



Stationary Points

IB Mathematics: Applications and Interpretation SL | SL 5.6

50 questions: fluency, interpretation and IB-style practice

Learning goal	Locate and classify stationary points; distinguish local extrema from greatest and least values on a stated domain.
Syllabus link	SL 5.6: zero gradients, solving $f'(x) = 0$, local maxima and minima, derivative graphs using technology, and domain comparisons.
Structure	1–15 short response; 16–30 extended response; 31–50 IB exam-style.
Technology	A GDC is permitted throughout, as in AI SL examinations.  marks tasks requiring numerical or graphical technology.

Instructions. Work on separate paper and show your method. Give exact answers unless a decimal is requested or a numerical method is used; give numerical results to 3 significant figures. Use radians for trigonometric functions. Classify stationary points using first-derivative signs or the behaviour of the graph. Check endpoints when comparing greatest and least values on a closed interval.

1. Short response

15 questions

1. At a stationary point of a differentiable curve, state (i) the value of the gradient and (ii) the direction of the tangent.

2. The derivative changes from positive to negative as x passes through a , and $f'(a) = 0$. Classify the stationary point.

3. The derivative changes from negative to positive as x passes through b , and $f'(b) = 0$. Classify the stationary point.

4. The sign table describes a differentiable function. Is its stationary point a local maximum, a local minimum, or neither? Explain.

x	$-\infty$	2	$+\infty$
$f'(x)$		0	
	$+$		$+$

5. Find the coordinates of the stationary point of $f(x) = x^2 - 8x + 9$.

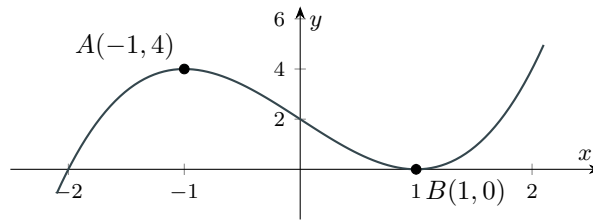
6. Find the stationary value of $f(x) = 12 + 6x - x^2$.

7. Given $f'(x) = 3(x - 1)(x + 2)$, find the x -coordinates of the stationary points.

8. Given $f'(x) = (x - 3)^2$, state the stationary x -coordinate and explain why this is not a local maximum or minimum.

9. Given $f'(x) = 2x + 4$ and $f(-2) = 7$, write down the coordinates of the stationary point.

10. The graph shows $f(x) = x^3 - 3x + 2$. Identify the local maximum and local minimum.

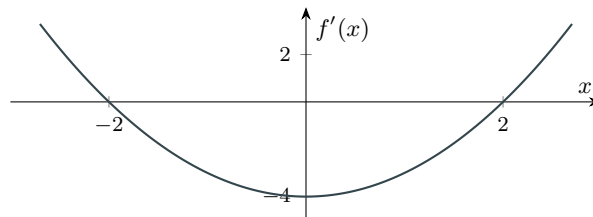


11. For $f(x) = 2x^3 - 9x^2 + 12x + 1$, write down the equation you would solve to find stationary points. Factorize it.

12. A function is defined on $[-2, 5]$. Its only interior stationary points and its endpoints are listed. State its greatest value on this domain.

Point type	x	$f(x)$
Left endpoint	-2	-8
Local maximum	1	5
Local minimum	3	1
Right endpoint	5	12

13. The graph shown is $y = f'(x)$, not $y = f(x)$. State the x -coordinate of each local extremum of f , and classify it.



14.  Use your GDC to find the stationary x -coordinates of $f(x) = x^3 - 2x^2 - 5x + 1$, to three significant figures.

15. A student claims: "Every solution of $f'(x) = 0$ is a local maximum or minimum. Use $f(x) = x^3$ to explain why this is false.

2. Extended response

15 questions

16. For $f(x) = x^3 - 6x^2 + 9x + 2$, find and classify both stationary points. State the intervals on which f is decreasing.

