



Introduction to Integration

IB Mathematics: Applications and Interpretation SL | SL 5.5

50 questions: fluency, applications and IB-style practice

Learning goal	Find antiderivatives, use a given point to determine a function, and model areas and accumulated quantities.
Syllabus link	SL 5.5: integer powers x^n ($n \neq -1$), boundary conditions, definite integrals using technology, and area above the x -axis.
Structure	1–15 short response; 16–30 extended response; 31–50 IB exam-style.
Technology	A GDC is allowed throughout, as in AI SL examinations.  marks questions requiring numerical integration or numerical equation solving.

Instructions. Work on separate paper and show your method. Include $+C$ for general antiderivatives. Give exact algebraic answers unless a decimal is requested. For GDC calculations, retain full precision and give non-exact final values to 3 significant figures, unless stated otherwise. Write the definite integral before calculating an area. All diagrams are schematic.

1. Short response

15 questions

1. Find $\int 8 \, dx$.

2. Find $\int 6x^2 \, dx$.

3. Find $\int (4x + 3) \, dx$.

4. Find $\int (5x^4 - 2x) \, dx$.

5. Find $\int 3x^{-2} \, dx$, for $x > 0$.

6. Find $\int \left(2 - \frac{8}{x^3}\right) \, dx$, for $x > 0$.

7. Given $f'(x) = 6x + 2$ and $f(0) = 5$, find $f(x)$.

8. Given $\frac{dy}{dx} = 3x^2$ and $y = 6$ when $x = 1$, find y in terms of x .

9. Find two different functions whose derivative is $4x$.

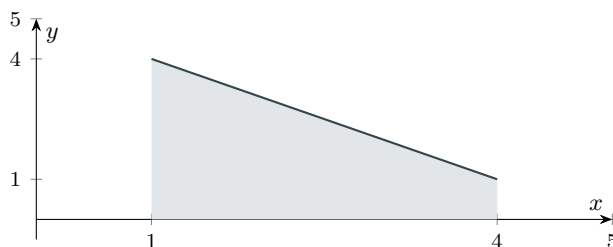
10. A student writes $\int 4x^3 \, dx = 12x^2 + C$. Explain the error and give the correct result.

11. The curve $y = x^2 + C$ passes through $(2, 7)$. Find C .

12. Complete the missing entries.

$f(x)$	One antiderivative $F(x)$
$2x$	...
...	$x^3 + 4$
$-6x^{-4}$	...

13. The shaded region is under $y = 5 - x$, between $x = 1$ and $x = 4$. Write down an integral for its area. Do not evaluate it.



14.  Use your GDC to evaluate $\int_0^3 (x^2 + 2) dx$.

15. Water enters a tank at a rate $r(t)$ litres per minute. State the units and meaning of $\int_0^5 r(t) dt$.

2. Extended response

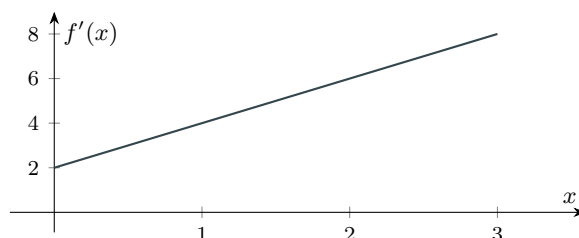
15 questions

16. A curve has gradient function $6x^2 - 4x + 3$ and passes through $(1, 8)$. Find its equation, then find its y -intercept.

17. For $x > 0$, $f'(x) = 4 + 6x^{-2}$ and $f(1) = 2$. Find $f(x)$ and $f(3)$.

18. A function satisfies $f'(x) = ax + 2$, $f(0) = 1$ and $f(2) = 13$. Find a and $f(x)$.

19. The straight-line graph shows $f'(x)$. Given $f(0) = 3$, find $f(x)$.



20. Three models are proposed for a quantity $Q(x)$.

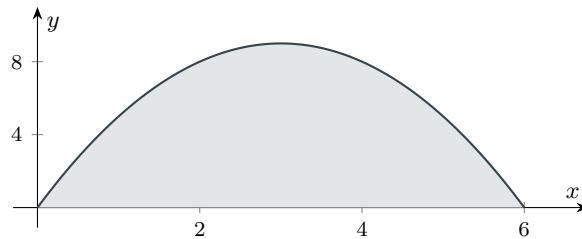
Model	$Q(x)$
A	$x^3 + 2x + 1$
B	$x^3 + 2x + 7$
C	$3x^2 + 2$

Which models satisfy $Q'(x) = 3x^2 + 2$? Which also satisfies $Q(1) = 10$? Explain.

