



Differentiation: the power rule

IB Mathematics: Applications and Interpretation SL

Integer powers, gradients and rates of change | 50 questions

Learning goal	Differentiate sums of integer powers and apply the derivative.
Syllabus link	SL 5.3: $\frac{d}{dx}(ax^n) = anx^{n-1}$, $n \in \mathbb{Z}$; sums of such terms.
Structure	15 short + 15 extended + 20 original IB-style questions; answer key.
Technology	A GDC is permitted throughout, as in AI SL Paper 1. The  GDC symbol identifies tasks requiring numerical technology.

Instructions. Work on separate paper and show your differentiation steps. Give exact answers unless instructed otherwise. Rewrite fractions using integer powers where useful. Reciprocal expressions have $x \neq 0$ unless a narrower domain is stated.

1 | Short response

1. Differentiate $f(x) = x^7$.
2. Differentiate $y = 6x^4$.
3. Find $f'(x)$ when $f(x) = -3x^5$.
4. Differentiate $y = 9x - 12$.
5. Find $g'(x)$ when $g(x) = 14$.
6. Differentiate $f(x) = \frac{1}{2}x^6$.
7. Differentiate $y = 4x^{-2}$.
8. Differentiate $h(x) = -5x^{-3}$.
9. Write $f(x) = \frac{7}{x}$ using a negative power. Hence find $f'(x)$.
10. Find $\frac{dy}{dx}$ when $y = \frac{3}{x^2} + 2x$.
11. Differentiate $f(x) = 2x^3 - 5x^2 + 4x - 8$.



12. Differentiate $g(x) = \frac{x^4}{4} - \frac{x^2}{2} + 6$.

13. Given $f(x) = 3x^2 - 2x + 1$, find $f'(2)$.

14. A student writes $\frac{d}{dx}(5x^3 + 8) = 15x^2 + 8$. Identify and correct the error.

15. A student writes $\frac{d}{dx}(x^{-2}) = -2x^{-1}$. Identify and correct the error.

2 | Extended response

16. Let $f(x) = 4x^3 - 7x^2 + 2x - 9$.

(a) Find $f'(x)$.

(b) Evaluate $f'(-1)$ and $f'(2)$.

17. Let $g(x) = \frac{6}{x^2} - \frac{4}{x} + 5$, $x \neq 0$.

(a) Rewrite $g(x)$ using negative powers and differentiate.

(b) Find the gradient of $y = g(x)$ at $x = 2$.

18. Let $h(x) = x(x^2 - 3x + 5)$.

(a) Expand $h(x)$ and find $h'(x)$.

(b) Find $h'(3)$.

19. The curve $y = \frac{2x^3 - 5x + 4}{x}$ is defined for $x \neq 0$.

(a) Simplify the expression for y and find $\frac{dy}{dx}$.

(b) Find the gradient at $x = -1$.

20. Let $p(x) = (x - 2)(x + 3)$.

(a) Expand $p(x)$ and differentiate.

(b) Find the value of x for which $p'(x) = 7$.

21. Let $f(x) = ax^3 + 2x$, where a is a constant. Given that $f'(2) = 38$, find a .

22. Let $g(x) = x^3 + bx^2 - 4x$. The gradient at $x = 1$ is 5. Find b .

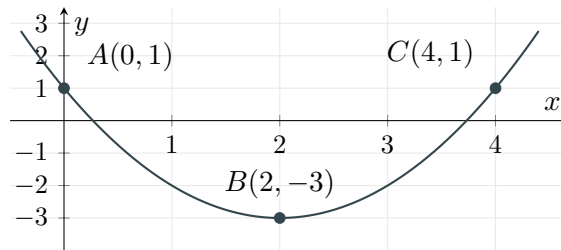


23. Let $f(x) = 2x^2 + k$ and $g(x) = 2x^2 - 7$, where k is a constant.

- (a) Show that $f'(x) = g'(x)$.
(b) Given that $f(3) = 23$, find k .

24. The graph shows $y = f(x) = x^2 - 4x + 1$.

- (a) Find $f'(x)$.
(b) Find the gradient at each of A , B and C .



25. Let $f(x) = x^3 - 6x^2 + 11x$. Find all values of x at which the gradient of the curve is 2.

26. The temperature of a liquid is modelled by $T(t) = 80 - 6t + 0.2t^2$ for $0 \leq t \leq 10$, where T is in $^{\circ}\text{C}$ and t is in minutes.