17. Find and classify every stationary point of $f(x) = x^4 - 8x^2 + 3$. Use the sign of the first derivative to justify your classifications.

18. Consider $f(x) = x^3 - 3x$ on $-2 \leq x \leq 3$. Find its local extrema, then its greatest and least values on the stated domain.

19. For $f(x) = (x - 2)^3 + 1$ on $[0, 4]$, find its stationary point. Explain why it is neither a local maximum nor a local minimum, and find the greatest value on the domain.

20. For $x > 0$, $f(x) = x + \frac{9}{x}$. Find and classify its stationary point using the sign of f' .

21. For $f(x) = -x^3 + 3x^2 + 9x$ on $[-2, 5]$, find all local extrema and the greatest and least values. Include endpoint checks.

22. The complete sign table of f' is shown. Classify each stationary point and state where f is decreasing.

x	$-\infty$	-3	1	4	$+\infty$
$f'(x)$	$-$	0	$+$	0	$-$

23.  Use technology to generate the derivative graph of $f(x) = 0.2x^4 - 2x^2 + x + 3$, for $-3 \leq x \leq 3$. Find and classify all stationary points.

24. A GDC shows the following values close to a zero of f' .

x	1.69	1.70	1.71
$f'(x)$	0.024	0.000	-0.026

What classification do these readings suggest at $x = 1.70$? Explain why three rounded readings alone do not prove the exact stationary coordinate.

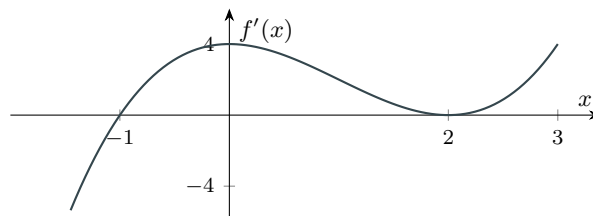
25.  For $f(x) = 2 \sin x + 0.4x$ on $0 \leq x \leq 2\pi$, use a GDC in radian mode to find and classify its interior stationary points. Then find its greatest and least values on the domain.

26. The function $f(x) = x^3 + ax^2 + bx$ has stationary points at $x = 1$ and $x = 3$. Find a and b , then classify the two points.

27. The curve $y = f(x)$ has a local maximum at $(2, 7)$ and a local minimum at $(5, 1)$. For $g(x) = f(x) + 4$, state the new extrema and explain whether the stationary x -coordinates change.

28.  For $f(x) = x^3 - 6x$ on $[-2, 2]$, find and classify its stationary points, to three significant figures. Find the greatest and least values on the domain.

29. The graph is $y = f'(x) = (x + 1)(x - 2)^2$. Explain the different behaviour of f at $x = -1$ and $x = 2$.



30.  Let $f(x) = x^3 - 4x^2 + 2$ on $[0, 5]$. Find the interior stationary point and classify it. Explain why the endpoint $x = 0$, although $f'(0) = 0$, must also be checked separately when finding the greatest and least values.



3. IB exam-style practice

20 questions

31.

[Maximum mark: 7]

Let $f(x) = x^3 - 3x^2 - 9x + 5$.

- (a) Find $f'(x)$. [2]
- (b) Find the coordinates of both stationary points. [3]
- (c) Classify each point using a change in sign of f' . [2]

32.

[Maximum mark: 7]

The temperature in a greenhouse is modelled by $T(t) = 18 + 4t - 0.5t^2$ degrees Celsius, where $0 \leq t \leq 10$ is the time in hours after 06:00.

- (a) Find $T'(t)$ and the stationary time. [2]
- (b) Classify the stationary point and find the temperature there. [2]
- (c) Find the greatest and least temperatures during the stated period. State when they occur as clock times. [3]

33.