21. Water enters a tank at rate $r(t) = 4t + 6$ litres per minute, for $0 \leq t \leq 5$. Initially the tank contains 20 litres. Find a model for the volume $V(t)$, and find $V(5)$.

22. A seedling grows at rate $h'(t) = 0.6t^2 + 0.4$ centimetres per day, for $0 \leq t \leq 4$. Its height at $t = 1$ is 5 cm. Find $h(t)$ and the increase in height from $t = 1$ to $t = 4$.

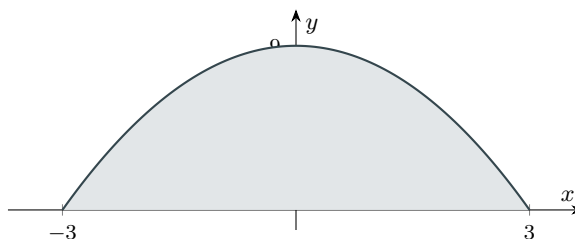
23.  The graph of $y = 6x - x^2$ meets the x -axis at O and P . Find the coordinates of P , then write and evaluate an integral for the shaded area.



24.  The area under $y = 2x + 1$ from $x = 0$ to $x = b$, where $b > 0$, is 12. Write an integral equation and use your GDC to find b .

25.  A strip of land is bounded by $y = 4 + 0.5x^2$, the x -axis, $x = 0$ and $x = 6$. Coordinates are in metres. Calculate its area and the cost of covering it at €18.50 per square metre.

26.  A pond is modelled by the region between $y = 9 - x^2$ and the x -axis. Coordinates are in metres. Find the two intercepts and the area of the pond.



27.  A panel is bounded by $y = (x - 1)(4 - x)$ and the x -axis, for $1 \leq x \leq 4$. Dimensions are in metres. Find its area. A rectangular sheet 3 m by 2.5 m is used to cut the panel. Find the percentage of the sheet used.

28.  The rate of production is $p(t) = 12 + 0.8t^2$ kilograms per hour for $0 \leq t \leq 5$. Find the mass produced during these five hours, and the time when 30 kg has been produced.

29. For $x > 0$, $f'(x) = kx^{-2}$, with $f(1) = 2$ and $f(2) = 5$. Find k and $f(x)$.

30. The table gives values of F .

x	0	1	2
$F(x)$	5	8	15

A student claims that $F'(x) = 4x + 1$. Find the function with this derivative and $F(0) = 5$. Does it agree with all three values?



3. IB exam-style practice

20 questions

31.

[Maximum mark: 5]

A function f satisfies $f'(x) = 9x^2 - 4x + 2$ and $f(1) = 6$.

- (a) Find $f(x)$. [4]
(b) Find $f(2)$. [1]

32.

[Maximum mark: 5]

A student is finding an antiderivative of $8x^3 - 6x$.

- (a) Write down the general antiderivative. [2]
(b) Find the particular antiderivative whose graph passes through $(1, 5)$. [2]
(c) Explain why the condition in part (b) is necessary to select one function. [1]

33.

[Maximum mark: 5]

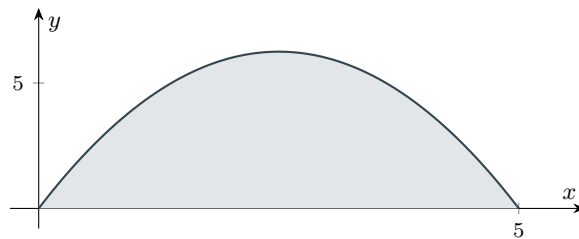
For $x > 0$, $\frac{dy}{dx} = 5 - \frac{4}{x^2}$. The curve passes through $(2, 7)$.

- (a) Find an expression for y . [4]
(b) Find y when $x = 4$. [1]

34. 

[Maximum mark: 5]

The region shown is bounded by $y = x(5 - x)$ and the x -axis.



- (a) Write down an integral for its area. [2]
(b) Calculate the area. [1]
(c) A triangle has the same area and base 5 units. Find its perpendicular height. [2]

35. 

[Maximum mark: 6]

The upper edge of a gate is modelled by $h(x) = 2.4 - 0.15x^2$, for $-2 \leq x \leq 2$. The lower edge lies on the x -axis. Coordinates are in metres.

- (a) Find the height at each end of the gate. [1]
(b) Write down an integral for the area of one face. [2]
(c) Calculate the cost of painting both faces at €12 per square metre. [3]

36. 

