}$ for $0 \leq x \leq 9$ and $g(x) = 6 - x$ for $2 \leq x \leq 7$. Let $h = f + g$.

(a) State the domain on which h can be graphed.
(b) Use technology to sketch h on that domain. Label its endpoints and the point with $x = 4$, giving non-exact coordinates to three significant figures.

26. Check the angle setting

The height of a marker is $H(t) = 3 + 2\sin(90t)$ metres, for $0 \leq t \leq 8$ seconds. The angle $90t$ is in degrees. Supplied points include $(0, 3)$, $(1, 5)$, $(2, 3)$, $(3, 1)$ and $(4, 3)$; this pattern repeats.

(a) State the required angle mode. Use technology to sketch H over the stated interval, showing both complete cycles.
(b) Label the endpoints and all highest and lowest points.

27. Discrete sales

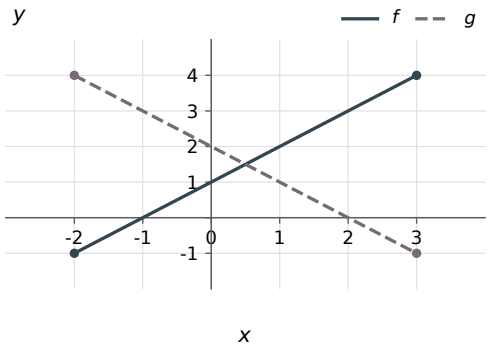
A club sells n badges, where n is a whole number and $0 \leq n \leq 20$. Revenue is $R(n) = 15n$ euros and cost is $C(n) = 60 + 3n$ euros.

(a) Write the profit function $P(n) = R(n) - C(n)$.
(b) Sketch its graph, labelling the supplied points $(0, -60)$, $(5, 0)$ and $(20, 180)$. Should the graph be a continuous line or separate points? Explain.

28. Combine two supplied graphs

The graphs shown are $f(x) = x + 1$ and $g(x) = 2 - x$, both for $-2 \leq x \leq 3$.

(a) Use values at $x = -2, 0, 3$ to sketch $h = f + g$ on separate axes.
(b) On another pair of axes, sketch $k = f - g$, labelling the three corresponding points.



29. Correct a difference graph

For $f(x) = x^2$ and $g(x) = 3x + 4$, a student enters $x^2 - 3x + 4$ to graph $f - g$.

(a) Explain the sign error and write the correct expression.
(b) Use the value at $x = 0$ to check your correction.
(c) Use technology to sketch the correct graph on $-2 \le x \le 5$. You are given the vertex $(1.5, -6.25)$ and intercepts $(-1, 0)$, $(4, 0)$ and $(0, -4)$. Label them.

30. An unfamiliar function

Let $f(x) = \dfrac{x^2 + 1}{x^2 + 4}$ for $-4 \le x \le 4$.

x	-4	-2	0	2	4
$f(x)$					

(a) Use technology to complete the table to three significant figures.
(b) Choose a useful window and sketch the graph, labelling the endpoints and $(0, f(0))$.
(c) Why is a vertical window from -10 to 10 less helpful here?

3. IB exam-style practice

Questions 31–50 | GDC required

Original short-response questions, modelled on AI SL Paper 1. The bracketed numbers are marks.

31. Transferring a graph with two low points

[Maximum mark: 5]

Consider $f(x) = (x^2 - 1)^2$, for $-1.5 \le x \le 1.5$. Its graph has low points $(-1, 0)$ and $(1, 0)$, a high point between them at $(0, 1)$, and endpoints $(-1.5, 1.5625)$ and $(1.5, 1.5625)$.

(a) Use technology to graph f. Sketch the graph on paper, labelling the given points. [4]
(b) Explain why drawing a single U-shaped curve would be incorrect. [1]

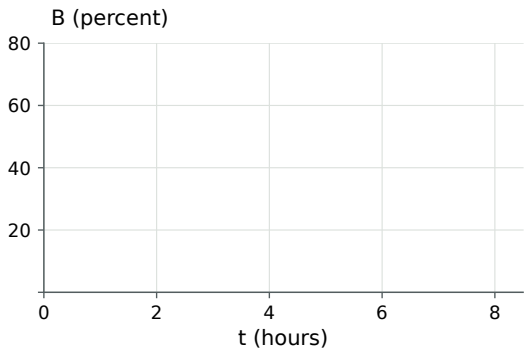
32. An accurately drawn charge graph

[Maximum mark: 4]

The table gives a battery charge B, as a percentage, at time t hours. Between successive readings, assume that B changes at a constant rate.

t (hours)	0	2	4	6	8
B (%)	10	40	70	70	20

(a) Draw the graph of B against t accurately, using suitable scales. [3]
(b) Explain why a single smooth curve is inappropriate for this model. [1]



33. A window for production data [Maximum mark: 6]

A factory model gives $P(t) = 500 + 80t + 20t^2$, for $0 \le t \le 5$, where P is the cumulative number of items produced and t is time in hours. A student uses $-10 \le x \le 10$, $-10 \le y \le 10$ and sees no graph.

(a) Explain why no part of this model appears in the window, and state a suitable replacement window. [2]
(b) Calculate $P(2)$. [1]
(c) Use technology to sketch the model. Label $(0, 500)$, $(5, 1400)$ and your point from part (b). [3]

34. An unfamiliar function [Maximum mark: 6]

Let $r(x) = \dfrac{1}{1+x^2}$, for $-4 \le x \le 4$. The graph passes through $(0, 1)$, $(-1, \frac{1}{2})$ and $(1, \frac{1}{2})$.

(a) Calculate $r(-2)$ and $r(4)$. [2]
(b) Use technology to sketch $y = r(x)$. Label the three given points and both endpoints. [3]
(c) A student uses $-100 \le y \le 100$. Explain why a smaller vertical interval is more useful here. [1]

35. A response that levels off [Maximum mark: 6]

A device response is modelled by $S(t) = 10(1 - 0.5^t)$, for $0 \le t \le 6$. If the same formula were continued, its graph would approach the line $S = 10$.

(a) Calculate $S(0)$, $S(2)$ and $S(6)$; round $S(6)$ to three significant figures. [2]
(b) Use technology to sketch S over the stated domain. Label the endpoints and draw the given line $S = 10$ dashed. [3]
(c) Explain why the sketch of the stated model should not be extended past $t = 6$. [1]

36. A calculator entry error [Maximum mark: 6]

The intended function is $f(x) = \dfrac{12}{x+2}$, for $0 \le x \le 10$. A student enters `12/x+2` instead of `12/(x+2)`.