[Maximum mark: 7]

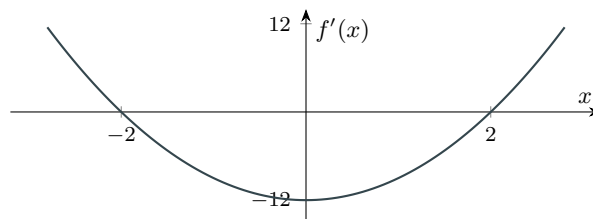
Consider $f(x) = -2x^3 + 9x^2 - 12x + 7$.

- (a) Show that $f'(x) = -6(x-1)(x-2)$. [2]
- (b) Find and classify the stationary points. [4]
- (c) State the interval on which f is increasing. [1]

34.

[Maximum mark: 5]

The graph shows $y = f'(x) = 3x^2 - 12$. It is known that $f(-2) = 9$ and $f(2) = -23$.



- (a) Write down the stationary points of f . [2]
- (b) Use the graph to classify them. [2]
- (c) State where f is decreasing. [1]

35.

[Maximum mark: 8]

Let $f(x) = x^3 - 6x^2 + 9x + 4$, for $-1 \leq x \leq 4$.

- (a) Find and classify the interior stationary points. [4]
- (b) Find the greatest and least values of f on the domain. [3]
- (c) Explain why a local minimum need not be the least value on a restricted domain. [1]

36.

[Maximum mark: 9]

The pressure in a test chamber is modelled by $P(t) = -0.4t^3 + 3t^2 - 4t + 20$ kPa, for $0 \leq t \leq 7$ minutes.

- (a) Find $P'(t)$. [2]
- (b) Find and classify the two interior stationary points. [4]
- (c) Find the greatest and least pressures during the experiment. [3]

**37.**

[Maximum mark: 7]

A function is defined by $f(x) = x + \frac{4}{x}$, for $1 \leq x \leq 5$.

- (a) Find $f'(x)$. [2]
- (b) Find and classify the interior stationary point. [3]
- (c) Find the greatest and least values on the domain. [2]

38.

[Maximum mark: 7]

Let $f(x) = (x - 1)^4 + 2$.

- (a) Expand $f(x)$ and hence find $f'(x)$. [3]
- (b) Show that the only stationary point is $(1, 2)$. [2]
- (c) A student cannot see a sign change on a very coarse GDC display. Classify the point using $f'(x) = 4(x - 1)^3$. [2]

39. 

[Maximum mark: 9]

Consider $f(x) = e^{-x^2} + 0.15x$, for $-2 \leq x \leq 2$. Use technology to generate and inspect f' .

- (a) Find all interior stationary points, giving coordinates to three significant figures. [4]
- (b) Classify each point using the sign of the derivative. [2]
- (c) Determine the greatest and least values of f on this domain. [3]

40.

[Maximum mark: 6]

A differentiable function on $[-4, 5]$ has the sign table shown.

x	-4	-2	0	3	5			
$f'(x)$		+	0	-	0	-	0	+

x	-4	-2	0	3	5
$f(x)$	1	7	4	-2	9

- (a) Classify each interior stationary point. [3]
- (b) Find the greatest and least values of f on the domain. [2]
- (c) Explain why the stationary point at $x = 0$ is not a local extremum. [1]

41.

[Maximum mark: 8]

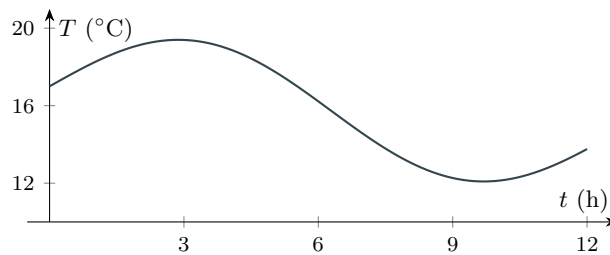
The function $f(x) = x^3 + kx + 2$ has a stationary point at $x = 2$.

