



Tangents and normals

IB Mathematics: Applications and Interpretation SL

Gradients, line equations and geometric applications | 50 questions

Learning goal	Find tangent and normal equations using derivatives and a GDC.
Syllabus link	SL 5.4: tangents and normals at a given point, and their equations; analytic and technology approaches.
Structure	15 short + 15 extended + 20 original IB-style questions; answer key.
Technology	A GDC is permitted throughout, as in AI SL Paper 1. The  GDC symbol identifies tasks requiring numerical technology.

Instructions. Work on separate paper. Show the point, gradient and line equation. Give exact answers unless a GDC is specified; then use three significant figures and retain unrounded values in working. A horizontal tangent has a vertical normal. For numerical derivatives on a TI-Nspire CX: use *Menu* → *Calculus* → *Numerical Derivative at a Point*.

1 | Short response

1. A curve has gradient 4 at P . State the gradients of the tangent and normal at P .

2. A tangent has gradient $-\frac{2}{3}$. Find the gradient of the corresponding normal.

3. A normal has gradient 5. Find the gradient of the corresponding tangent.

4. A curve passes through $P(2, 7)$ and has gradient 3 there. Find the tangent equation.

5. The tangent at $P(-1, 4)$ has gradient -2 . Find its equation.

6. The tangent at $P(3, -2)$ has gradient $\frac{1}{2}$. Find the normal equation.

7. For $f(x) = x^2 + 3$, find the point on the curve where $x = 2$, and the tangent gradient there.

8. For $f(x) = 3x^2 - 4x + 1$, find the normal gradient at $x = 1$.

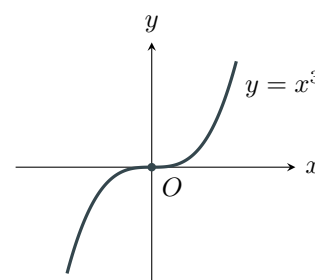
9. A curve has zero gradient at $P(5, 2)$. Write the equations of its tangent and normal at P .

10. The tangent at $P(1, 3)$ is parallel to $y = -4x + 9$. Find its equation.



11. The tangent at $P(2, 5)$ is perpendicular to $y = 2x - 7$. Find its equation.
12. A tangent has equation $3x + 2y = 12$. Find the gradient of the corresponding normal.
13. A student finds $f'(2) = 6$ and writes the tangent as $y = 6x$. What information and step are missing?
14. A student claims that a normal has gradient -3 whenever the tangent has gradient 3 . Correct the claim.

15. A student says a tangent can never cross the curve at the point of contact. The diagram shows $y = x^3$ near the origin. Use its gradient at O and the graph to explain why the claim is false.



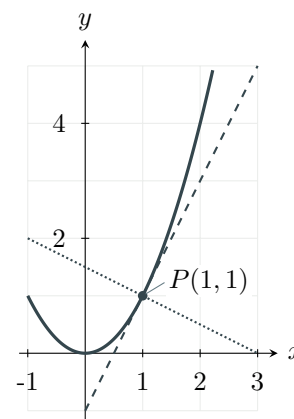
2 | Extended response

16. Let $f(x) = x^2 - 2x + 3$.
- (a) Find $f(2)$ and $f'(2)$.
- (b) Find the tangent and normal equations at $x = 2$.
17. Let $f(x) = x^3 - 2x + 4$.
- (a) Find the tangent gradient at $x = -1$.
- (b) Find the tangent and normal equations there.
18. Let $f(x) = \frac{8}{x}$, $x \neq 0$.
- (a) Find $f'(x)$.
- (b) Find the tangent and normal at $x = 2$.
19. The curve is $y = x^2 - 4x + 7$.
- (a) Find the point where the tangent is horizontal.
- (b) Find the tangent and normal equations at that point.



20. The graph shows $y = x^2$ and two lines through $P(1, 1)$.

- (a) Identify the tangent and normal by their line styles. Justify your choice using gradients.
- (b) Find the equation of each line.



