Normal distribution

IB Mathematics: Applications and Interpretation SL | Section 4.9

50 questions | Normal models, probabilities and inverse normal

Learning goal	Interpret normal models; calculate probabilities, percentiles and cutoffs.
Syllabus link	AI SL 4.9: properties, diagrams, the empirical rule, normal and inverse-normal calculations with given mean and standard deviation.
Structure	1–15 short response; 16–30 extended response; 31–40 Paper 1-style; 41–50 Paper 2-style; answer key at the back.
Technology	A GDC is allowed throughout, as in both AI examination papers. GDC marks numerical technology use; REASON marks questions normally solved by reasoning or the empirical rule.

Instructions. Work on separate paper and show your method. Use $X \sim \mathrm{N}(\mu, \sigma^2)$: the second parameter is the *variance*. Give exact answers where appropriate; otherwise use 3 significant figures unless instructed differently. Retain unrounded values in working; round estimated counts to the nearest whole number. Use 68%, 95% and 99.7% only when the empirical rule is requested.
GDC reminder. For normal Cdf, enter the lower bound, upper bound, mean and *standard deviation*. For inverse normal, use the left-tail area with the given mean and standard deviation. No z-scores are required. Sketch on separate paper unless axes are supplied.

1. Short response

1. REASON

Let $X \sim \mathrm{N}(42, 3.5^2)$. Write down:

(a) the mean;

(b) the standard deviation;

(c) the variance.

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2. REASON

A normal random variable has mean 68 and standard deviation 4.

(a) Write down its median and mode.

(b) State the equation of the vertical line of symmetry of its density curve.

(c) State the total area under the density curve.

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3. REASON

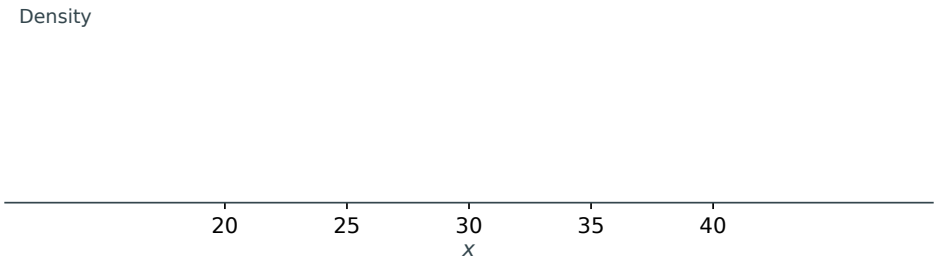
Let $A \sim \mathrm{N}(20, 4)$ and $B \sim \mathrm{N}(20, 16)$.

(a) Find the standard deviation of each variable.

(b) Which density curve is narrower and has a higher peak? Give a reason.

4. REASON

Let $X \sim \mathrm{N}(30, 5^2)$. Use the supplied axes.



(a) Sketch its density curve and shade the region representing $\mathrm{P}(X > 35)$.

(b) Write down $\mathrm{P}(X < 30)$.

5. REASON

Let $X \sim \mathrm{N}(100, 8^2)$. Use the 68–95–99.7 rule to estimate:

(a) $\mathrm{P}(92 < X < 108)$;

(b) $\mathrm{P}(84 < X < 116)$;

(c) $\mathrm{P}(X > 124)$.

6. REASON

Let $T \sim \mathrm{N}(50, 6^2)$. Use the 68–95–99.7 rule to estimate:

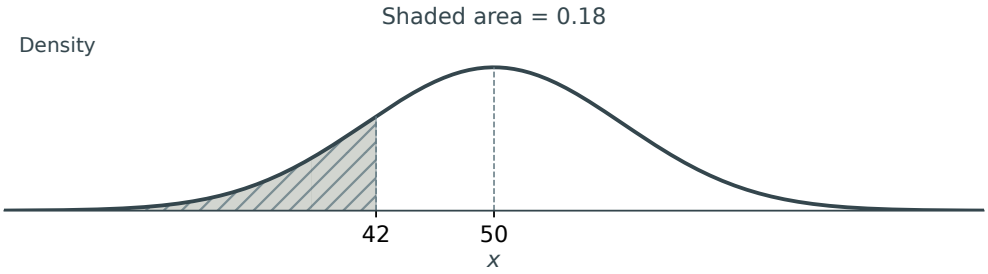
(a) $\mathrm{P}(50 < T < 56)$;

(b) $\mathrm{P}(56 < T < 62)$;

(c) $\mathrm{P}(T < 38)$.

7. REASON

X is normally distributed with mean 50. The diagram shows that $\mathrm{P}(X < 42) = 0.18$.



(a) Find $\mathrm{P}(X > 58)$.

(b) Find $\mathrm{P}(42 < X < 58)$.

8. REASON

Let $X \sim \text{N}(12, 2^2)$.

(a) Write down $\text{P}(X = 12)$.