- (a) Find k . [2]
- (b) Find the coordinates of both stationary points. [3]
- (c) Classify the points and state where f is decreasing. [3]

42. 

[Maximum mark: 7]

The temperature of a room is modelled by $T(t) = 17 + 3 \sin(0.5t) - 0.2t$ degrees Celsius, for $0 \leq t \leq 12$ hours. Angles are in radians.



- (a) Use your GDC to find and classify the interior stationary points. [4]
 (b) Find the greatest and least temperatures over the 12-hour period. [3]

43. 

[Maximum mark: 10]

Let $f(x) = x^4 - 4x^2 + x$, for $-2 \leq x \leq 2$.

- (a) Find $f'(x)$. [2]
 (b) Use your GDC to find and classify all stationary points. [5]
 (c) Find the greatest and least values on the domain, checking endpoints. [3]

44.

[Maximum mark: 6]

The table lists every interior stationary point and the endpoints of a differentiable model $H(t)$ on $[0, 8]$. The derivative signs immediately before and after each stationary point are also given.

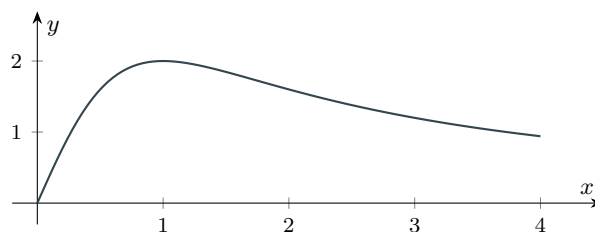
t	$H(t)$	Before: H'	After: H'
0	5	—	—
2	12	+	-
4	7	-	+
6	10	+	-
8	3	—	—

- (a) Identify the local maximum points and the local minimum point. [3]
 (b) Find the greatest and least values on the domain. [2]
 (c) Explain why the lower of the two local maximum values is still a local maximum. [1]

45. 

[Maximum mark: 7]

For $0 \leq x \leq 4$, let $f(x) = \frac{4x}{1+x^2}$. Use a GDC to generate the derivative graph.



- (a) Find the interior stationary point. [2]
 (b) Use the sign of the derivative to classify it. [2]
 (c) Find the greatest and least values on the domain. [3]



46.

[Maximum mark: 8]

Let $f(x) = \frac{x^3}{3} - 2x^2 + 3x + 7$, for $0 \leq x \leq 5$.

- (a) Find the stationary x -coordinates. [2]
- (b) Find the equation of the horizontal tangent at each stationary point. [2]
- (c) Classify these points and find the greatest value of f on the domain. [4]

47.

[Maximum mark: 8]

A function has derivative $f'(x) = -(x+1)(x-2)(x-5)$. It is known that $f(-1) = 10$, $f(2) = -\frac{41}{4}$ and $f(5) = 10$.

- (a) Find all stationary points. [2]
- (b) Construct a sign table for f' and classify the points. [4]
- (c) State the intervals on which f is increasing. [2]

48.

[Maximum mark: 11]

Consider $f(x) = 0.1x^5 - x^3 + x$, for $-3 \leq x \leq 3$.

- (a) Find $f'(x)$. [2]
- (b) Use your GDC to find and classify all interior stationary points. [6]
- (c) Determine the greatest and least values on the domain. [3]

49.

[Maximum mark: 6]

A differentiable function f has a local maximum at $(1, 8)$ and a local minimum at $(4, 2)$. Its derivative is positive for $x < 1$, negative for $1 < x < 4$, and positive for $x > 4$. Let $g(x) = -2f(x) + 3$.

- (a) Write down the stationary points of g . [2]
- (b) Classify them, explaining how multiplication by -2 changes the derivative signs. [3]
- (c) State where g is decreasing. [1]

50.

[Maximum mark: 10]

Let $f(x) = x^3 - 6x^2 + 8x + 1$, for $0 \leq x \leq 5$.