21. For $y = x^2 + x - 2$, find the point where the tangent is parallel to $y = 5x + 4$. Hence find the tangent equation.

22. For $y = x^2 + 1$, find the point where the normal is parallel to $y = -\frac{1}{4}x + 3$. Find the normal equation.

23. The tangent to $y = ax^2 + 3x - 1$ at $x = 1$ has gradient 7. Find a and the tangent equation.

24. The normal to $y = x^2 + bx + 2$ at $x = 1$ has gradient $-\frac{1}{5}$. Find b and the normal equation.

25. Find the tangent to $y = x^2 + 2x$ at $x = 1$. Find its intercepts with both coordinate axes.

26. Find the normal to $y = x^2$ at $x = 2$. Find where this normal meets the x -axis.

27.  GDC Let $f(x) = 2^x$.

- (a) Use your GDC to find $f(1.5)$ and the numerical value of $f'(1.5)$.
- (b) Find the tangent and normal equations in point-slope form at $x = 1.5$.

28.  GDC Let $f(x) = \ln(x + 2)$, $x > -2$.

- (a) Use your GDC to find $f(1)$ and $f'(1)$.
- (b) Find the tangent and normal equations at $x = 1$, in point-slope form.

29. The tangent to $y = x^2$ at $x = 1$ and the tangent at $x = 3$ meet at Q . Find both tangent equations and the coordinates of Q .



30. For $f(x) = x^2 - 6x + 10$, find the point where the tangent is perpendicular to $y = -\frac{1}{2}x + 7$. Find the tangent and normal equations there.

3 | IB AI SL exam-style practice

31. [Maximum mark: 7]

Consider $f(x) = 2x^2 - 3x + 4$.

- (a) Find $f(2)$ and $f'(x)$. [3]
- (b) Find the tangent equation at $x = 2$. [2]
- (c) Find the normal equation there. [2]

32. [Maximum mark: 7]

The curve $y = x^3 + 2x + 1$ contains the point $P(1, 4)$.

- (a) Find $\frac{dy}{dx}$. [2]
- (b) Find the equation of the normal at P . [3]
- (c) Find the x -intercept of this normal. [2]

33. [Maximum mark: 7]

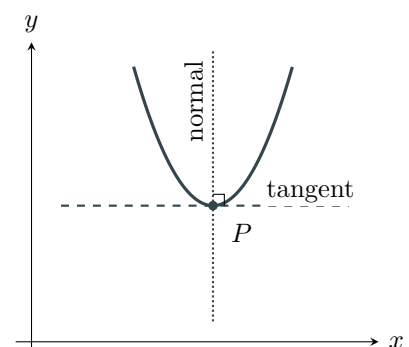
Let $f(x) = \frac{12}{x}$, $x > 0$.

- (a) Find the tangent equation at $x = 3$. [4]
- (b) Find the area of the triangle enclosed by this tangent and the coordinate axes. [3]

34. [Maximum mark: 5]

Let $f(x) = x^2 - 8x + 19$. The diagram shows a horizontal tangent and the corresponding normal at P .

- (a) Find the coordinates of P . [3]
- (b) Write down the tangent and normal equations at P . [2]



35. [Maximum mark: 7]

A tangent to $y = x^2 - 2x + 5$ is parallel to $y = 4x - 7$.

- (a) Find the point of contact. [3]
- (b) Find the tangent equation. [2]
- (c) Find the normal equation at the same point. [2]

**36.** [Maximum mark: 5]

A normal to $y = x^2 + 4$ is parallel to $y = \frac{1}{2}x - 3$.

- (a) Find the point of contact. [3]
(b) Find the normal equation. [2]

37. [Maximum mark: 7]

The tangent to $y = ax^2 + bx$ at $P(1, 5)$ has gradient 8.