(b) Explain why $\text{P}(X < 15) = \text{P}(X \leq 15)$.

9. GDC

Let $X \sim \text{N}(70, 9^2)$. Find $\text{P}(X < 76)$.

10. GDC

Let $X \sim \text{N}(120, 15^2)$. Find $\text{P}(X > 142)$.

11. GDC

Let $X \sim \text{N}(24, 3^2)$. Find $\text{P}(20 < X < 28)$.

12. GDC

Let $X \sim \text{N}(500, 18^2)$. Find the value of a such that $\text{P}(X < a) = 0.90$.

13. GDC

Let $T \sim \text{N}(45, 7^2)$. Find b such that $\text{P}(T > b) = 0.12$.

14. GDC

Let $X \sim \text{N}(60, 8^2)$. Find the endpoints of the central 90% interval.

15. REASON

Give one reason why a normal model might be unsuitable for each variable.

(a) The number of messages received in one hour.

(b) The waiting time for a bus when many waits are short but a few are very long.

2. Extended response

16. GDC

The mass M grams of a packet is modelled by $M \sim \mathrm{N}(500, 6^2)$.

(a) Find the probability that a packet has mass below 490 g.

(b) Find the probability that its mass is between 494 g and 508 g.

(c) In a batch of 2000 packets, estimate the number with mass below 490 g.

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17. GDC

A journey time T minutes is modelled by $T \sim \mathrm{N}(32, 4.5^2)$.

(a) Find $\mathrm{P}(T > 40)$.

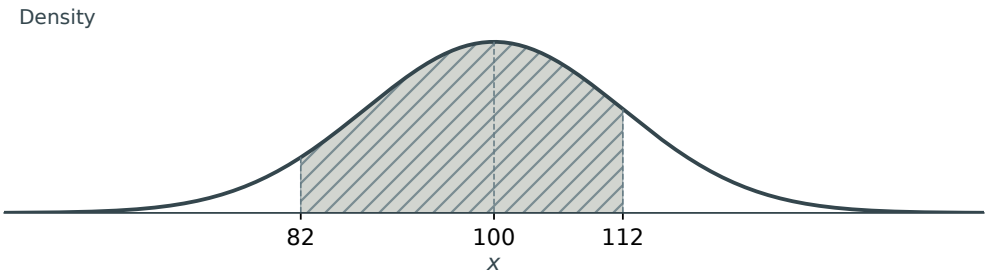
(b) Find the time exceeded by only 5% of journeys.

(c) A traveller allows 40 minutes. Does this give at least a 95% chance of arriving on time? Justify your answer.

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18. GDC

Let $X \sim \mathrm{N}(100, 12^2)$. The shaded region is shown below.



(a) Write the shaded area as a probability.

(b) Calculate this probability.

(c) Find the probability represented by the unshaded area under the curve.

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19. GDC

The heights H centimetres of plants are modelled by $H \sim \mathrm{N}(72, 8^2)$.

(a) Find the lower quartile and the upper quartile.

(b) Hence find the interquartile range.

(c) Interpret the interquartile range in this context.

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20. REASON

Test scores are modelled by $S \sim \mathrm{N}(64, 10^2)$. There are 800 candidates. Use the 68–95–99.7 rule.

(a) Estimate the number with scores between 54 and 74.

(b) Estimate the number with scores above 84.

(c) Explain why the model does not guarantee these exact counts.

21. GDC

Bottle volumes V millilitres are modelled by $V \sim \mathrm{N}(330, 4^2)$. Bottles with volume below 324 ml or above 337 ml are rejected.

(a) Find the probability that a bottle is rejected.

(b) Estimate the number accepted in a production run of 5000 bottles.

(c) Which rejection tail is larger? Support your answer with probabilities.

22. GDC

The lifetime L hours of a component is modelled by $L \sim \mathrm{N}(1500, 180^2)$.

(a) Find the probability that it lasts longer than 1750 hours.

(b) A warranty is set so that only 2% of components fail before it expires. Find the warranty duration.

(c) If the duration must be a whole number of hours and the failure probability must not exceed 2%, state the longest allowable warranty.

23. GDC

Two machines produce rods with lengths (in mm) modelled by $A \sim \mathrm{N}(50, 0.4^2)$ and $B \sim \mathrm{N}(50, 0.7^2)$.

(a) Without calculating, state which machine has a higher proportion of lengths between 49.5 and 50.5 mm. Explain.

(b) Find both proportions.

(c) For a 1000-rod run from each machine, estimate the difference in the numbers outside this tolerance.