[Maximum mark: 6]

A tank initially contains 50 litres. Water enters at rate $r(t) = 3t^2 + 2$ litres per minute for $0 \leq t \leq 5$; none leaves.

- Find a formula for the volume $V(t)$. [3]
- Find $V(4)$. [1]
- The tank holds 120 litres. Find the time at which it becomes full. [2]

37.

[Maximum mark: 6]

A factory models its total production cost by $C(x)$ euros, where x is the number of items. It uses $C'(x) = 0.06x + 4$, for $0 \leq x \leq 100$, and has fixed cost $C(0) = 150$.

- Find $C(x)$. [3]
- Find the cost of producing 50 items. [1]
- Find the additional cost of increasing production from 50 to 60 items. [2]

38. 

[Maximum mark: 5]

The power from a solar panel is $P(t) = 0.04t(10 - t)$ kilowatts, for $0 \leq t \leq 10$, where t is hours after sunrise. Energy equals the integral of power with respect to time.

- Write down an integral for the energy generated. [2]
- Calculate this energy, in kWh. [1]
- Each kWh saves €0.28. Find the saving, to the nearest cent. [2]

39. 

[Maximum mark: 5]

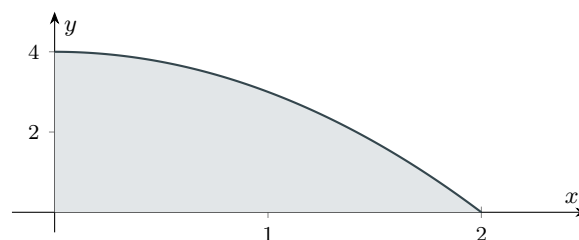
A decorative panel is the region below $y = 0.5x^2 + 1$ and above the x -axis, from $x = 0$ to $x = 4$. Coordinates are in metres.

- Calculate the area of the panel, first stating the integral used. [3]
- A rectangular opening 1.2 m by 0.8 m is cut out. Find the remaining area. [2]

40. 

[Maximum mark: 5]

The shaded region is below $y = 4 - x^2$, above the x -axis and to the right of the y -axis.



- Find its area. State your integral. [3]
- A rectangle has vertices $(0, 0)$, $(2, 0)$, $(2, 4)$ and $(0, 4)$. Find the percentage of the rectangle occupied by the shaded region. [2]

41. 

[Maximum mark: 5]

Consider $f(x) = \frac{12}{x^2}$, for $x > 0$.

- Find the general antiderivative of f . [2]
- Write down an integral for the area under $y = f(x)$ from $x = 1$ to $x = 4$. [2]
- Use your GDC to find the area. [1]

**42.**

[Maximum mark: 5]

The function $f(x) = kx^2 + 2$, where $k > 0$, encloses area 14 with the x -axis, $x = 0$ and $x = 2$.

- (a) Write an integral equation involving k . [2]
- (b) Find k . [2]
- (c) Find $f(2)$. [1]

43.

[Maximum mark: 5]

The area under $y = x^2 + 1$ from $x = 0$ to $x = b$ is 10, where $b > 0$.

- (a) Write an equation for b using a definite integral. [2]
- (b) Find b , correct to three significant figures. [2]
- (c) Find the height of the curve at this value of b . [1]

44.

[Maximum mark: 5]

A function satisfies $f'(x) = ax^2 + 2$. The graph passes through $(0, 1)$ and $(3, 25)$.

- (a) Find an expression for $f(x)$ in terms of a . [2]
- (b) Find a . [2]
- (c) Find $f(2)$. [1]

45.

[Maximum mark: 6]

The derivative of f is represented by the line through $(0, 3)$ and $(2, 7)$. Also, $f(1) = 8$.

- (a) Find $f'(x)$. [2]
- (b) Find $f(x)$. [3]
- (c) Find the y -intercept of $y = f(x)$. [1]

46.

[Maximum mark: 5]

A function has derivative $F'(x) = 3x^2 - 2x + 1$. The table gives two known values.

x	0	2
$F(x)$	4	10

- (a) Find $F(x)$ using $F(0) = 4$. [3]
- (b) Verify that your function gives the second value in the table. [1]
- (c) Find $F(-1)$. [1]

**47.**

[Maximum mark: 5]

A wall has height $h(x) = 2 + 0.25x^2$, for $0 \leq x \leq 3.6$, where all lengths are in metres.

- (a) Find the area of one face, first stating the integral. [3]
- (b) Two coats of paint are required on this face. One tin covers 8 m^2 with one coat. Find the minimum number of tins needed. [2]

48.

[Maximum mark: 6]

The region R is enclosed by $y = 5 - 0.8x^2$ and the x -axis.