- (a) Write down two equations involving a and b . [3]
(b) Find a and b . [2]
(c) Find the tangent equation at P . [2]

38. [Maximum mark: 6]

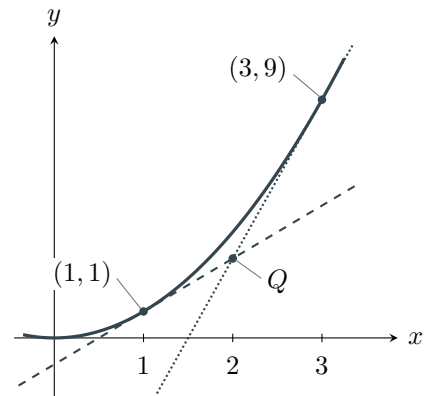
The curve $y = ax^2 + c$ passes through $P(2, 7)$. Its normal at P has gradient $-\frac{1}{8}$.

- (a) Find a and c . [4]
(b) Find the normal equation. [2]

39. [Maximum mark: 7]

The diagram shows $y = x^2$ and tangents at $x = 1$ and $x = 3$.

- (a) Find the equations of the two tangents. [4]
(b) Find their intersection point Q . [2]
(c) Show that Q does not lie on the curve. [1]

**40.** [Maximum mark: 6]

The normal to $y = x^2$ at $P(1, 1)$ meets the curve again at Q .

- (a) Find the normal equation. [3]
(b) Find the coordinates of Q . [3]

41. [Maximum mark: 8]

 GDC Let $f(x) = 3^x + 1$.

- (a) Use your GDC to find $f(1)$ and $f'(1)$. [2]
(b) Find the tangent and normal equations at $x = 1$, in point-slope form. [4]
(c) Find the x -intercept of the tangent. [2]



42. [Maximum mark: 7]

 GDC Consider $f(x) = \sqrt{x+3}$, $x \geq -3$.

- (a) Use your GDC to find $f(1)$ and $f'(1)$. [2]
 - (b) Find the normal equation at $x = 1$. [3]
 - (c) Find where the normal meets the x -axis. [2]
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43. [Maximum mark: 6]

The curve is $y = x^3 - 3x^2 + 5$.

- (a) Find all points where the tangent is horizontal. [4]
 - (b) Write down the normal equation at each point. [2]
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44. [Maximum mark: 7]

A tangent to $y = x^3$ is parallel to $y = 12x + 1$.

- (a) Find both possible points of contact. [4]
 - (b) Find the equation of each tangent. [3]
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45. [Maximum mark: 7]

Let $f(x) = x^2 + 3x - 2$. A tangent is perpendicular to $2x + y = 5$.

- (a) Find the tangent gradient. [2]
 - (b) Find the point of contact. [3]
 - (c) Find the tangent equation. [2]
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46. [Maximum mark: 8]

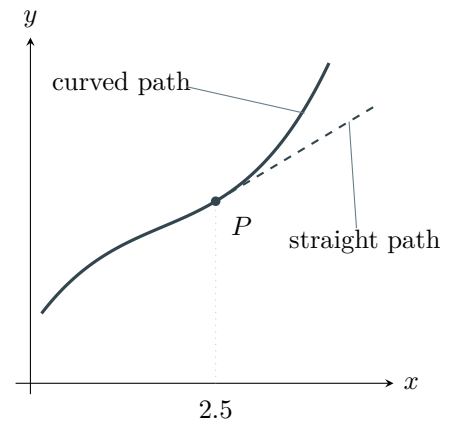
The height of a balloon, in metres, is modelled by $h(t) = 20 + 6t - t^2$ for $0 \leq t \leq 5$, where t is in seconds.

- (a) Find the equation of the tangent to the height–time graph at $t = 2$. [4]
- (b) Use this tangent to estimate $h(2.1)$. [2]
- (c) Find the model value $h(2.1)$ and state whether the tangent estimate is too high or too low. [2]

47. [Maximum mark: 7]

GDC A curved path is modelled by $y = 0.15x^3 - 0.8x^2 + 2x + 1$. Both coordinates are measured in metres. A straight path leaves the curve tangentially at P , where $x = 2.5$.