- (a) Find $T'(t)$.
(b) Find and interpret $T'(4)$, including units.

27. Let $f(x) = x^4 - 2x^2$.

- (a) Find $f(2)$ and $f'(2)$.
(b) Explain what these two values represent on the graph of $y = f(x)$.

28. Let $f(x) = 3x^2 + \frac{12}{x}$, $x > 0$.

- (a) Find $f'(x)$.
(b)  GDC Use your GDC to evaluate $f'(1.7)$. Give your answer to three significant figures.

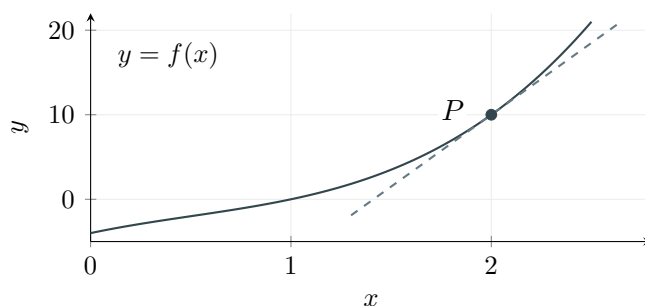
29. A student differentiates $y = \frac{x^3 + 4}{x^2}$ and obtains $\frac{3x^2}{2x}$.

- (a) Explain why this method is invalid.
(b) Rewrite y as a sum of powers and find the correct derivative.

30. Let $f(x) = ax^2 + bx + 4$. Given that $f'(1) = 7$ and $f'(3) = 15$, find a and b .



3 | IB exam-style practice

31. [Maximum mark: 4]Consider $f(x) = 2x^3 - 3x^2 + 5x - 4$.The dashed line is the tangent at P , where $x = 2$.

- (a) Find $f'(x)$. [2]
- (b) Find the gradient of the curve $y = f(x)$ at the point P , where $x = 2$. [2]

32. [Maximum mark: 4]Consider $g(x) = \frac{9}{x} + 2x^2$, $x \neq 0$.

- (a) Find $g'(x)$. [3]
- (b) Find $g'(3)$. [1]

33. [Maximum mark: 4]Let $h(x) = \frac{1}{4}x^4 - 2x^2 + 7$.

- (a) Find $h'(x)$. [2]
- (b) Find $h(2)$. [1]
- (c) Show that the gradient at $x = 2$ is zero. [1]

34. [Maximum mark: 4]Let $f(x) = ax^3 - 5x + 2$, where a is a constant.

- (a) Find $f'(x)$ in terms of a . [2]
- (b) Given that $f'(2) = 19$, determine a . [2]

35. [Maximum mark: 5]A curve has equation $y = (x + 1)(x^2 - 2x + 3)$.

- (a) Expand the expression for y . [2]
- (b) Find $\frac{dy}{dx}$. [2]
- (c) Find the gradient at $x = -1$. [1]

**3 | IB exam-style practice (continued)****36.** [Maximum mark: 5]

A curve has equation $y = \frac{3x^4 - 2x^2 + 5}{x^2}$, $x \neq 0$.

- (a) Write y as a sum of terms with integer powers of x . [2]
- (b) Find $\frac{dy}{dx}$. [2]
- (c) Find the gradient at $x = 1$. [1]

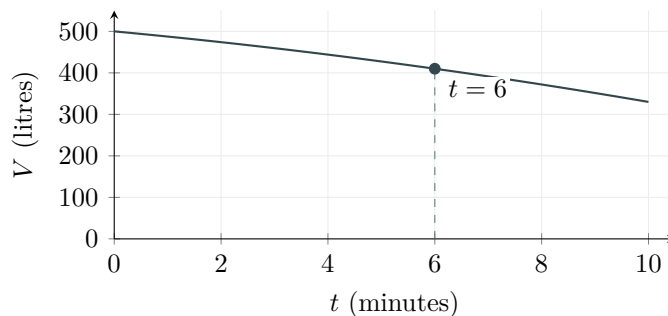
37. [Maximum mark: 4]

The height of a plant is modelled by $H(t) = 12 + 1.5t + 0.2t^2$ for $0 \leq t \leq 8$. Here H is in centimetres and t is in weeks.