- (a) Find $f'(x)$. [2]
- (b) Find and classify the two interior stationary points. [4]
- (c) Complete a comparison of the stationary values and endpoint values. Hence find the greatest and least values of f . [3]
- (d) A student reports the local maximum as the greatest value. Explain the error. [1]



Answer key and solution outlines

AI SL 5.6

Equivalent forms are accepted. Numerical coordinates are rounded to 3 significant figures; use unrounded values for comparisons.

Section 1: Short response

1. (i) 0; (ii) horizontal.
2. A local maximum: f increases before a and decreases after a .
3. A local minimum: f decreases before b and increases after b .
4. Neither: $f'(2) = 0$, but f is increasing on both sides.
5. $f'(x) = 2x - 8 = 0 \Rightarrow x = 4$; $f(4) = -7$, so $(4, -7)$.
6. $6 - 2x = 0 \Rightarrow x = 3$; the stationary value is $f(3) = 21$.
7. $x = -2$ and $x = 1$.
8. $x = 3$; the derivative is positive on both sides, so there is no change between increasing and decreasing.
9. $2x + 4 = 0 \Rightarrow x = -2$; the point is $(-2, 7)$.
10. Local maximum $A(-1, 4)$; local minimum $B(1, 0)$.
11. $f'(x) = 6x^2 - 18x + 12 = 0$, or $6(x - 1)(x - 2) = 0$.
12. The greatest value is 12, at the right endpoint $x = 5$.
13. $x = -2$: local maximum $(+ \rightarrow -)$. $x = 2$: local minimum $(- \rightarrow +)$.
14. Solve $3x^2 - 4x - 5 = 0$: $x = -0.786$ and $x = 2.12$.
15. $f'(x) = 3x^2 = 0$ at $x = 0$, but the derivative is positive on both sides; $(0, 0)$ is neither a local maximum nor a local minimum.

Section 2: Extended response

16. $f' = 3(x - 1)(x - 3)$. Signs $+$, $-$, $+$: maximum $(1, 6)$; minimum $(3, 2)$. Decreasing for $1 < x < 3$.
17. $f' = 4x(x - 2)(x + 2)$. Signs $-$, $+$, $-$, $+$: minima $(-2, -13)$ and $(2, -13)$; maximum $(0, 3)$.
18. $f' = 3(x - 1)(x + 1)$: local maximum $(-1, 2)$ and minimum $(1, -2)$. Endpoint values: $f(-2) = -2$, $f(3) = 18$. Greatest 18 at $x = 3$; least -2 at $x = -2$ and $x = 1$.
19. $f' = 3(x - 2)^2$, so the stationary point is $(2, 1)$. The derivative is positive on both sides. The function increases throughout; greatest value $f(4) = 9$.
20. $f' = 1 - 9/x^2 = 0 \Rightarrow x = 3$. It is negative for $0 < x < 3$ and positive for $x > 3$, giving a minimum at $(3, 6)$.
21. $f' = -3(x - 3)(x + 1)$: local minimum $(-1, -5)$ and maximum $(3, 27)$. $f(-2) = 2$, $f(5) = -5$. Greatest 27 at 3; least -5 at -1 and 5.
22. $x = -3$: local minimum; $x = 1$: neither; $x = 4$: local maximum. Decreasing for $x < -3$ and $x > 4$.
23. $f' = 0.8x^3 - 4x + 1$. Solve $f' = 0$ numerically and inspect its signs: local minimum $(-2.35, -4.30)$; local maximum $(0.253, 3.13)$; local minimum $(2.10, 0.170)$.
24. They suggest a local maximum $(+ \rightarrow -)$. A displayed 0.000 may be a small nonzero number; use a root calculation and inspect the derivative locally with sufficient precision.
25. Generate f' with the GDC; its zeros give local maximum $(1.77, 2.67)$; local minimum $(4.51, -0.155)$. Endpoints: $f(0.00) = 0.00$, $f(2\pi) = 2.51$. Greatest value 2.67 at $x = 1.77$; least value -0.155 at $x = 4.51$.
26. $f' = 3x^2 + 2ax + b = 3(x - 1)(x - 3)$, so $a = -6$, $b = 9$. Signs $+$, $-$, $+$: maximum $(1, 4)$ and minimum $(3, 0)$.
27. Maximum $(2, 11)$; minimum $(5, 5)$. Adding a constant shifts the graph vertically without changing its gradients, so the stationary x -coordinates are unchanged.
28. $f' = 3x^2 - 6 = 0 \Rightarrow x = \pm\sqrt{2}$. Maximum $(-1.41, 5.66)$; minimum $(1.41, -5.66)$. Endpoints give 4, -4 , so greatest $4\sqrt{2} \approx 5.66$; least $-4\sqrt{2} \approx -5.66$.
29. At -1 the derivative changes from negative to positive: a local minimum of f . At 2 it touches zero but stays positive: a stationary point which is neither a maximum nor a minimum.