- (a) Find the coordinates of P and the tangent gradient. [3]
- (b) Give the line equation in point-slope form. [2]
- (c) Use this equation to estimate y when $x = 2.7$. [2]



48. [Maximum mark: 7]

The line $y = 6x - 5$ is tangent to $y = x^2 + k$, where k is a constant.

- (a) Find the x -coordinate of the point of contact. [2]
- (b) Find k and the point of contact. [3]
- (c) Find the normal equation at that point. [2]

49. [Maximum mark: 7]

GDC The tangent to $f(x) = x^3 + x$ at $x = a$ has gradient 7.

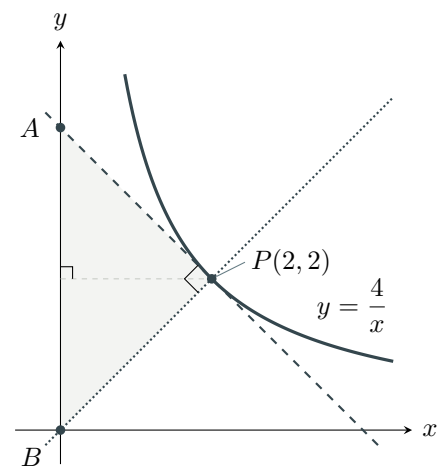
- (a) Find both possible values of a . [3]
- (b) Find both tangent equations. Give coefficients to three significant figures. [4]

50. [Maximum mark: 9]

The curve is $y = \frac{4}{x}$, $x > 0$. Its tangent and normal at $P(2, 2)$ meet the y -axis at A and B , respectively.

- (a) Find the tangent and normal equations. [4]
- (b) Find the coordinates of A and B . [2]
- (c) Find the area of triangle APB . [3]

The shaded triangle is APB . The thin horizontal segment is perpendicular to AB .





Answer key and solution outlines

1 | Short response

Use $m_T = f'(a)$ and $y - f(a) = m(x - a)$. For $m_T \neq 0$, $m_N = -1/m_T$; when $m_T = 0$, the normal is $x = a$.

1. $m_T = 4$, $m_N = -\frac{1}{4}$.
2. $m_N = -1/(-2/3) = \frac{3}{2}$.
3. $m_T = -\frac{1}{5}$.
4. $y - 7 = 3(x - 2)$, so $y = 3x + 1$.
5. $y - 4 = -2(x + 1)$, so $y = -2x + 2$.
6. $m_N = -2$; $y + 2 = -2(x - 3)$, so $y = -2x + 4$.
7. $P(2, 7)$; $f'(x) = 2x$, so $m_T = 4$.
8. $f'(1) = 6 - 4 = 2$, so $m_N = -\frac{1}{2}$.
9. Tangent $y = 2$; normal $x = 5$. The normal is vertical.
10. $m_T = -4$; $y - 3 = -4(x - 1)$, so $y = -4x + 7$.
11. $m_T = -\frac{1}{2}$; $y - 5 = -\frac{1}{2}(x - 2)$, so $y = -\frac{1}{2}x + 6$.
12. $y = -\frac{3}{2}x + 6$ gives $m_T = -\frac{3}{2}$ and $m_N = \frac{2}{3}$.
13. The point $(2, f(2))$ is needed. Use $y - f(2) = 6(x - 2)$; $y = 6x$ is valid only if $f(2) = 12$.
14. The gradients are negative reciprocals: $m_N = -\frac{1}{3}$, not -3 .
15. $f'(x) = 3x^2$, so the tangent at $(0, 0)$ is $y = 0$. Since x^3 changes sign through 0, the curve crosses this tangent.