- (a) Find $H'(t)$. [2]
- (b) Find and interpret $H'(5)$. [2]

38. [Maximum mark: 4]

For $0 \leq t \leq 10$, the volume of water in a tank is modelled by $V(t) = 500 - 12t - 0.5t^2$, where V is in litres and t is in minutes.



The model for the volume of water remaining in the tank.

- (a) Find $V'(t)$. [2]
- (b) Find $V'(6)$ and explain the meaning of its sign. [2]

39. [Maximum mark: 6]

Let $f(x) = x^3 + bx^2 + cx$. The gradients at $x = 1$ and $x = 2$ are 0 and 5, respectively.

- (a) Find $f'(x)$ in terms of b and c . [2]
- (b) Determine b and c . [4]

40. [Maximum mark: 6]

Let $g(x) = 4x^{-1} + bx + c$, $x > 0$. The gradient at $x = 2$ is 3, and the curve passes through $(2, 11)$.

- (a) Find $g'(x)$. [2]
- (b) Determine b . [2]
- (c) Determine c . [2]

**3 | IB exam-style practice (continued)****41.** [Maximum mark: 5]Consider $f(x) = 2x^3 - 9x^2 + 12x + 5$.

- (a) Find $f'(x)$. [2]
- (b) Find the values of x for which the curve has gradient 12. [3]
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42. [Maximum mark: 5]Consider $f(x) = \frac{12}{x^2} + 3x$, $x > 0$.

- (a) Find $f'(x)$. [2]
- (b) Find the positive value of x at which the gradient is zero. [3]
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43. [Maximum mark: 5]The cost, in euros, of producing q items is modelled by $C(q) = 250 + 18q + 0.04q^2$, for $0 \leq q \leq 200$. Treat q as a continuous variable.

- (a) Find $C'(q)$. [2]
- (b) Find $C'(50)$ and state its units. [2]
- (c) Interpret your answer in the context of the model. [1]
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44. [Maximum mark: 5]Let $p(x) = x^3 - 2x$ and $q(x) = 2x^2 + 4x$.

- (a) Find $p'(x)$ and $q'(x)$. [2]
- (b)  GDC Find both values of x at which the two curves have the same gradient. Give your answers to three significant figures. [3]
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45. [Maximum mark: 6]Let $f(x) = ax^{-2} + bx$, $x > 0$. It is given that $f'(1) = -5$ and $f'(2) = 2$.

- (a) Find $f'(x)$. [2]
- (b) Determine a and b . [4]

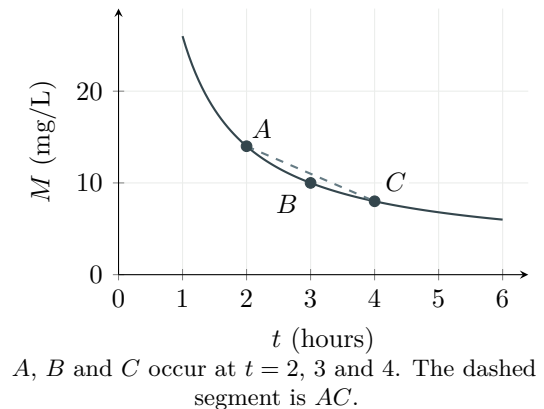


3 | IB exam-style practice (continued)

46. [Maximum mark: 6]

For $1 \leq t \leq 6$, the concentration of a substance is modelled by $M(t) = \frac{24}{t} + 2$, where M is in mg/L and t is in hours.

- (a) Find $M'(t)$. [2]
- (b) Find and interpret $M'(3)$. [2]
- (c) Calculate the average rate of change from $t = 2$ to $t = 4$ (the gradient of AC). [2]



47. [Maximum mark: 5]

GDC Let $f(x) = 0.3x^4 - 1.7x^2 + 2.4x$.

- (a) Find $f'(x)$ analytically. [2]
- (b) Calculate $f'(1.8)$, giving your answer to three significant figures. [2]
- (c) Use your GDC's numerical derivative feature to check your answer. [1]

48. [Maximum mark: 5]

Two models are $A(t) = 100 + 8t + 0.5t^2$ and $B(t) = 120 + 2t + t^2$, for $0 \leq t \leq 10$. Both give quantities in grams, with t in minutes.