- (a) Find the x -coordinates of both intercepts. [2]
- (b) Write and evaluate an integral for the area of R . [3]
- (c) Find the area of the part of R to the right of the y -axis. [1]

49.

[Maximum mark: 5]

A curve satisfies $\frac{dy}{dx} = 6x^2 + k$, where k is a constant. It passes through $(0, 2)$ and $(2, 26)$.

- (a) Find k and the equation of the curve. [4]
- (b) Find the gradient of the curve at $x = 1$. [1]

50.

[Maximum mark: 10]

A curve has derivative $f'(x) = 4 - 2x$ and passes through the origin.

- (a) Find $f(x)$. [3]
- (b) Find the second x -intercept. [2]
- (c) Calculate the area enclosed by the curve and the x -axis, stating the integral used. [3]
- (d) A triangle has base 8 units and the same area. Find its perpendicular height. [2]



Answer key and solution outlines

AI SL 5.5

Equivalent algebraic forms are accepted. For numerical answers, use unrounded intermediate values.

Section 1: Short response

1. $8x + C$.
2. $2x^3 + C$.
3. $2x^2 + 3x + C$.
4. $x^5 - x^2 + C$.
5. $-3x^{-1} + C = -\frac{3}{x} + C$.
6. $2x + 4x^{-2} + C$.
7. $f(x) = 3x^2 + 2x + C$; $C = 5$, so $f(x) = 3x^2 + 2x + 5$.
8. $y = x^3 + C$; $6 = 1 + C$, so $y = x^3 + 5$.
9. For example, $2x^2$ and $2x^2 + 1$. Any two functions $2x^2 + C$ with different constants work.
10. The student differentiated. An antiderivative is $x^4 + C$, because $\frac{d}{dx}(x^4 + C) = 4x^3$.
11. $7 = 4 + C$, so $C = 3$.
12. x^2 ; $3x^2$; $2x^{-3}$, respectively. An arbitrary constant may be added to either antiderivative.
13. $A = \int_1^4 (5 - x) dx$.
14. 15.
15. Litres; the volume of water entering the tank during the first five minutes.

Section 2: Extended response

16. $y = 2x^3 - 2x^2 + 3x + C$. At $(1, 8)$, $8 = 3 + C$, so $C = 5$. Thus $y = 2x^3 - 2x^2 + 3x + 5$, with y -intercept $(0, 5)$.
17. $f(x) = 4x - 6x^{-1} + C$; $2 = -2 + C$, so $C = 4$. Therefore $f(3) = 12 - 2 + 4 = 14$.
18. $f(x) = \frac{a}{2}x^2 + 2x + C$, $C = 1$. Then $13 = 2a + 5$, giving $a = 4$ and $f(x) = 2x^2 + 2x + 1$.
19. The gradient function is $f'(x) = 2x + 2$. Hence $f(x) = x^2 + 2x + C = x^2 + 2x + 3$.
20. A and B have derivative $3x^2 + 2$. Only B also satisfies $Q(1) = 10$, since $1 + 2 + 7 = 10$.
21. $V(t) = 2t^2 + 6t + C$ and $V(0) = 20$, so $V(t) = 2t^2 + 6t + 20$. Thus $V(5) = 100$ litres.
22. $h(t) = 0.2t^3 + 0.4t + C$; $C = 4.4$. The increase is $h(4) - h(1) = 18.8 - 5 = 13.8$ cm.
23. $x(6 - x) = 0$, so $P = (6, 0)$. $A = \int_0^6 (6x - x^2) dx = 36$ square units.
24. $\int_0^b (2x + 1) dx = 12$ gives $b = 3$. (Equivalently, $b^2 + b = 12$ and $b > 0$.)
25. $A = \int_0^6 (4 + 0.5x^2) dx = 60 \text{ m}^2$. Cost $= 60 \times 18.50 = \text{€}1110$.
26. The intercepts are $(-3, 0)$ and $(3, 0)$. $A = \int_{-3}^3 (9 - x^2) dx = 36 \text{ m}^2$.
27. $A = \int_1^4 (x - 1)(4 - x) dx = 4.5 \text{ m}^2$. Sheet area $= 7.5 \text{ m}^2$, so 60% is used.
28. $M = \int_0^5 (12 + 0.8t^2) dt = 93.3 \text{ kg}$ (3 s.f.). Solve $\int_0^T (12 + 0.8t^2) dt = 30$: $T = 2.25 \text{ h}$ (3 s.f.).
29. $f(x) = -k/x + C$. The conditions give $-k + C = 2$ and $-k/2 + C = 5$. Subtraction gives $k/2 = 3$, so $k = 6$, $C = 8$ and $f(x) = 8 - 6/x$.
30. $F(x) = 2x^2 + x + C$ and $C = 5$. This gives $F(1) = 8$ and $F(2) = 15$, so it agrees with every value. The table alone does not prove the derivative formula for all x .