24. REASON

A calculator gives the following probabilities for a normal random variable X.

x	12	18	24
$\mathrm{P}(X \leq x)$	0.10	0.50	0.90

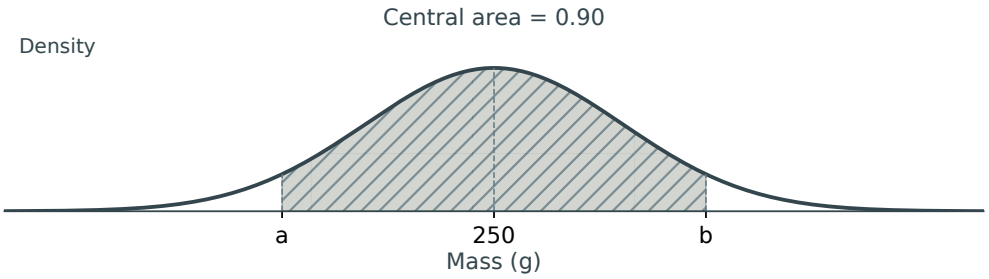
(a) Find $\mathrm{P}(12 < X < 24)$.

(b) Find $\mathrm{P}(X > 18)$.

(c) State the mean of X and justify your answer.

25. GDC

Product masses M grams follow $M \sim \mathrm{N}(250, 12^2)$. The central 90% of masses lie between a and b, as shown.



(a) Find a and b.
(b) Find the width $b - a$ of this interval.
(c) State the percentage of products with mass greater than b.

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26. GDC

The time T seconds for a runner to complete a race follows $T \sim \mathrm{N}(14.2, 0.6^2)$.
(a) Find the time below which the fastest 8% of performances lie.
(b) Find the time above which the slowest 15% lie.
(c) Find the percentage of performances between these two cutoffs.

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27. GDC

Let $X \sim \mathrm{N}(80, 5^2)$. A student uses 0.07 as the left-tail input to find c in $\mathrm{P}(X > c) = 0.07$.
(a) Explain the error and state the correct left-tail input.
(b) Find the correct value of c.
(c) Explain, without further calculation, where the student's incorrect cutoff lies relative to 80.

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28. GDC

The daily output X kilograms of a process is modelled by $X \sim \mathrm{N}(18, 2.5^2)$.
(a) Find $\mathrm{P}(15 < X < 22)$.
(b) Find the output exceeded on 90% of days.
(c) Estimate the number of days in a 240-day year with output above 22 kg.

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29. GDC

Let $X \sim \mathrm{N}(40, 6^2)$. Find the positive number d such that $\mathrm{P}(40 - d < X < 40 + d) = 0.92$.
(a) State the probability in each tail.
(b) Find d.
(c) Find $\mathrm{P}(X < 40 - d \text{ or } X > 40 + d)$.

30. GDC

A model for the mass M kilograms of a small parcel is $M \sim \mathrm{N}(0.40, 0.25^2)$.

(a) Find $\mathrm{P}(M < 0)$.

(b) Explain why this probability reveals a limitation of the model.

(c) Would this model be convincing for estimating very small parcel masses? Justify your answer.

3. IB exam-style questions

All questions are original. Marks are shown in brackets. **31–40: Paper 1 short-response style. 41–50: Paper 2 extended-response style. A GDC is allowed throughout.**

31. REASON Maximum mark: 5

A measurement X is normally distributed with mean 24. It is given that $\mathrm{P}(X < 19) = 0.12$.

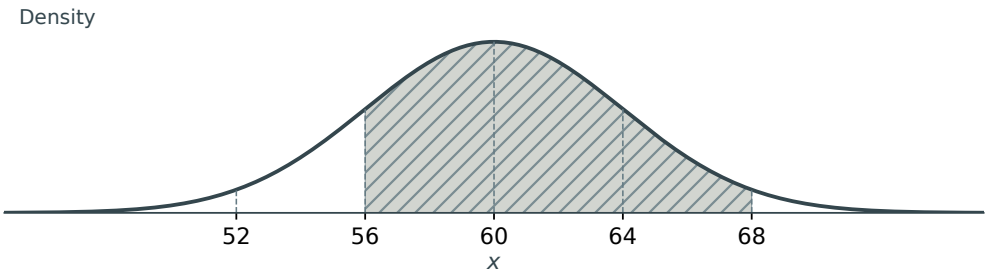
(a) Find $\mathrm{P}(X > 29)$. [1]

(b) Find $\mathrm{P}(19 < X < 29)$. [2]

(c) Find $\mathrm{P}(24 < X < 29)$. [2]

32. REASON Maximum mark: 6

Let $X \sim \mathrm{N}(60, 4^2)$. Use the 68–95–99.7 rule.



(a) Estimate the probability represented by the shaded region. [3]

(b) Estimate $\mathrm{P}(X > 68)$. [2]

(c) For 400 observations, estimate the number lying between 56 and 68. [1]

33. REASON Maximum mark: 5

A normal random variable X has mean μ and standard deviation $\sigma > 0$.