- (a) Find $A'(t)$ and $B'(t)$. [2]
- (b) Find when the two quantities have the same instantaneous rate of change. [2]
- (c) Find that common rate, with units. [1]

49. [Maximum mark: 6]

Let $f(x) = ax^3 + bx^2 + c$. It is given that $f(0) = 7$, $f'(1) = 5$ and $f'(2) = 16$.

- (a) Write down c and find $f'(x)$. [2]
- (b) Determine a and b . [4]

50. [Maximum mark: 8]

Let $f(x) = \frac{k}{x^2} + mx + n$, $x > 0$. The curve passes through $(1, 8)$ and has gradients -6 at $x = 1$ and 1 at $x = 2$.

- (a) Find $f'(x)$ in terms of k and m . [2]
- (b) Determine k and m . [4]
- (c) Hence determine n . [2]

**Answer key and solution outlines**

Use $\frac{d}{dx}(ax^n) = anx^{n-1}$ term by term. Constants differentiate to zero.

1 | Short response

1. $f'(x) = 7x^6$.
2. $\frac{dy}{dx} = 24x^3$.
3. $f'(x) = -15x^4$.
4. $\frac{dy}{dx} = 9$.
5. $g'(x) = 0$; the derivative of a constant is zero.
6. $f'(x) = 3x^5$.
7. $\frac{dy}{dx} = -8x^{-3} = -\frac{8}{x^3}$.
8. $h'(x) = 15x^{-4} = \frac{15}{x^4}$.
9. $f(x) = 7x^{-1}$, so $f'(x) = -7x^{-2} = -\frac{7}{x^2}$.
10. $\frac{dy}{dx} = -6x^{-3} + 2$.
11. $f'(x) = 6x^2 - 10x + 4$.
12. $g'(x) = x^3 - x$.
13. $f'(x) = 6x - 2$; $f'(2) = 12 - 2 = 10$.
14. The constant 8 differentiates to 0, so the derivative is $15x^2$.
15. Subtract 1 from the exponent: $-2 - 1 = -3$. The derivative is $-2x^{-3}$.



Answer key and solution outlines

2 | Extended response

16. (a) $f'(x) = 12x^2 - 14x + 2$. (b) $f'(-1) = 28$; $f'(2) = 22$.
17. (a) $g = 6x^{-2} - 4x^{-1} + 5$; $g' = -12x^{-3} + 4x^{-2}$. (b) $g'(2) = -\frac{12}{8} + \frac{4}{4} = -\frac{1}{2}$.
18. (a) $h = x^3 - 3x^2 + 5x$; $h' = 3x^2 - 6x + 5$. (b) $h'(3) = 14$.
19. (a) $y = 2x^2 - 5 + 4x^{-1}$; $\frac{dy}{dx} = 4x - 4x^{-2}$. (b) Gradient $= -4 - 4 = -8$.
20. (a) $p = x^2 + x - 6$; $p' = 2x + 1$. (b) $2x + 1 = 7 \Rightarrow x = 3$.
21. $f' = 3ax^2 + 2$. Thus $12a + 2 = 38$, giving $a = 3$.
22. $g' = 3x^2 + 2bx - 4$; $3 + 2b - 4 = 5 \Rightarrow b = 3$.
23. (a) Both derivatives are $4x$ because constants differentiate to zero. (b) $18 + k = 23 \Rightarrow k = 5$.
24. (a) $f' = 2x - 4$. (b) At A : -4 ; at B : 0 ; at C : 4 .
25. $f' = 3x^2 - 12x + 11$. Set $f' = 2$: $3x^2 - 12x + 9 = 0$, so $(x - 1)(x - 3) = 0$. Thus $x = 1, 3$.
26. (a) $T' = -6 + 0.4t$. (b) $T'(4) = -4.4^\circ\text{C/min}$: after 4 minutes the liquid is cooling at 4.4°C/min .
27. (a) $f(2) = 8$; $f' = 4x^3 - 4x$ gives $f'(2) = 24$. (b) The point is $(2, 8)$ and the gradient there is 24 .
28. (a) $f' = 6x - 12x^{-2}$. (b) $f'(1.7) = 10.2 - 12/(1.7)^2 \approx 6.05$.
29. (a) Differentiating numerator and denominator separately and dividing does not give the derivative of the quotient. (b) $y = x + 4x^{-2}$; $\frac{dy}{dx} = 1 - 8x^{-3}$.
30. $f' = 2ax + b$. Hence $2a + b = 7$, $6a + b = 15$. Subtracting gives $4a = 8$, so $a = 2$ and $b = 3$.