30. $f' = x(3x - 8)$, so the interior point is $(8/3, -202/27)$, a minimum $(- \rightarrow +)$. The endpoint $x = 0$ has no two-sided neighbourhood in the restricted domain. Compare $f(0) = 2$, $f(5) = 27$ and $f(8/3) = -202/27$: greatest 27; least $-202/27 \approx -7.48$.

Section 3: IB exam-style practice

- 31.** (a) $f' = 3x^2 - 6x - 9 = 3(x + 1)(x - 3)$. (b) $(-1, 10)$ and $(3, -22)$. (c) Signs $+, -, +$: local maximum at $(-1, 10)$ and local minimum at $(3, -22)$.
- 32.** (a) $T' = 4 - t = 0$ at $t = 4$. (b) $+ \rightarrow -$, so local maximum 26°C . (c) $T(0) = 18$, $T(10) = 8$: greatest 26°C at 10:00; least 8°C at 16:00.
- 33.** (a) $f' = -6x^2 + 18x - 12 = -6(x - 1)(x - 2)$. (b) Signs $-, +, -$: minimum $(1, 2)$; maximum $(2, 3)$. (c) $1 < x < 2$.
- 34.** (a) $(-2, 9)$ and $(2, -23)$. (b) Maximum at $-2 (+ \rightarrow -)$; minimum at $2 (- \rightarrow +)$. (c) $-2 < x < 2$.
- 35.** (a) $f' = 3(x - 1)(x - 3)$: maximum $(1, 8)$, minimum $(3, 4)$. (b) $f(-1) = -12$, $f(4) = 8$: greatest 8 at 1 and 4; least -12 at -1 . (c) Endpoint values can be smaller than an interior local minimum.
- 36.** (a) $P'(t) = -1.2t^2 + 6t - 4$. (b) local minimum $(0.792, 18.5)$; local maximum $(4.21, 26.5)$. (c) Endpoints: $P(0.00) = 20.0$, $P(7.00) = 1.80$. Greatest value 26.5 at $t = 4.21$; least value 1.80 at $t = 7.00$. Pressure is in kPa; time is in minutes.
- 37.** (a) $f' = 1 - 4/x^2$. (b) $x = 2$, $f(2) = 4$; signs $- \rightarrow +$ give a minimum. (c) $f(1) = 5$, $f(5) = 29/5$: greatest $29/5$ at 5; least 4 at 2.
- 38.** (a) $f = x^4 - 4x^3 + 6x^2 - 4x + 3$; $f' = 4x^3 - 12x^2 + 12x - 4$. (b) $4(x - 1)^3 = 0 \Rightarrow x = 1$; $f(1) = 2$. (c) Negative for $x < 1$, positive for $x > 1$: local minimum.
- 39.** (a), (b) Solve $f'(x) = 0$ numerically and inspect the derivative signs: local maximum $(0.0754, 1.01)$; local minimum $(1.78, 0.309)$. (c) Endpoints: $f(-2.00) = -0.282$, $f(2.00) = 0.318$. Greatest value 1.01 at $x = 0.0754$; least value -0.282 at $x = -2.00$.
- 40.** (a) $x = -2$: maximum; $x = 0$: neither; $x = 3$: minimum. (b) Greatest 9 at 5; least -2 at 3. (c) The derivative is negative on both sides of 0, so f continues decreasing.