(a) Write down $\mathrm{P}(X < \mu)$. [1]

(b) Explain why $\mathrm{P}(X = \mu) = 0$, although the density curve is highest at μ. [2]

(c) Use the 68–95–99.7 rule to estimate $\mathrm{P}(\mu + \sigma < X < \mu + 2\sigma)$. [2]

34. REASON Maximum mark: 6

The models $A \sim \mathrm{N}(100, 9)$ and $B \sim \mathrm{N}(100, 36)$ describe two sets of measurements.

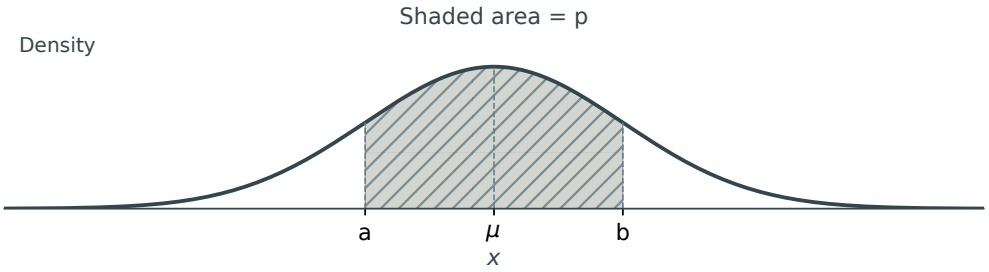
(a) Write down both standard deviations. [2]

(b) Sketch both density curves on the same axes, labelling the common mean and identifying each curve. [2]

(c) Use the 68–95–99.7 rule to estimate $\mathrm{P}(94 < A < 106)$ and $\mathrm{P}(94 < B < 106)$. [2]

35. REASON Maximum mark: 4

The diagram shows a normal density curve. The points a and b are equally far from the mean μ. The shaded area is p.



(a) Write down an expression for $a + b$ in terms of μ. [1]

(b) Find $\mathrm{P}(X > b)$ in terms of p. [2]

(c) Find $\mathrm{P}(\mu < X < b)$ when $p = 0.84$. [1]

36. GDC Maximum mark: 6

The net mass M grams of a cereal packet follows $M \sim \mathrm{N}(750, 12^2)$.

(a) Find $\mathrm{P}(M < 735)$. [2]

(b) Find $\mathrm{P}(740 < M < 770)$. [2]

(c) Estimate how many packets in a batch of 2400 have mass below 735 g. [2]

37. GDC Maximum mark: 7

The arrival time T minutes after 08:00 of a train follows $T \sim \mathrm{N}(6, 2.5^2)$.

(a) Find the probability that the train arrives before 08:05. [2]

(b) Find the probability that it arrives between 08:04 and 08:09. [2]

(c) Find the arrival time that is later than 95% of arrivals. Give your answer to the nearest second. [3]

38. GDC Maximum mark: 6

The diameter D millimetres of a manufactured part follows $D \sim \mathrm{N}(8.00, 0.06^2)$. A part is acceptable if $7.90 \leq D \leq 8.08$.

(a) Find the probability that a part is acceptable. [2]

(b) Find the probability that a part is rejected because it is too large. [2]

(c) Estimate the total number rejected in 10 000 parts. [2]

39. GDC Maximum mark: 5

The time T minutes needed to complete a task follows $T \sim \mathrm{N}(28, 5^2)$.

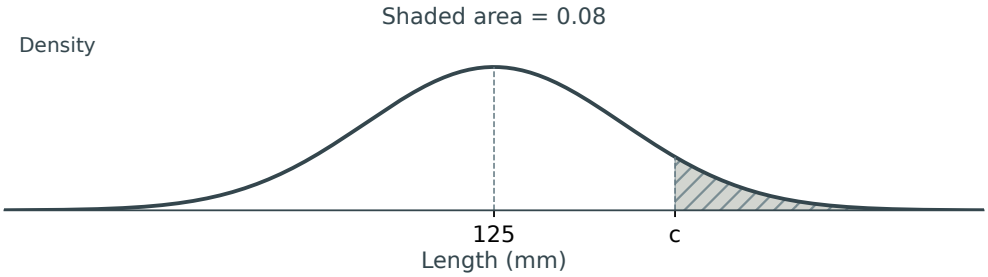
(a) Find the time below which the quickest 20% of completions lie. [2]

(b) Find the time above which the slowest 10% lie. [2]

(c) Find the percentage of completions between these two times. [1]

40. GDC Maximum mark: 6

The length L millimetres of a leaf follows $L \sim \mathrm{N}(125, 8^2)$. Leaves longer than c are classified as large. The shaded area is 0.08.



(a) Find c. [3]

(b) Find $\mathrm{P}(120 < L < c)$. [2]

(c) In a sample of 350 leaves, estimate the number classified as large. [1]

Paper 2-style extended-response questions | 41–50

41. GDC Maximum mark: 10

A company models the lifetime L hours of its rechargeable batteries by $L \sim \mathrm{N}(11.5, 1.2^2)$.