Answer key and solution outlines

2 | Extended response

16. (a) $f(2) = 3$, $f'(x) = 2x - 2$, $m_T = 2$. (b) $T : y = 2x - 1$; $N : y = -\frac{1}{2}x + 4$.
17. (a) $f'(x) = 3x^2 - 2$ gives $m_T = 1$. (b) $P(-1, 5)$; $T : y = x + 6$; $N : y = -x + 4$.
18. (a) $f'(x) = -8x^{-2}$. (b) $P(2, 4)$; $m_T = -2$, $m_N = \frac{1}{2}$; $T : y = -2x + 8$; $N : y = \frac{1}{2}x + 3$.
19. (a) $dy/dx = 2x - 4 = 0$ gives $x = 2$, $y = 3$. (b) $T : y = 3$; $N : x = 2$.
20. (a) $f'(1) = 2$: the rising dashed line is the tangent; the falling dotted line is the normal. (b) $T : y = 2x - 1$; $N : y = -\frac{1}{2}x + \frac{3}{2}$.
21. $2x + 1 = 5 \Rightarrow x = 2$. The point is $(2, 4)$; $y - 4 = 5(x - 2)$ gives $y = 5x - 6$.
22. $m_N = -\frac{1}{4}$ implies $m_T = 4$. Thus $2x = 4$, so $P(2, 5)$ and $N : y = -\frac{1}{4}x + \frac{11}{2}$.
23. $dy/dx = 2ax + 3$. Thus $2a + 3 = 7$, $a = 2$. The point is $(1, 4)$; $T : y = 7x - 3$.
24. $m_T = 5$, so $2 + b = 5$ and $b = 3$. The point is $(1, 6)$; $N : y = -\frac{1}{5}x + \frac{31}{5}$.
25. $P(1, 3)$, $m_T = 4$; $T : y = 4x - 1$. Intercepts: $(0, -1)$ and $(\frac{1}{4}, 0)$.
26. $P(2, 4)$, $m_N = -\frac{1}{4}$; $N : y = -\frac{1}{4}x + \frac{9}{2}$. Setting $y = 0$ gives $(18, 0)$.
27. (a) $f(1.5) \approx 2.828427$, $f'(1.5) \approx 1.960516$. (b) $T : y - 2.83 = 1.96(x - 1.5)$; $N : y - 2.83 = -0.510(x - 1.5)$ (3 s.f.). Use unrounded values in working.
28. (a) $f(1) = \ln 3 \approx 1.098612$, $m_T \approx 0.333333$. (b) $T : y - 1.10 = 0.333(x - 1)$; $N : y - 1.10 = -3(x - 1)$ (3 s.f.).
29. At $x = 1$: $y = 2x - 1$. At $x = 3$: $y = 6x - 9$. Equating gives $4x = 8$, so $Q(2, 3)$.
30. $m_T = 2$, so $2x - 6 = 2$ gives $x = 4$, $f(4) = 2$. $T : y = 2x - 6$; $N : y = -\frac{1}{2}x + 4$.