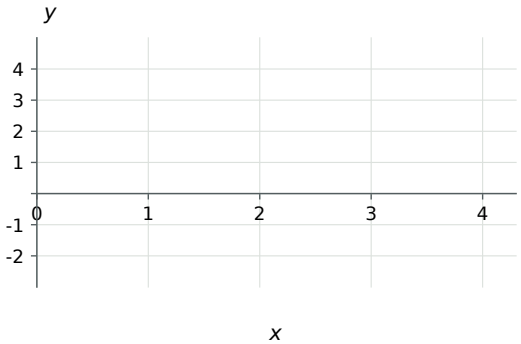
(a) Calculate the value produced by each entry when $x = 2$. [2]
(b) Use technology to sketch the correct graph, labelling its endpoints and the point with $x = 2$. [3]
(c) State what happens to the incorrect expression at $x = 0$. [1]

37. Sums and differences from a table [Maximum mark: 8]

The functions f and g are linear between successive values in the table. Let $h = f + g$ and $d = f - g$, for $0 \le x \le 4$.

x	0	1	2	3	4
$f(x)$	1	2	1	0	1
$g(x)$	3	1	-1	1	3

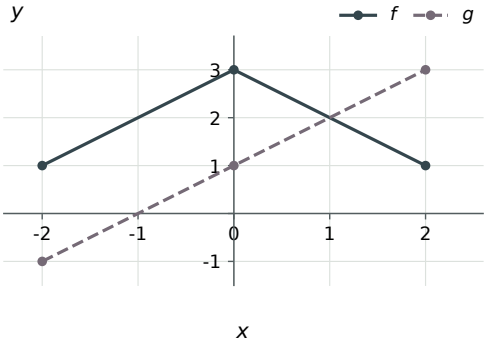
(a) Complete a table of values for h and d at each listed value of x. [4]
(b) On the same axes, draw the graphs of h and d, labelling each graph clearly. [4]



38. Subtracting two supplied graphs

[Maximum mark: 5]

The diagram shows piecewise linear graphs of f and g, for $-2 \le x \le 2$. Let $d(x) = f(x) - g(x)$.



(a) Write down $d(-2)$, $d(0)$ and $d(2)$. [2]

(b) Draw the graph of d. [2]

(c) Explain why d is constant on $-2 \le x \le 0$. [1]

39. A common domain for a sum

[Maximum mark: 5]

The functions $f(x) = \sqrt{9 - x}$ and $g(x) = x - 1$ are defined for $0 \le x \le 9$ and $2 \le x \le 8$, respectively. Let $h = f + g$. On its domain, h passes through approximately $(2, 3.65)$, $(5, 6)$ and $(8, 8)$.

(a) State the domain of h. [1]

(b) Write down an expression for $h(x)$. [1]

(c) Use technology to sketch h, showing only its domain and labelling the given points. [3]

40. Reversing a subtraction

[Maximum mark: 7]

Let $f(x) = 3x + 2$ and $g(x) = x^2 + 2$, for $-1 \le x \le 4$. Write $h = f - g$ and $k = g - f$. The graph of h has endpoints $(-1, -4)$ and $(4, -4)$, zeros at $x = 0$ and $x = 3$, and a high point $(1.5, 2.25)$.

(a) Show that $k(x) = -h(x)$. [2]

(b) Use technology to sketch h and k on the same axes, labelling the supplied features and their corresponding points on k. [4]

(c) Describe the relationship between the two graphs. [1]

41. Adding at the same input

[Maximum mark: 5]

Two stalls have sales-rate models $A(t) = 40 + 6t$ and $B(t) = 100 - 4t$, for $0 \le t \le 10$, where t is time in hours and each output is in euros per hour. Their combined sales rate is $T = A + B$.

(a) Find $T(3)$. [2]

(b) Use technology to draw the graph of T over the stated domain, labelling both endpoints. [2]

(c) A student adds the points $(3, A(3))$ and $(3, B(3))$ to obtain $(6, 146)$. Explain the error. [1]

42. Comparing two journeys [Maximum mark: 6]

Two hikers start at the same place and move along the same route. Their distances from the start, in kilometres, are $A(t) = 2t$ and $B(t) = \frac{1}{4}t^2$, for $0 \le t \le 8$ hours. Let $D = A - B$. You are given $D(0) = D(8) = 0$ and the highest point $(4, 4)$ on its graph.

(a) Write down an expression for $D(t)$. [1]
(b) Use technology to sketch D, labelling the supplied features. [3]
(c) Calculate $D(6)$ and interpret this value in context. [2]

43. A negative output with a useful meaning [Maximum mark: 6]

Water enters a tank at $I(t) = 8 - t$ litres per minute and leaves at $O(t) = 2 + 0.5t$ litres per minute, for $0 \le t \le 6$. The net inflow rate is $N = I - O$. You are given that its graph crosses the horizontal axis at $(4, 0)$.

(a) Find an expression for $N(t)$. [2]
(b) Draw the graph of N for the stated domain, labelling its endpoints and the given crossing. [3]
(c) Explain the meaning of $N(6) = -3$. [1]

44. The sum of two square-root graphs [Maximum mark: 6]

Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{4 - x}$, for $0 \le x \le 4$, and let $h = f + g$.

(a) Use technology to construct a table for $h(0)$, $h(1)$, $h(2)$, $h(3)$ and $h(4)$, to three significant figures where necessary. [2]
(b) Use technology to sketch h, labelling those five points. [3]
(c) Explain why the graph of h is not the straight line segment joining its endpoints. [1]

45. Checking the scale of a difference [Maximum mark: 6]

Use technology to explore the unfamiliar functions $f(x) = \ln(x+1)$ and $g(x) = 0.25x$, for $0 \le x \le 7$, and let $h = f - g$. The graph of h has a highest point at approximately $(3, 0.636)$. Use the `ln` key for the natural logarithm.

(a) Calculate $h(0)$, $h(1)$ and $h(7)$, to three significant figures where necessary. [2]
(b) State a suitable graphing window and use technology to sketch h, labelling the given highest point and your three calculated points. [4]

46. A table that misses the shape [Maximum mark: 6]

Let $f(x) = \sin(180x^\circ)$, for $0 \le x \le 4$. A student evaluates f only at the integers $0, 1, 2, 3, 4$, obtains zero each time, and draws the horizontal line $y = 0$. Set your calculator to degree mode.

(a) Find $f(0.5)$ and $f(1.5)$. [2]
(b) Use technology to sketch the correct graph, showing the full interval and labelling its highest and lowest points. [3]
(c) Explain why the student's table is insufficient. [1]

47. When points should remain separate [Maximum mark: 6]

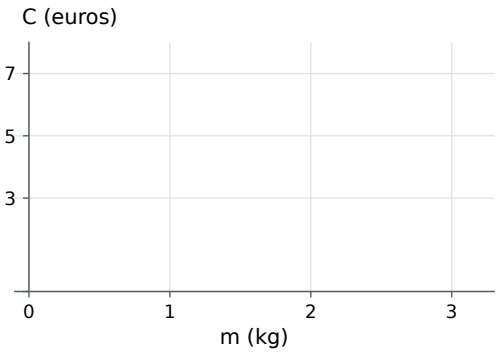
A service charges a monthly fee of €4 plus €2.50 per download. For a customer who makes at most six downloads, the cost model is $C(n) = 4 + 2.5n$, with $n \in \{0, 1, 2, 3, 4, 5, 6\}$.