(a) Find the probability that a battery lasts at least 13 hours. [2]

(b) The company claims that at least 90% last 10 hours or longer. Determine whether the model supports the claim. [3]

(c) Find the lifetime exceeded by 95% of batteries. [2]

(d) In a batch of 2500 batteries, estimate the number lasting less than 10 hours. [2]

(e) Explain why this estimate need not equal the observed number in the batch. [1]

42. GDC Maximum mark: 11

The time T minutes taken to service a bicycle is modelled by $T \sim \mathrm{N}(35, 4^2)$. The central 98% of service times lie between a and b.

(a) State the probability in each tail outside $[a, b]$. [1]

(b) Find a and b. [4]

(c) Find the probability that a service takes between 32 and 40 minutes. [2]

(d) Estimate the number of services outside $[a, b]$ among 200 services. [2]

(e) The manager says that every service will take between a and b minutes. Explain why the model does not justify this statement. [2]

43. GDC Maximum mark: 11

Adult plant heights H centimetres follow $H \sim \mathrm{N}(90, 11^2)$. A nursery uses the following classification.

Label	Height interval	Proportion
Small	$H < a$	Shortest 15%
Medium	$a \leq H \leq b$	All remaining plants
Tall	$H > b$	Tallest 10%

(a) Find a and b. [4]

(b) Find the percentage labelled medium. [1]

(c) Estimate the number of medium plants among 600 plants. [2]

(d) Find the probability that a plant is taller than 100 cm. [2]

(e) A student claims that every plant taller than 100 cm is labelled tall. Determine whether the claim is correct. [2]

44. GDC Maximum mark: 13

A sensor measurement X has a normal distribution with mean 200 and standard deviation 16. A technician compares an empirical-rule estimate with a calculator result.

(a) Use the 68–95–99.7 rule to estimate $\mathrm{P}(184 < X < 216)$. [2]

(b) Use technology to calculate $\mathrm{P}(184 < X < 216)$. Explain the small difference from part (a). [3]

(c) Find the endpoints of the central 95% interval using technology. [4]

(d) Find the probability that a measurement exceeds 230. [2]

(e) Estimate the number exceeding 230 in 1500 measurements. [2]

45. GDC Maximum mark: 11

The completion time T seconds for a chemical process follows $T \sim \mathrm{N}(52, 6^2)$. If the process exceeds its time limit, an additional inspection costing €12 is required.

(a) Find the probability that the process is finished within 60 seconds. [2]

(b) Find the 97th percentile of T. [2]

(c) Hence find the least whole-number limit m for which $\mathrm{P}(T \leq m) \geq 0.97$. [2]

(d) For 600 processes with a 60-second limit, estimate the number requiring an additional inspection. [2]

(e) Estimate the reduction in total inspection cost if the limit is increased from 60 to 64 seconds. Give your answer to the nearest euro. [3]

46. GDC Maximum mark: 12

Two filling machines produce bottle volumes in millilitres according to the models below.

Machine	Mean (ml)	Standard deviation (ml)
A	1000	5
B	1002	4

A bottle is accepted when its volume lies in $[995, 1008]$.

(a) Find the acceptance probability for each machine. [4]

(b) Determine which machine is expected to produce fewer rejected bottles. [1]

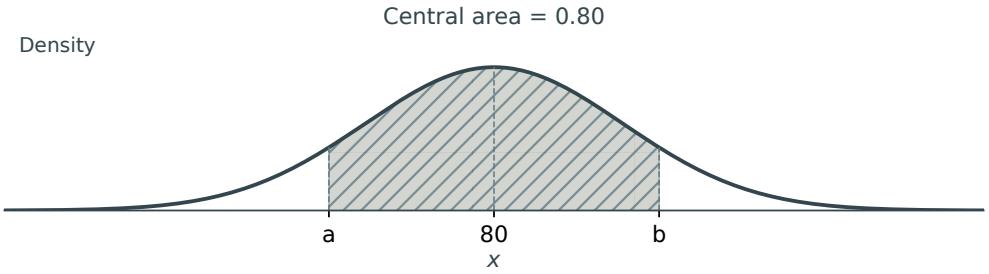
(c) For 4000 bottles from each machine, estimate the difference in the numbers rejected. [2]

(d) Find the volume below which the lowest 5% of machine B's bottles lie. [2]

(e) A new rule rejects bottles from machine B with volume below 996 ml. Is the rejection probability at most 5%? Justify your answer. [3]

47. GDC Maximum mark: 12

A performance measurement X is modelled by $X \sim \mathrm{N}(80, 9^2)$. The points a and b bound the central 80% of the distribution, as shown.



(a) State $\mathrm{P}(X < a)$ and $\mathrm{P}(X > b)$. [1]

(b) Find a and b. [4]

(c) Find $\mathrm{P}(X < 70 \text{ or } X > b)$. [2]

(d) Find the width of the central 95% interval. [3]

(e) Explain why the central 95% interval is wider than the central 80% interval. [2]

48. GDC Maximum mark: 11

For each delivery, the delay D minutes relative to the scheduled time follows $D \sim \mathrm{N}(2, 3^2)$. A negative delay means an early arrival.