Answer key and solution outlines

3 | IB exam-style: 31–40

31. (a) $f(2) = 6$, $f' = 4x - 3$. (b) $m_T = 5$, $T : y = 5x - 4$. (c) $m_N = -\frac{1}{5}$, $N : y = -\frac{1}{5}x + \frac{32}{5}$.
32. (a) $dy/dx = 3x^2 + 2$. (b) $m_T = 5$, $m_N = -\frac{1}{5}$; $N : y = -\frac{1}{5}x + \frac{21}{5}$. (c) $(21, 0)$.
33. (a) $P(3, 4)$; $f' = -12x^{-2}$, so $m_T = -\frac{4}{3}$. $T : y = -\frac{4}{3}x + 8$. (b) Intercepts $(6, 0)$, $(0, 8)$; area $= \frac{1}{2}(6)(8) = 24$.
34. (a) $f' = 2x - 8 = 0 \Rightarrow x = 4$; $P(4, 3)$. (b) $T : y = 3$; $N : x = 4$.
35. (a) $2x - 2 = 4 \Rightarrow x = 3$, $y = 8$. (b) $T : y = 4x - 4$. (c) $N : y = -\frac{1}{4}x + \frac{35}{4}$.
36. (a) $m_T = -2$, so $2x = -2$ gives $P(-1, 5)$. (b) $N : y = \frac{1}{2}x + \frac{11}{2}$.
37. (a) From P : $a + b = 5$; from the gradient: $2a + b = 8$. (b) Subtraction gives $a = 3$, $b = 2$. (c) $T : y = 8x - 3$.
38. (a) $m_T = 8$, $dy/dx = 2ax$ gives $4a = 8$, so $a = 2$. Then $4a + c = 7 \Rightarrow c = -1$. (b) $N : y = -\frac{1}{8}x + \frac{29}{4}$.
39. (a) $T_1 : y = 2x - 1$, $T_2 : y = 6x - 9$. (b) Equating gives $Q(2, 3)$. (c) On the curve, $y = 2^2 = 4 \neq 3$.
40. (a) $m_T = 2$, $N : y = -\frac{1}{2}x + \frac{3}{2}$. (b) $x^2 = -\frac{1}{2}x + \frac{3}{2} \Rightarrow (2x + 3)(x - 1) = 0$. Excluding P gives $Q(-\frac{3}{2}, \frac{9}{4})$.



Answer key and solution outlines

3 | IB exam-style: 41–50

41. (a) $f(1) = 4$, $m_T \approx 3.295837$. (b) $T : y - 4 = 3.30(x - 1)$; $N : y - 4 = -0.303(x - 1)$. (c) $x = 1 - 4/3.295837 \approx -0.214$, giving $(-0.214, 0)$.
42. (a) $f(1) = 2$, $m_T = 0.25$. (b) $m_N = -4$; $N : y = -4x + 6$. (c) $(1.5, 0)$.
43. (a) $dy/dx = 3x(x - 2) = 0$, so $(0, 5)$ and $(2, 1)$. (b) Normals $x = 0$ and $x = 2$.
44. (a) $3x^2 = 12 \Rightarrow x = \pm 2$, giving $(2, 8)$ and $(-2, -8)$. (b) $T : y = 12x - 16$ and $T : y = 12x + 16$, respectively.
45. (a) The line has gradient -2 , so $m_T = \frac{1}{2}$. (b) $2x + 3 = \frac{1}{2} \Rightarrow x = -\frac{5}{4}$; $y = -\frac{67}{16}$. (c) $T : y = \frac{1}{2}x - \frac{57}{16}$.
46. (a) $h(2) = 28$, $h'(2) = 6 - 4 = 2$; tangent $h = 2t + 24$. (b) Estimate 28.2 m. (c) Model $= 20 + 12.6 - 4.41 = 28.19$ m; estimate is 0.01 m too high.
47. (a) $y = 3.34375$, $dy/dx = 0.45x^2 - 1.6x + 2$ gives $m_T = 0.8125$. (b) $y - 3.34 = 0.813(x - 2.5)$ (3 s.f.). (c) Using unrounded values: $y = 3.34375 + 0.8125(0.2) = 3.50625 \approx 3.51$ m.
48. (a) $2x = 6 \Rightarrow x = 3$. (b) On the line $y = 13$, so $9 + k = 13$, $k = 4$ and $P(3, 13)$. (c) $N : y = -\frac{1}{6}x + \frac{27}{2}$.
49. (a) $3a^2 + 1 = 7 \Rightarrow a = \pm\sqrt{2} \approx \pm 1.41$. (b) $f(a) = 3a$, so $y - 3a = 7(x - a)$ gives $y = 7x - 4a$. Tangents: $y = 7x - 5.66$ and $y = 7x + 5.66$.
50. (a) $dy/dx = -4x^{-2}$ gives $m_T = -1$, $m_N = 1$. $T : y = -x + 4$; $N : y = x$. (b) $A(0, 4)$, $B(0, 0)$. (c) Base $AB = 4$, perpendicular height from P to the y -axis $= 2$, so area $= \frac{1}{2}(4)(2) = 4$.