(a) Calculate $C(0)$, $C(2)$ and $C(6)$. [2]
(b) Draw the graph of this model, showing all allowed inputs. [3]
(c) Explain why the points should not be joined. [1]

48. A graph with jumps [Maximum mark: 6]

A parcel service charges €3 for $0 < m \leq 1$ kg, €5 for $1 < m \leq 2$ kg, and €7 for $2 < m \leq 3$ kg. Let $C(m)$ be the charge in euros.

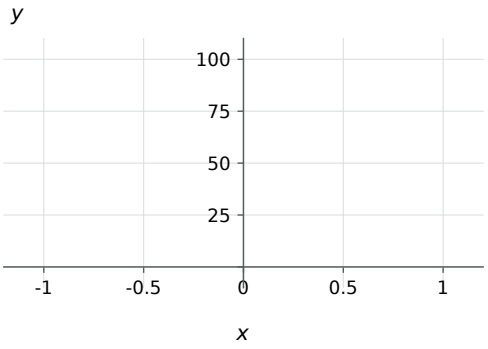
(a) Draw the graph of C against m, using open and closed circles to show the interval endpoints. [5]
(b) Write down $C(2)$. [1]



49. Choosing a useful horizontal scale [Maximum mark: 6]

The function $f(x) = 100x^2$ is defined for $-1 \leq x \leq 1$. Its graph passes through $(-1, 100)$, $(-0.5, 25)$, $(0, 0)$, $(0.5, 25)$ and $(1, 100)$. Two possible windows are A: $-1.2 \leq x \leq 1.2$, $-10 \leq y \leq 110$; and B: $-10 \leq x \leq 10$, $-10 \leq y \leq 110$.

(a) State which window is more useful and justify your choice. [2]
(b) Use technology to sketch the graph over its stated domain, labelling all five given points. [4]



50. Adding and subtracting varying functions [Maximum mark: 8]

Let $f(t) = 4 + 2\sin(60t°)$ and $g(t) = 1 + 2\sin(60t°)$, for $0 \leq t \leq 6$. Set your calculator to degree mode. Let $s = f + g$ and $d = f - g$.

(a) Write down expressions for $s(t)$ and $d(t)$. [2]
(b) Use technology to sketch s, labelling $(0, 5)$, $(1.5, 9)$, $(3, 5)$, $(4.5, 1)$ and $(6, 5)$. [3]
(c) On the same axes, draw the graph of d. [2]
(d) Explain why the difference is constant even though both original functions vary. [1]

Answer key and solution outlines

Reference graphs included

Sketches may vary in scale. Accept the correct overall shape, correct domain and clear labels. Where a drawing is requested, plotted points and scale must also be accurate. Decimal graph labels are rounded where needed.

Section 1: Short response

1. Draw or sketch?

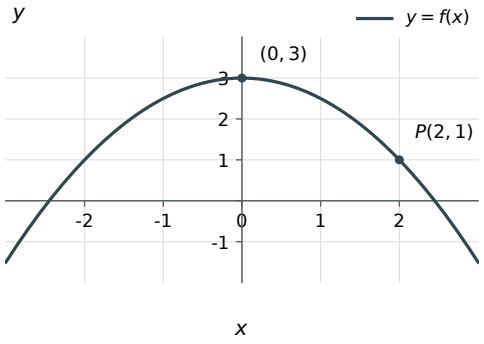
(a) *Draw* requires accurate plotting and a consistent scale.
(b) A sketch needs the correct overall shape and clearly labelled axes and relevant features (such as given intercepts or endpoints).

2. Coordinates on a function graph

(a) $(-1,-5)$, $(0,-3)$, $(3,3)$. Each point is $(x, f(x))$.
(b) P and R: $f(2)=1$ and $f(-2)=-7$. Q is not on the graph because $f(1)=-1$, not 2.

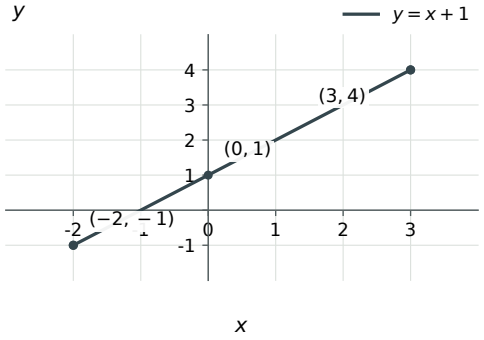
3. Finish the graph labels

(a) Horizontal axis x; vertical axis y; curve $y=f(x)$.
(b) Mark $(2,1)$ on the curve; $f(2)=1$.



4. Draw a straight-line segment

Plot $(-2,-1)$ and $(3,4)$ and join them with a straight segment. It also passes through $(0,1)$. Use filled endpoints because both ends are included.



5. A table before a curve

(a) The y-values are $5, 0, -3, -4, -3, 0, 5$, in order.
(b) The function is quadratic: straight joins would create corners and misrepresent the curved graph between the plotted points.

6. Choose a useful window

Window B shows the complete specified graph and leaves some margin. A misses the minimum and right-hand part because $x \leq 10$. C hides the part below the x-axis.

7. Enter the intended function

(a) `(x+1)/(x-2)`.
(b) They make the whole numerator $x+1$ divide by the whole denominator $x-2$. The other entries represent different functions.

8. From notation to points

(a) $(-2,5)$ and $(1,-3)$.
(b) The input is the horizontal coordinate. The student has swapped the input and output.

9. Sum and difference entries

(a) $h(x) = 2x + 1 + x^2$; $k(x) = 2x + 1 - x^2$.
(b) $h(2) = 5 + 4 = 9$; $k(2) = 5 - 4 = 1$.

10. Combine corresponding outputs

$f + g$: $3, 1, 1, 3, 7$.
$f - g$: $5, 1, -1, -1, 1$. Add or subtract values in the same column, so the input x stays fixed.

11. A point on a combined graph

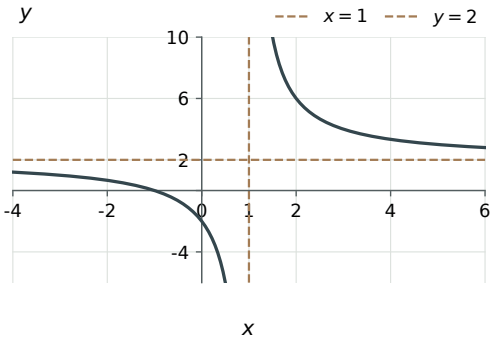
(a) $(3, 9)$ because $7 + 2 = 9$.
(b) $(3, 5)$ because $7 - 2 = 5$. The horizontal coordinate remains 3.

12. Subtracting a negative output

(a) $(f - g)(2) = 3 - (-5) = 8$; $(g - f)(2) = -5 - 3 = -8$.
(b) At the same input 2, the outputs are different. Reversing the subtraction changes the sign.

13. A misleading screen join

(a) No point of the curve has $x = 1$. The solid joining segment is a display artefact, not part of the function.
(b) Show $x = 1$ as a dashed asymptote, with two separate branches approaching it. Also label the given horizontal asymptote $y = 2$.



14. Axes in context

(a) Horizontal: t (minutes). Vertical: T (°C).
(b) $(5, 62)$; its coordinates mean 5 minutes and 62°C.