(a) Find the probability of an early arrival. [2]

(b) Find the probability of an arrival within 2 minutes of the scheduled time. [3]

(c) Find the delay exceeded by only 1% of deliveries. [2]

(d) In 1200 deliveries, estimate the number arriving within 2 minutes of the scheduled time. [2]

(e) Explain why negative values are meaningful in this model, whereas they would be unsuitable for a model of the total journey duration. [2]

49. GDC Maximum mark: 12

The breaking force F newtons of a wire follows $F \sim \mathrm{N}(420, 25^2)$. A wire is suitable if its breaking force is at least 380 N. Each unsuitable wire costs €1.80 to replace.

(a) Find the probability that a wire is suitable. [2]

(b) Find the breaking force below which the weakest 3% of wires lie. [2]

(c) Estimate the number unsuitable in 1200 wires. [2]

(d) Estimate the replacement cost for 1200 wires, to the nearest euro. Use the unrounded probability. [2]

(e) Find the endpoints of the central 90% interval for breaking force. [4]

50. GDC Maximum mark: 12

A test score S is modelled by $S \sim \mathrm{N}(58, 14^2)$, although actual scores range from 0 to 100. The model is used to set an award threshold.

(a) Find the probability assigned by the model to scores outside $[0, 100]$. [3]

(b) Find the score c such that $\mathrm{P}(S > c) = 0.12$. [2]

(c) An award is given for scores at least k, where k is a whole number. Find the least k for which the model predicts that at most 12% receive an award. [2]

(d) Estimate the number outside $[0, 100]$ predicted by the model in 2000 candidates. [2]

(e) Explain why part (d) reveals a limitation of the model. Does it prove that every prediction from the model is useless? [3]

Answer key and solution outlines

Decimal answers are normally rounded to 3 significant figures. Counts use unrounded probabilities. invNorm(p, μ, σ) below denotes inverse normal using **left-tail** area p. Normal Cdf uses the stated bounds and given mean and standard deviation. Graph descriptions specify the required features.

1. Short response | Questions 1–15

1.
(a) $\mu = 42$.
(b) $\sigma = 3.5$.
(c) $\sigma^2 = 12.25$.

2.
(a) Median $= 68$; mode $= 68$.
(b) $x = 68$.
(c) 1.

3.
(a) $\sigma_A = 2$; $\sigma_B = 4$.
(b) A: its standard deviation is smaller and both curves have total area 1.

4.
(a) Bell-shaped curve symmetric about $x = 30$; shade the area to the right of 35.
(b) 0.5.

5.
(a) 0.68.
(b) 0.95.
(c) $(1 - 0.997)/2 = 0.0015$.

6.
(a) $0.68/2 = 0.34$.
(b) $(0.95 - 0.68)/2 = 0.135$.
(c) $(1 - 0.95)/2 = 0.025$.

7.
(a) 0.18, by symmetry about 50.
(b) $1 - 2(0.18) = 0.64$.

8.
(a) 0, since X is continuous.
(b) $\mathrm{P}(X = 15) = 0$, so including the endpoint adds no probability.

9. Left-tail calculation: $\mathrm{P}(X < 76) = 0.748$.

10. $1 - \mathrm{P}(X \leq 142) = 0.0712$.

11. Area between 20 and 28: 0.818.

12. Inverse normal with left-tail area 0.90: $a = 523$.

13. Left-tail area $= 0.88$; $b = 53.2$.

14. Each tail has area 0.05. Endpoints: 46.8 and 73.2.

15.
(a) The count is discrete; a normal model is continuous. A suitable approximation would need justification.
(b) The stated distribution is right-skewed, whereas a normal distribution is symmetric.

2. Extended response | Questions 16–30

16.
(a) $\mathrm{P}(M < 490) = 0.0478$.
(b) $\mathrm{P}(494 < M < 508) = 0.750$.
(c) $2000\mathrm{P}(M < 490) = 95.6$, so about 96 packets.