Answer key and solution outlines

3 | IB exam-style practice

31. (a) $f' = 6x^2 - 6x + 5$. (b) $f'(2) = 24 - 12 + 5 = 17$.
32. (a) $g = 9x^{-1} + 2x^2 \Rightarrow g' = -9x^{-2} + 4x$. (b) $g'(3) = -1 + 12 = 11$.
33. (a) $h' = x^3 - 4x$. (b) $h(2) = 4 - 8 + 7 = 3$. (c) $h'(2) = 8 - 8 = 0$.
34. (a) $f' = 3ax^2 - 5$. (b) $12a - 5 = 19 \Rightarrow a = 2$.
35. (a) $y = x^3 - x^2 + x + 3$. (b) $\frac{dy}{dx} = 3x^2 - 2x + 1$. (c) Gradient = 6.
36. (a) $y = 3x^2 - 2 + 5x^{-2}$. (b) $\frac{dy}{dx} = 6x - 10x^{-3}$. (c) Gradient = -4.
37. (a) $H' = 1.5 + 0.4t$. (b) $H'(5) = 3.5$ cm/week: at week 5 the plant's height is increasing at this rate.
38. (a) $V' = -12 - t$. (b) $V'(6) = -18$ L/min. The negative sign means the volume is decreasing (at 18 L/min).
39. (a) $f' = 3x^2 + 2bx + c$. (b) $3 + 2b + c = 0$ and $12 + 4b + c = 5$. Subtract: $9 + 2b = 5$, so $b = -2$; then $c = 1$.
40. (a) $g' = -4x^{-2} + b$. (b) $-1 + b = 3 \Rightarrow b = 4$. (c) $2 + 8 + c = 11 \Rightarrow c = 1$.
41. (a) $f' = 6x^2 - 18x + 12$. (b) $6x^2 - 18x + 12 = 12 \Rightarrow 6x(x - 3) = 0$, so $x = 0, 3$.
42. (a) $f' = -24x^{-3} + 3$. (b) $-24/x^3 + 3 = 0 \Rightarrow x^3 = 8$, so $x = 2$.
43. (a) $C' = 18 + 0.08q$. (b) $C'(50) = 22$ euros/item. (c) Near 50 items, each additional item increases the cost by approximately €22.
44. (a) $p' = 3x^2 - 2$; $q' = 4x + 4$. (b) $3x^2 - 4x - 6 = 0$. Thus $x = \frac{2 \pm \sqrt{22}}{3} \approx -0.897, 2.23$.
45. (a) $f' = -2ax^{-3} + b$. (b) $-2a + b = -5$; $-a/4 + b = 2$. Subtract: $7a/4 = 7$, giving $a = 4$, $b = 3$.
46. (a) $M' = -24t^{-2}$. (b) $M'(3) = -8/3$ mg/L per hour: concentration is falling at 8/3 mg/L per hour at hour 3. (c) $[M(4) - M(2)]/(4 - 2) = (8 - 14)/2 = -3$ mg/L per hour.
47. (a) $f' = 1.2x^3 - 3.4x + 2.4$. (b) $f'(1.8) = 3.2784 \approx 3.28$. (c) Numerical derivative of the original f at 1.8 gives 3.2784, agreeing with (b).
48. (a) $A' = 8 + t$; $B' = 2 + 2t$. (b) $8 + t = 2 + 2t \Rightarrow t = 6$ min. (c) Common rate = 14 g/min.
49. (a) $c = 7$; $f' = 3ax^2 + 2bx$. (b) $3a + 2b = 5$, $12a + 4b = 16$. Subtract twice the first equation: $6a = 6$, so $a = 1$; then $b = 1$.
50. (a) $f' = -2kx^{-3} + m$. (b) $-2k + m = -6$; $-k/4 + m = 1$. Subtract: $7k/4 = 7$, so $k = 4$ and $m = 2$. (c) $4 + 2 + n = 8 \Rightarrow n = 2$.