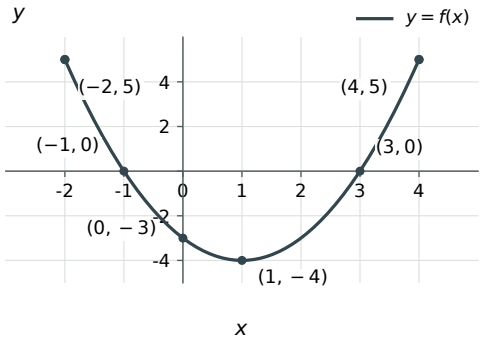
15. Window versus domain

(a) A window selects which part of a graph is visible. It does not restrict the function itself.
(b) All real x: $x \in \mathbb{R}$.

Section 2: Extended response

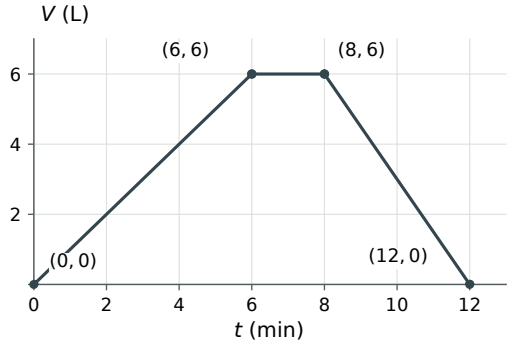
16. Transfer a quadratic graph

(a) An upward-opening parabola through all six supplied points; see the reference graph.
(b) The domain was explicitly restricted to $[-2, 4]$. Both endpoints are included, so use filled dots.



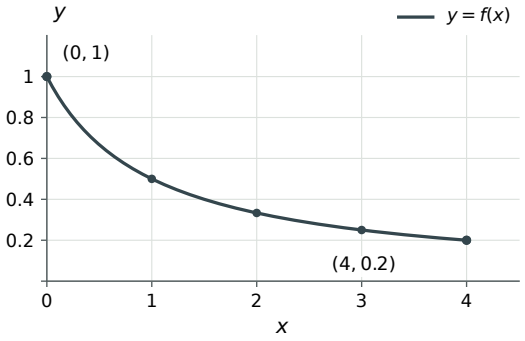
17. Sketch from a filling story

(a) Join $(0, 0)$, $(6, 6)$, $(8, 6)$ and $(12, 0)$ with straight segments. Emptying takes $6 \div 1.5 = 4$ minutes.
(b) From $t = 6$ to $t = 8$, the volume stays at 6 litres.



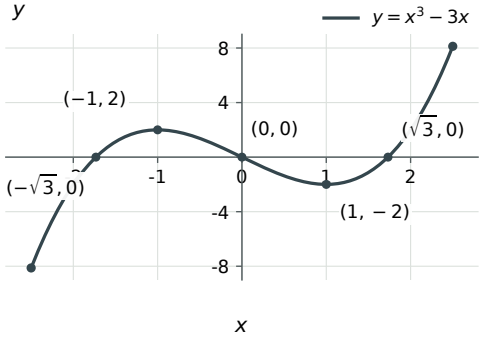
18. An accurate curved graph

(a) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$.
(b) The curve decreases smoothly through the five plotted points and gradually becomes less steep. See the reference graph.



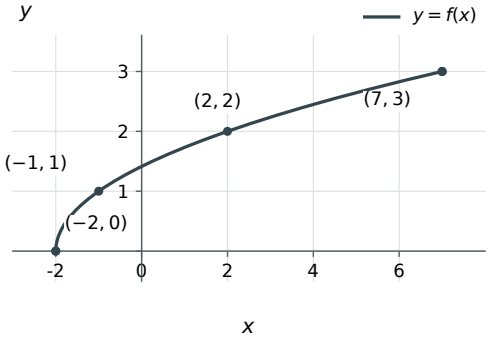
19. Preserve the shape of a cubic

(a) A smooth cubic passing through the supplied intercepts, with the given local maximum and minimum; see the reference graph.
(b) For example $-9 \leq y \leq 9$. The endpoint values are -8.125 and 8.125.



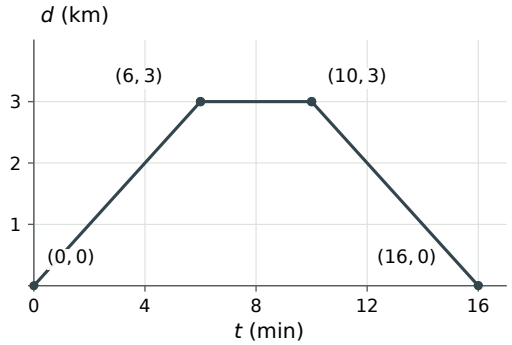
20. Choose a window, then sketch

(a) B uses more of the screen for the relevant curve. A contains the graph but compresses it into a small part of the display.
(b) An increasing curve, becoming less steep, from the filled endpoint $(-2, 0)$ to $(7, 3)$. See the reference graph.



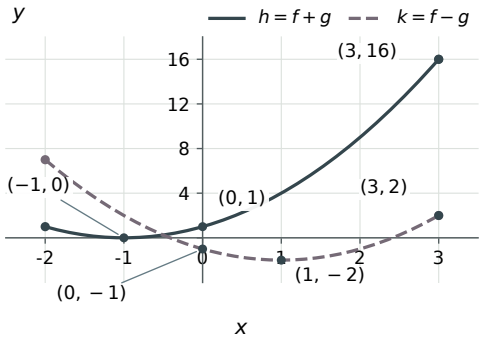
21. Distance from home

(a) Join $(0, 0)$, $(6, 3)$, $(10, 3)$ and $(16, 0)$ by straight segments.
(b) The graph represents distance *from home*, which decreases during the return journey; it does not represent total distance travelled.



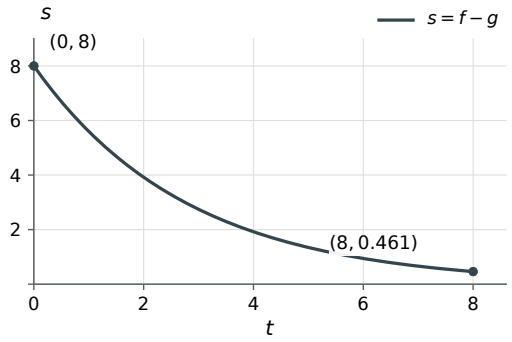
22. Graph a sum and a difference

(a) $h(x) = x^2 + 2x + 1$; $k(x) = x^2 - 2x - 1$. For the difference enter `x^2-(2*x+1)`.
(b) Both curves open upwards. Their vertices are among the supplied points: $(-1, 0)$ for h and $(1, -2)$ for k. See the reference graph.