17.
(a) $\mathrm{P}(T > 40) = 0.0377$.
(b) Left-tail area 0.95 gives 39.4 minutes.
(c) Yes: $\mathrm{P}(T \leq 40) = 0.962 > 0.95$.
18.
(a) $\mathrm{P}(82 < X < 112)$.
(b) 0.775.
(c) $1 - \mathrm{P}(82 < X < 112) = 0.225$.
19.
(a) Left-tail areas 0.25 and 0.75: $Q_1 = 66.6$ cm, $Q_3 = 77.4$ cm.
(b) $Q_3 - Q_1 = 10.8$ cm, using unrounded quartiles.
(c) The middle 50% of heights span about 10.8 cm, from 66.6 to 77.4 cm.
20.
(a) $800(0.68) = 544$.
(b) $800(0.025) = 20$.
(c) The percentages are approximate model probabilities, and observed counts vary.
21.
(a) $\mathrm{P}(V < 324) + \mathrm{P}(V > 337) = 0.107$.
(b) $5000\mathrm{P}(324 \leq V \leq 337) = 4.47 \times 10^3$; about 4466 bottles.
(c) Lower tail: 0.0668; upper tail: 0.0401. The lower tail is larger.
22.
(a) $\mathrm{P}(L > 1750) = 0.0824$.
(b) Left-tail area 0.02: $w = 1.13 \times 10^3$ hours (unrounded 1130.325...).
(c) 1130 hours; round the cutoff down.
23.
(a) Machine A: smaller spread about the same mean, so more mass lies in this centred interval.
(b) $p_A = 0.789$; $p_B = 0.525$.
(c) $1000(p_A - p_B) = 264$; machine B has about 264 more outside tolerance.
24.
(a) $0.90 - 0.10 = 0.80$.
(b) $1 - 0.50 = 0.50$.
(c) 18: the mean and median coincide for a normal distribution.
25.
(a) Use left-tail areas 0.05 and 0.95: $a = 230$ g, $b = 270$ g.
(b) $b - a = 39.5$ g.
(c) 5%.
26.
(a) Left-tail area 0.08: 13.4 s.
(b) Left-tail area 0.85: 14.8 s.
(c) $100(0.85 - 0.08) = 77\%$.
27.
(a) 0.07 is a right-tail area. The left-tail input is $1 - 0.07 = 0.93$.
(b) $c = 87.4$.
(c) Below 80, since a left-tail area below 0.5 gives a cutoff below the mean.
28.
(a) 0.830.
(b) $\mathrm{P}(X > a) = 0.90$ gives left-tail area 0.10; $a = 14.8$ kg.
(c) $240\mathrm{P}(X > 22) = 13.2$; about 13 days.
29.
(a) $(1 - 0.92)/2 = 0.04$.
(b) The upper endpoint is the 96th percentile: $40 + d = 50.5$, so $d = 10.5$.
(c) 0.08.

30.
(a) $\mathrm{P}(M < 0) = 0.0548$.
(b) Negative mass is impossible, yet the model assigns it about 5.48% probability.
(c) No: a substantial part of the lower tail is physically impossible, making lower-tail predictions questionable.

3. IB exam-style questions | Questions 31–50

31.
(a) 0.12, by symmetry.
(b) $1 - 2(0.12) = 0.76$.
(c) $0.76/2 = 0.38$.
32.
(a) $\mathrm{P}(56 < X < 68) \approx 0.68/2 + 0.95/2 = 0.815$.
(b) $(1 - 0.95)/2 = 0.025$.
(c) $400(0.815) = 326$.
33.
(a) 0.5.
(b) Probability is area. A single point has zero width and hence zero area; height is not a point probability.
(c) $(0.95 - 0.68)/2 = 0.135$.
34.
(a) $\sigma_A = 3$; $\sigma_B = 6$.
(b) Both curves are centred on 100; A is narrower and taller, B wider and lower.
(c) For A, the interval is $\mu \pm 2\sigma$: approximately 0.95. For B, it is $\mu \pm \sigma$: approximately 0.68.
35.
(a) $a + b = 2\mu$.
(b) $(1 - p)/2$.
(c) $p/2 = 0.42$.
36.
(a) $\mathrm{P}(M < 735) = 0.106$.
(b) $\mathrm{P}(740 < M < 770) = 0.750$.
(c) $2400\mathrm{P}(M < 735) = 254$; about 254 packets.
37.
(a) $\mathrm{P}(T < 5) = 0.345$.
(b) $\mathrm{P}(4 < T < 9) = 0.673$.
(c) $t = \mathrm{invNorm}(0.95, 6, 2.5) = 10.1121\ldots$ minutes after 08:00, so 08:10:07.
38.
(a) $\mathrm{P}(7.90 \leq D \leq 8.08) = 0.861$.
(b) $\mathrm{P}(D > 8.08) = 0.0912$.
(c) $10\,000[1 - \mathrm{P}(7.90 \leq D \leq 8.08)] = 1.39 \times 10^3$; about 1390 parts.
39.
(a) Left-tail area 0.20: 23.8 minutes.
(b) Left-tail area 0.90: 34.4 minutes.
(c) $100(0.90 - 0.20) = 70\%$.
40.
(a) Left-tail area 0.92: $c = 136$ mm.
(b) $0.92 - \mathrm{P}(L \leq 120) = 0.654$.
(c) $350(0.08) = 28$ leaves.

41.

(a) $\mathrm{P}(L \geq 13) = 1 - \mathrm{P}(L < 13) = 0.106$.

(b) $\mathrm{P}(L \geq 10) = 0.894$, below 0.90; the model does not support the claim.