Section 3: IB exam-style practice

31. (a) $f(x) = 3x^3 - 2x^2 + 2x + C$; $6 = 3 + C$, so $C = 3$. (b) $f(2) = 24 - 8 + 4 + 3 = 23$.
32. (a) $2x^4 - 3x^2 + C$. (b) $5 = 2 - 3 + C$, so $C = 6$: $2x^4 - 3x^2 + 6$. (c) Adding any constant leaves the derivative unchanged; the point fixes that constant.
33. (a) $y = 5x + 4/x + C$. $7 = 10 + 2 + C$, so $C = -5$. (b) $y = 20 + 1 - 5 = 16$.
34. (a) $A = \int_0^5 x(5 - x) dx$. (b) $A = 20.8$ square units (unrounded $125/6$). (c) $\frac{1}{2}(5)h = 125/6$, so $h = 8.33$ units (3 s.f.).
35. (a) $h(-2) = h(2) = 1.8$ m. (b) $A = \int_{-2}^2 (2.4 - 0.15x^2) dx$. (c) $A = 8.8$ m²; cost = $2(8.8)(12) = \text{€}211.20$.
36. (a) $V(t) = t^3 + 2t + C$, $C = 50$. (b) $V(4) = 122$ litres. (c) Solve $t^3 + 2t + 50 = 120$: $t = 3.96$ min (3 s.f.).
37. (a) $C(x) = 0.03x^2 + 4x + 150$. (b) $C(50) = \text{€}425$. (c) $C(60) - C(50) = 498 - 425 = \text{€}73$.
38. (a) $E = \int_0^{10} 0.04t(10 - t) dt$. (b) $E = 6.67$ kWh (3 s.f.). (c) Use the unrounded value $20/3$: saving = $\text{€}1.87$.
39. (a) $A = \int_0^4 (0.5x^2 + 1) dx = 14.7$ m² (unrounded $44/3$). (b) $44/3 - 0.96 = 13.7$ m² (3 s.f.).
40. (a) $A = \int_0^2 (4 - x^2) dx = 5.33$ square units. (b) $\frac{16/3}{8} \times 100 = 66.7\%$.
41. (a) $-12/x + C$. (b) $A = \int_1^4 12x^{-2} dx$. (c) $A = 9$ square units.
42. (a) $\int_0^2 (kx^2 + 2) dx = 14$. (b) GDC solution $k = 3.75$. (c) $f(2) = 17$.
43. (a) $\int_0^b (x^2 + 1) dx = 10$. (b) $b = 2.79$ (3 s.f.). (c) Using the unrounded b , the height is 8.77 units (3 s.f.).
44. (a) $f(x) = \frac{a}{3}x^3 + 2x + 1$. (b) $25 = 9a + 7$, so $a = 2$. (c) $f(2) = \frac{16}{3} + 5 = \frac{31}{3}$.
45. (a) Slope = $(7 - 3)/2 = 2$, so $f'(x) = 2x + 3$. (b) $f(x) = x^2 + 3x + C$, $8 = 4 + C$, hence $C = 4$. (c) $(0, 4)$.
46. (a) $F(x) = x^3 - x^2 + x + 4$. (b) $F(2) = 8 - 4 + 2 + 4 = 10$. (c) $F(-1) = -1 - 1 - 1 + 4 = 1$.
47. (a) $A = \int_0^{3.6} (2 + 0.25x^2) dx = 11.088$ m². (b) $2A/8 = 2.772$, so 3 tins are needed.
48. (a) $x = \pm 2.5$. (b) $A = \int_{-2.5}^{2.5} (5 - 0.8x^2) dx = 16.7$ square units (3 s.f.). (c) By symmetry, half the area: 8.33 square units.
49. (a) $y = 2x^3 + kx + C$, $C = 2$. $26 = 16 + 2k + 2$, so $k = 4$ and $y = 2x^3 + 4x + 2$. (b) Gradient = $6 + 4 = 10$.
50. (a) $f(x) = 4x - x^2 + C$, $C = 0$. (b) $x(4 - x) = 0$, so $(4, 0)$. (c) $A = \int_0^4 (4x - x^2) dx = 10.7$ square units (unrounded $32/3$). (d) $4h = 32/3$, so $h = 8/3 = 2.67$ units.