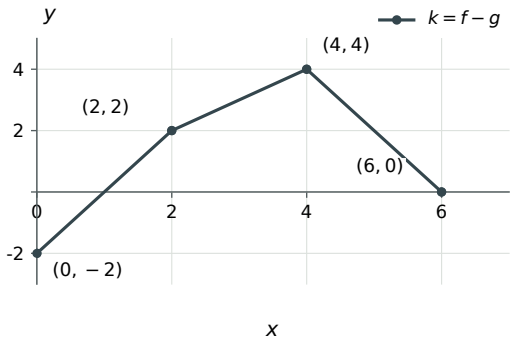
23. Remove a background signal

(a) $s(t) = 8(0.7)^t$. The values are 8, 3.92, 1.9208 and 0.46118408 (approximately 1.92 and 0.461 where appropriate).
(b) A smooth decreasing curve from $(0, 8)$ to approximately $(8, 0.461)$. Subtracting 2 removes the constant background from every output.



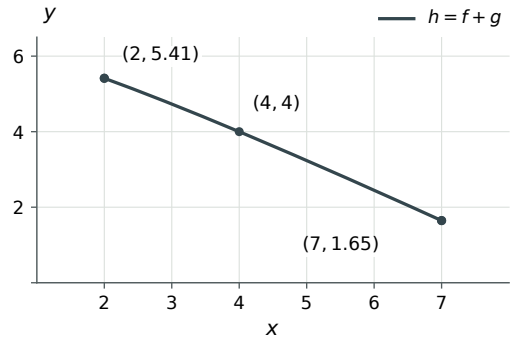
24. Combine piecewise linear graphs

(a) h: $4, 8, 6, 2$. k: $-2, 2, 4, 0$.
(b) Join $(0, -2)$, $(2, 2)$, $(4, 4)$ and $(6, 0)$ with straight segments. The difference is linear on each shared interval.



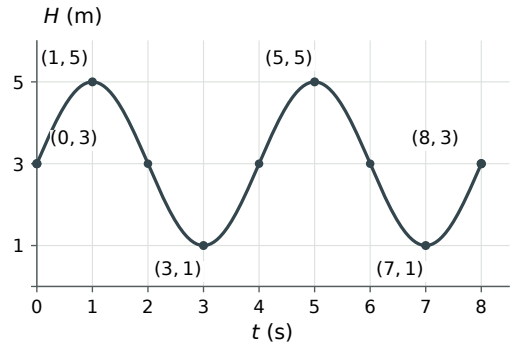
25. Use the common domain

(a) $2 \le x \le 7$, where both original functions are defined.
(b) $h(x) = \sqrt{x} + 6 - x$. Endpoints: $(2, 4 + \sqrt{2}) \approx (2, 5.41)$ and $(7, \sqrt{7} - 1) \approx (7, 1.65)$; also $(4, 4)$. The graph is smooth and decreasing on this interval.



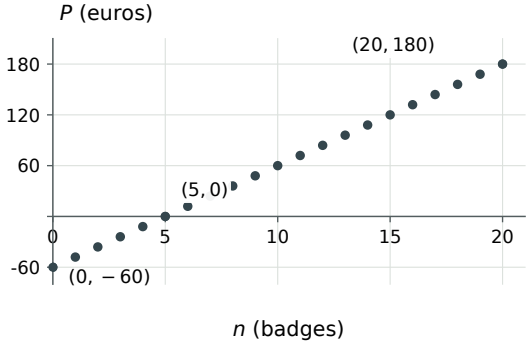
26. Check the angle setting

(a) Degree mode. A radian setting would graph a different function of t.
(b) A smooth wave through $(0, 3), (1, 5), (2, 3), (3, 1), (4, 3), (5, 5), (6, 3), (7, 1), (8, 3)$. See the reference graph.



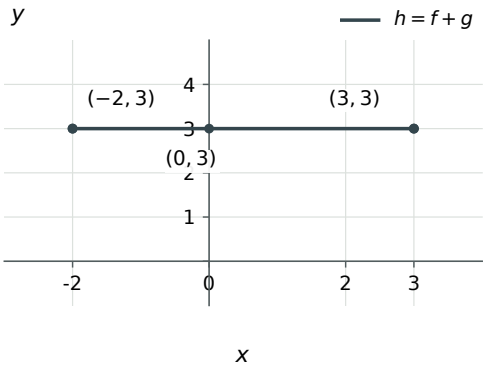
27. Discrete sales

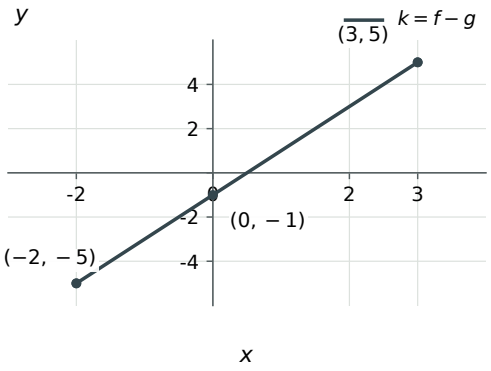
(a) $P(n) = 15n - (60 + 3n) = 12n - 60$.
(b) Plot separate points for integer inputs $0, 1, \ldots, 20$. Fractions of a badge are not possible, so a continuous segment would include inputs outside the domain.



28. Combine two supplied graphs

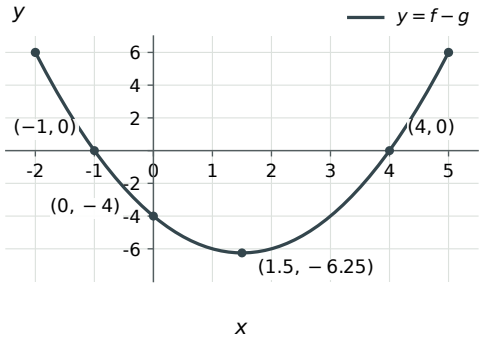
(a) $h(x) = 3$. Draw the horizontal segment from $(-2, 3)$ to $(3, 3)$, through $(0, 3)$.
(b) $k(x) = 2x - 1$. Draw the segment through $(-2, -5)$, $(0, -1)$ and $(3, 5)$. Both graphs use only $-2 \leq x \leq 3$.





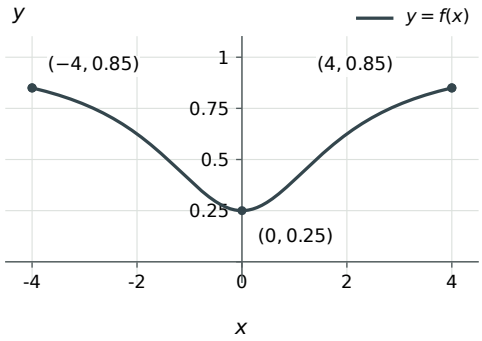
29. Correct a difference graph

(a) $f - g = x^2 - (3x + 4) = x^2 - 3x - 4$. The constant 4 must also be subtracted.
(b) $f(0) - g(0) = 0 - 4 = -4$, while the incorrect entry gives $+4$.
(c) An upward-opening parabola with the supplied vertex and intercepts, restricted to $[-2, 5]$. See the reference graph.