(c) An exceeded lifetime has 0.95 to its right, so use left-tail area 0.05: $\text{invNorm}(0.05, 11.5, 1.2) = 9.53$ hours.

(d) $2500\mathrm{P}(L < 10) = 264$; about 264 batteries.

(e) The calculation gives a model-based expected count. Random variation and imperfect modelling can make the observed count different.

42.

(a) Each tail has probability $(1 - 0.98)/2 = 0.01$.

(b) Left-tail inputs 0.01 and 0.99 give $a = 25.7$ min and $b = 44.3$ min.

(c) $\mathrm{P}(32 < T < 40) = 0.668$.

(d) $200(0.02) = 4$ services.

(e) The model assigns 2% probability outside the interval. A central interval is not a guaranteed range for every observation.

43.

(a) Use left-tail areas 0.15 and 0.90: $a = 78.6$ cm, $b = 104$ cm.

(b) $100(1 - 0.15 - 0.10) = 75\%$.

(c) $600(0.75) = 450$ plants.

(d) $\mathrm{P}(H > 100) = 0.182$.

(e) False. The tall cutoff is $b = 104.097\ldots$ cm; plants between 100 cm and this cutoff are medium.

44.

(a) $184 = 200 - 16$ and $216 = 200 + 16$, so the empirical estimate is 0.68.

(b) Normal Cdf with bounds $184, 216$, mean 200 and standard deviation 16 gives 0.683. The empirical percentage is rounded.

(c) Each tail has area 0.025. Inverse normal with left-tail areas 0.025 and 0.975 gives 169 and 231.

(d) $\mathrm{P}(X > 230) = 0.0304$.

(e) $1500\mathrm{P}(X > 230) = 45.6$; about 46 measurements.

45.

(a) $\mathrm{P}(T \leq 60) = 0.909$.

(b) $\text{invNorm}(0.97, 52, 6) = 63.2848\ldots$ s ≈ 63.3 s.

(c) $m = 64$ s. Round the unrounded percentile up to meet the probability requirement.

(d) $600\mathrm{P}(T > 60) = 54.7$; about 55 processes.

(e) Use unrounded probabilities: $600(12)[\mathrm{P}(T > 60) - \mathrm{P}(T > 64)] = 493$. The expected saving is about €493.

46.

(a) $p_A = \mathrm{P}(995 \leq A \leq 1008) = 0.787$; $p_B = 0.893$.

(b) Machine B, because its acceptance probability is higher.

(c) $4000[(1 - p_A) - (1 - p_B)] = 426$; about 426 more rejects from A.

(d) $\text{invNorm}(0.05, 1002, 4) = 995$ ml.

(e) No. $\mathrm{P}(B < 996) = 0.0668$, or about 6.68%, which exceeds 5%.

47.

(a) Each tail has probability $(1 - 0.80)/2 = 0.10$.

(b) Left-tail areas 0.10 and 0.90 give $a = 68.5$ and $b = 91.5$.

(c) The events are disjoint: $\mathrm{P}(X < 70) + 0.10 = 0.233$.

(d) Use left-tail areas 0.025 and 0.975: width $= \text{invNorm}(0.975, 80, 9) - \text{invNorm}(0.025, 80, 9) = 35.3$.

(e) Including more area symmetrically about the mean requires both endpoints to be farther from the mean.

48.

(a) $\mathrm{P}(D < 0) = 0.252$.

(b) Within 2 minutes means $-2 \leq D \leq 2$: $\mathrm{P}(-2 \leq D \leq 2) = 0.409$.

(c) Use left-tail area 0.99: $\text{invNorm}(0.99, 2, 3) = 8.98$ min.

(d) $1200\mathrm{P}(-2 \leq D \leq 2) = 491$; about 491 deliveries.

(e) A negative delay means arrival before the scheduled time. An elapsed journey duration cannot be negative.

49.

(a) $\mathrm{P}(F \geq 380) = 0.945$.

(b) $\mathrm{invNorm}(0.03, 420, 25) = 373$ N.

(c) $1200\mathrm{P}(F < 380) = 65.8$; about 66 wires.

(d) $1200(1.80)\mathrm{P}(F < 380) = 118$; about €118.

(e) Each tail is 0.05. The endpoints are 379 N and 461 N.

50.

(a) $\mathrm{P}(S < 0) + \mathrm{P}(S > 100) = 0.00137$.

(b) Use left-tail area 0.88: $c = \mathrm{invNorm}(0.88, 58, 14) = 74.4$ (unrounded $74.4498\ldots$).

(c) $k = 75$: round the unrounded cutoff up so that the upper tail is no greater than 0.12.

(d) $2000[\mathrm{P}(S < 0) + \mathrm{P}(S > 100)] = 2.73$; about 3 candidates.

(e) Scores outside the allowed range are impossible, yet the model assigns them nonzero probability. This warns against those tail predictions; it does not by itself rule out useful approximations within the allowed range.