30. An unfamiliar function

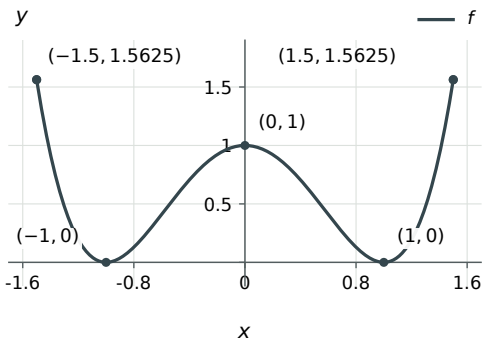
(a) $0.850, 0.625, 0.250, 0.625, 0.850$.
(b) For example $x \in [-4.5, 4.5]$, $y \in [-0.1, 1.1]$. Draw a smooth, symmetric trough through $(0, 0.25)$, with endpoints $(-4, 0.85)$ and $(4, 0.85)$.
(c) The relevant outputs lie between 0.25 and 0.85, so a very tall window makes the curve appear almost flat.



Section 3: IB exam-style practice

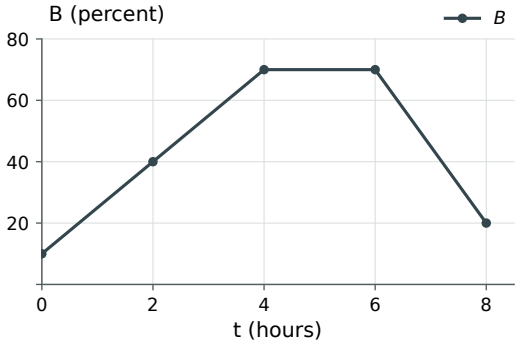
31. Transferring a graph with two low points

(a) The sketch is W-shaped, through both low points, the central high point and the two endpoints; see the reference graph. Label both axes and restrict the curve to the stated domain. [4]
(b) A single U-shaped curve has only one low point and cannot show the rise to $(0, 1)$ between the two low points. [1]



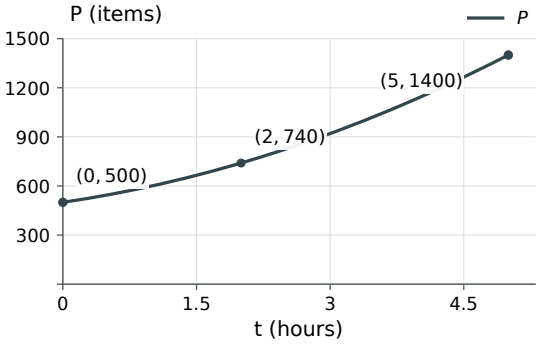
32. An accurately drawn charge graph

(a) Plot $(0, 10)$, $(2, 40)$, $(4, 70)$, $(6, 70)$ and $(8, 20)$ on linear, labelled scales, then join consecutive points with straight segments. [3]
(b) A constant rate on each interval gives a straight segment; the rate changes abruptly at $t = 4$ and $t = 6$, so corners are appropriate. [1]



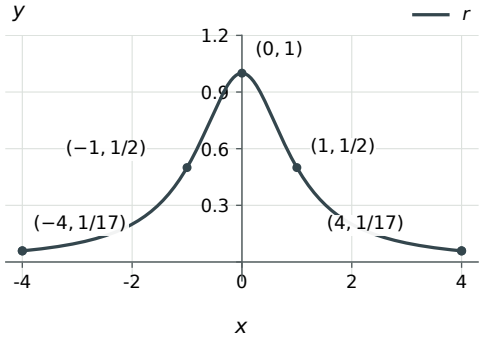
33. A window for production data

(a) All model values are at least 500, outside the displayed vertical interval. One suitable window is $0 \le t \le 5$, $0 \le P \le 1500$. Other suitable windows are acceptable. [2]
(b) $P(2) = 500 + 160 + 80 = 740$. [1]
(c) Draw an increasing curve that becomes progressively steeper, through $(0, 500)$, $(2, 740)$ and $(5, 1400)$; label time (hours) and cumulative production (items). [3]



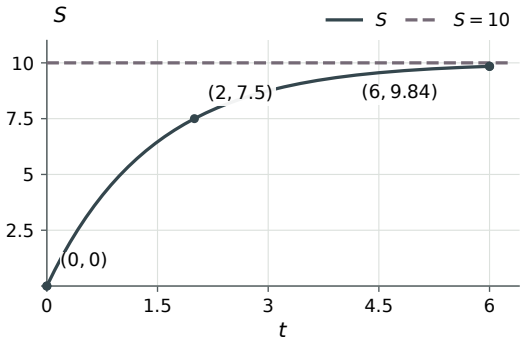
34. An unfamiliar function

(a) $r(-2) = 1/5 = 0.2$ and $r(4) = 1/17$. [2]
(b) The curve rises smoothly from $(-4, 1/17)$ to $(0, 1)$ and falls symmetrically to $(4, 1/17)$, passing through $(\pm 1, 1/2)$. [3]
(c) The values lie between $1/17$ and 1; a window such as $-0.1 \le y \le 1.2$ shows the shape clearly instead of compressing it near the horizontal axis. [1]



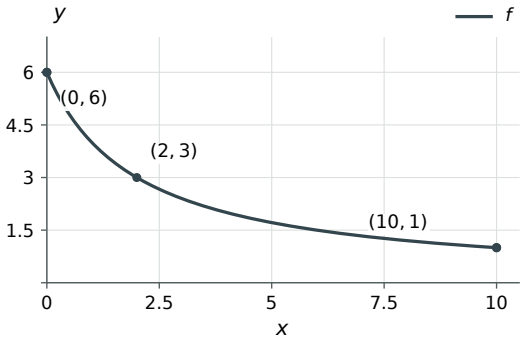
35. A response that levels off

(a) $S(0) = 0$, $S(2) = 7.5$, $S(6) = 9.84375 \approx 9.84$. [2]
(b) Draw an increasing curve that gradually flattens, from $(0, 0)$ to $(6, 9.84)$, through $(2, 7.5)$ and below the dashed line $S = 10$. [3]
(c) The model is specified only for $0 \leq t \leq 6$; its mathematical continuation is outside that stated domain. [1]



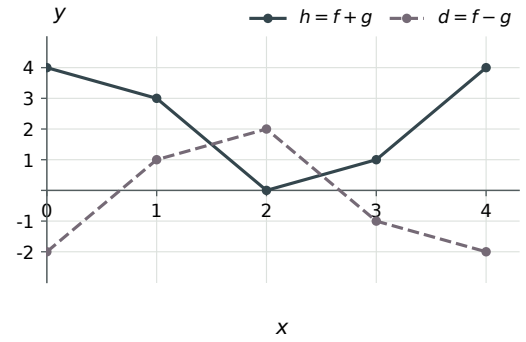
36. A calculator entry error

(a) Correct: $12/(2 + 2) = 3$. Incorrect: $12/2 + 2 = 8$. [2]
(b) The correct graph decreases smoothly from $(0, 6)$ through $(2, 3)$ to $(10, 1)$, becoming less steep. [3]
(c) The incorrect expression is undefined at $x = 0$, because it involves division by zero. [1]



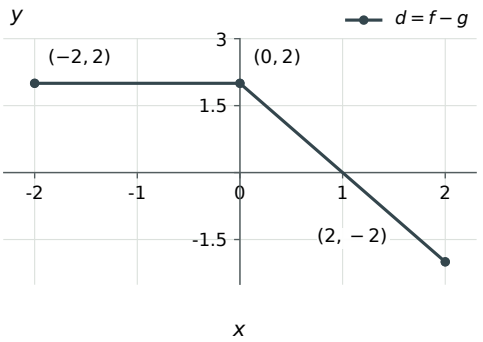
37. Sums and differences from a table

(a) At $x = 0, 1, 2, 3, 4$, the values of h are $4, 3, 0, 1, 4$ and those of d are $-2, 1, 2, -1, -2$. Add or subtract outputs at the same input. [4]
(b) Plot those points and join consecutive points with straight segments. Use solid and dashed lines, or clear labels, to distinguish the two graphs. [4]



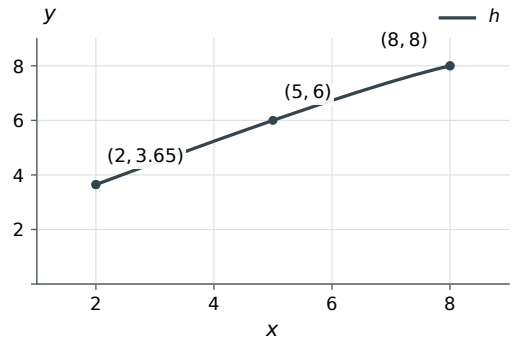
38. Subtracting two supplied graphs

(a) $d(-2) = 1 - (-1) = 2$, $d(0) = 3 - 1 = 2$, and $d(2) = 1 - 3 = -2$. [2]
(b) Join $(-2, 2)$ to $(0, 2)$ and then to $(2, -2)$ with straight segments. [2]
(c) On the first interval the two original lines are parallel, and their vertical separation is always 2. [1]



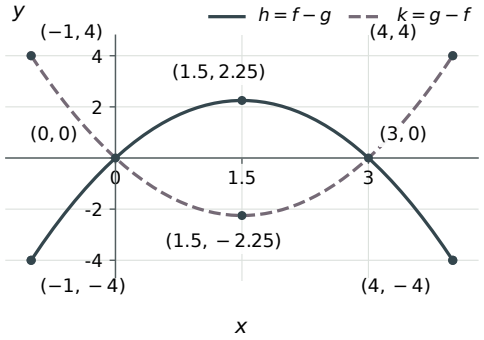
39. A common domain for a sum

(a) Both functions must be defined at the same input, so $2 \le x \le 8$. [1]
(b) $h(x) = \sqrt{9-x} + x - 1$. [1]
(c) Draw a smooth increasing curve from $(2, 3.65)$ through $(5, 6)$ to $(8, 8)$, with closed endpoints. It becomes less steep. Do not extend it outside $[2, 8]$. [3]



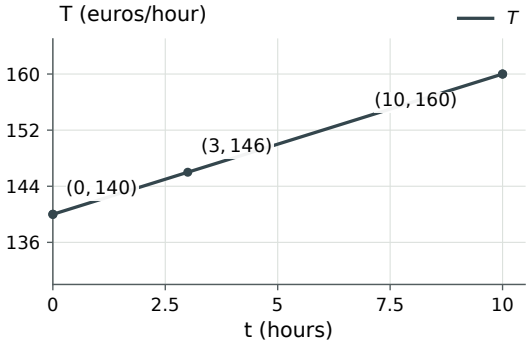
40. Reversing a subtraction

(a) $h(x) = (3x+2) - (x^2+2) = -x^2 + 3x$ and $k(x) = x^2 - 3x = -h(x)$. [2]
(b) h is a downward-opening parabola. The corresponding points on k are $(-1, 4)$, $(0, 0)$, $(1.5, -2.25)$, $(3, 0)$ and $(4, 4)$. [4]
(c) Each graph is the reflection of the other in the x-axis: each output changes sign. [1]



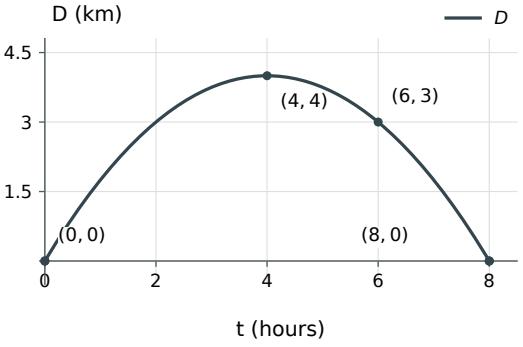
41. Adding at the same input

(a) $A(3) = 58$ and $B(3) = 88$, so $T(3) = 146$ euros per hour. [2]
(b) $T(t) = 140 + 2t$: draw the straight segment from $(0, 140)$ to $(10, 160)$. [2]
(c) Addition of functions adds their outputs at the same input; the time remains 3. The correct point is $(3, 146)$, not $(6, 146)$. [1]



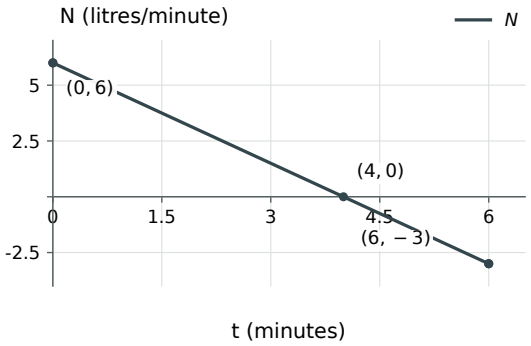
42. Comparing two journeys

(a) $D(t) = 2t - \frac{1}{4}t^2$. [1]
(b) Draw a downward-opening curve from $(0, 0)$ to $(8, 0)$ with high point $(4, 4)$. Label time (hours) and distance difference (km). [3]
(c) $D(6) = 12 - 9 = 3$: after 6 hours, hiker A is 3 km farther along the route than hiker B. [2]



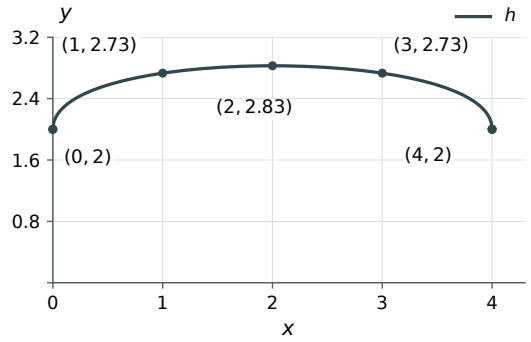
43. A negative output with a useful meaning

(a) $N(t) = (8 - t) - (2 + 0.5t) = 6 - 1.5t$. [2]
(b) Draw the straight segment from $(0, 6)$ through $(4, 0)$ to $(6, -3)$. [3]
(c) At 6 minutes, water leaves the tank 3 litres per minute faster than it enters; the amount of water is decreasing. This is a negative rate, not a negative volume. [1]



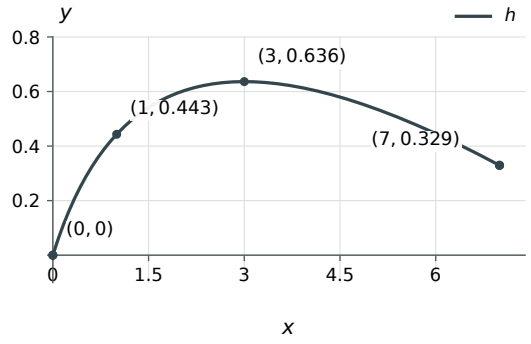
44. The sum of two square-root graphs

(a) The values are 2, $1 + \sqrt{3} \approx 2.73$, $2\sqrt{2} \approx 2.83$, $1 + \sqrt{3} \approx 2.73$, and 2. [2]
(b) Draw a smooth arch, symmetric about $x = 2$, through those points, with endpoints $(0, 2)$ and $(4, 2)$. [3]
(c) The line joining the endpoints is $y = 2$, but $h(2) = 2\sqrt{2} > 2$; the middle of the graph lies above that line. [1]



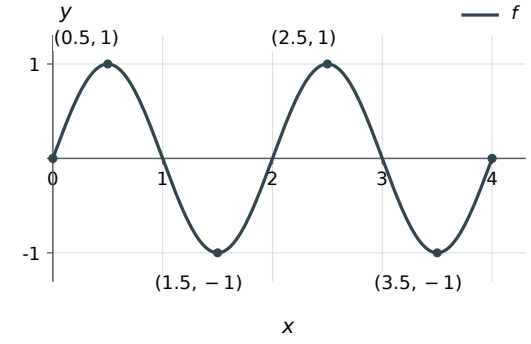
45. Checking the scale of a difference

(a) $h(0) = 0$, $h(1) = \ln 2 - 0.25 \approx 0.443$, and $h(7) = \ln 8 - 1.75 \approx 0.329$. [2]
(b) For example, $0 \le x \le 7$ and $-0.1 \le y \le 0.8$. The curve rises from $(0, 0)$ through $(1, 0.443)$ to $(3, 0.636)$, then falls to $(7, 0.329)$. It remains above the horizontal axis after $x = 0$. Other sensible windows are acceptable. [4]



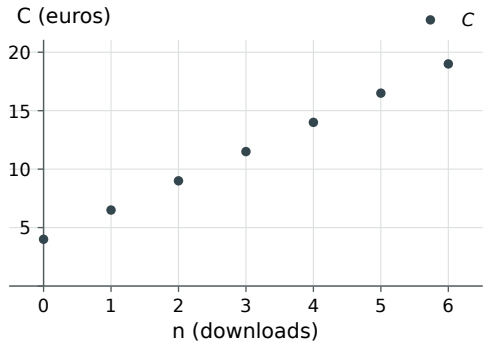
46. A table that misses the shape

(a) $f(0.5) = \sin 90^\circ = 1$ and $f(1.5) = \sin 270^\circ = -1$. [2]
(b) Draw two complete sine waves: zeros at $x = 0, 1, 2, 3, 4$, high points $(0.5, 1)$ and $(2.5, 1)$, and low points $(1.5, -1)$ and $(3.5, -1)$. [3]
(c) The samples occur only at zeros, so they miss all positive and negative values between them. [1]



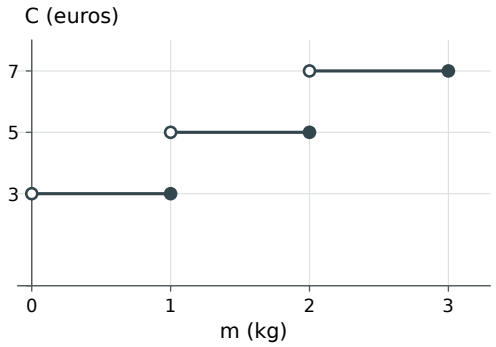
47. When points should remain separate

(a) $C(0) = 4$, $C(2) = 9$, $C(6) = 19$ euros. [2]
(b) Plot the seven separate points $(0, 4)$, $(1, 6.5)$, $(2, 9)$, $(3, 11.5)$, $(4, 14)$, $(5, 16.5)$ and $(6, 19)$. Label the input as number of downloads and the output as cost (euros). [3]
(c) Only whole numbers of downloads are allowed; joining the points would incorrectly include fractional inputs. [1]



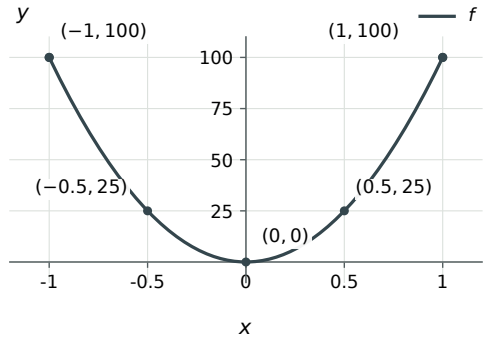
48. A graph with jumps

(a) Draw horizontal segments at $C = 3$ for $0 < m \leq 1$, $C = 5$ for $1 < m \leq 2$, and $C = 7$ for $2 < m \leq 3$. Use an open circle at each left endpoint and a closed circle at each right endpoint. Do not draw vertical joining segments. [5]
(b) $C(2) = 5$ euros: mass exactly 2 kg belongs to the middle interval. [1]



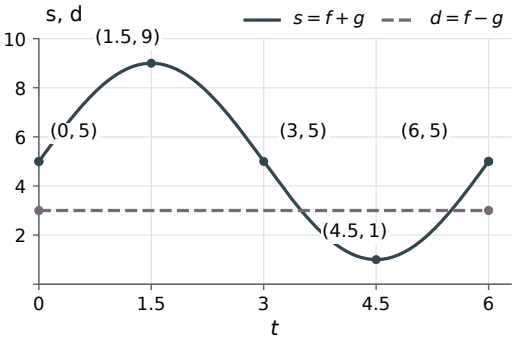
49. Choosing a useful horizontal scale

(a) Window A uses most of the screen width for the actual domain and leaves a small margin. Window B compresses the entire graph into a narrow strip near $x = 0$. [2]
(b) Draw a U-shaped parabola with low point $(0, 0)$ and the four other labelled points; show no curve outside $[-1, 1]$. The axes can have different scales, but their labels must make those scales clear. [4]



50. Adding and subtracting varying functions

(a) $s(t) = 5 + 4\sin(60t^\circ)$ and $d(t) = 3$. [2]
(b) Draw one complete sine wave through the five supplied points, starting at height 5 and rising initially. [3]
(c) Draw the horizontal segment $d = 3$ for $0 \leq t \leq 6$. [2]
(d) Both functions contain the same varying term $2\sin(60t^\circ)$; this term cancels when they are subtracted. [1]



Source alignment. IB *Mathematics: applications and interpretation guide*, SL 2.3 (p. 36); AI SL assessment guidance (pp. 74 and 76). All questions and diagrams are original Radford Mathematics material.