- 41.** (a) $f' = 3x^2 + k$; $12 + k = 0$, so $k = -12$. (b) $3x^2 - 12 = 0$: $(-2, 18)$ and $(2, -14)$. (c) Maximum at -2 , minimum at 2; decreasing for $-2 < x < 2$.
- 42.** (a) The numerical derivative graph gives local maximum $(2.87, 19.4)$; local minimum $(9.69, 12.1)$. (b) Endpoints: $T(0.00) = 17.0$, $T(12.0) = 13.8$. Greatest value 19.4 at $t = 2.87$; least value 12.1 at $t = 9.69$. Temperature is in $^\circ\text{C}$; time is in hours.
- 43.** (a) $f'(x) = 4x^3 - 8x + 1$. (b) local minimum $(-1.47, -5.44)$; local maximum $(0.126, 0.0627)$; local minimum $(1.35, -2.62)$. (c) Endpoints: $f(-2.00) = -2.00$, $f(2.00) = 2.00$. Greatest value 2.00 at $x = 2.00$; least value -5.44 at $x = -1.47$.
- 44.** (a) Maxima $(2, 12)$ and $(6, 10)$; minimum $(4, 7)$. (b) Greatest 12 at 2; least 3 at 8. (c) A local maximum is greater than nearby values; it need not exceed values elsewhere on the domain.
- 45.** (a) $(1, 2)$. (b) The derivative is positive for $0 < x < 1$ and negative for $1 < x < 4$: local maximum. (c) $f(0) = 0$, $f(4) = 16/17$; greatest 2 at 1, least 0 at 0.
- 46.** (a) $f' = (x - 1)(x - 3)$: $x = 1, 3$. (b) $y = 25/3$ and $y = 7$. (c) Signs $+, -, +$: maximum $(1, 25/3)$; minimum $(3, 7)$. Compare $f(0) = 7$, $f(5) = 41/3$: greatest $41/3$ at 5.
- 47.** (a) $(-1, 10)$, $(2, -41/4)$, $(5, 10)$. (b) Signs $+, -, +, -$: maxima at -1 and 5; minimum at 2. (c) Increasing for $x < -1$ and $2 < x < 5$.
- 48.** (a) $f'(x) = 0.5x^4 - 3x^2 + 1$. (b) local maximum $(-2.38, 3.47)$; local minimum $(-0.595, -0.392)$; local maximum $(0.595, 0.392)$; local minimum $(2.38, -3.47)$. (c) Endpoints: $f(-3.00) = -0.300$, $f(3.00) = 0.300$. Greatest value 3.47 at $x = -2.38$; least value -3.47 at $x = 2.38$.
- 49.** (a) $(1, -13)$ and $(4, -1)$. (b) $g' = -2f'$, so every nonzero sign reverses: minimum at $(1, -13)$, maximum at $(4, -1)$. (c) $x < 1$ and $x > 4$.
- 50.** (a) $f'(x) = 3x^2 - 12x + 8$. (b) local maximum $(0.845, 4.08)$; local minimum $(3.15, -2.08)$. (c) Endpoints: $f(0.00) = 1.00$, $f(5.00) = 16.0$. Greatest value 16.0 at $x = 5.00$; least value -2.08 at $x = 3.15$. (d) The endpoint value at $x = 5$ exceeds the local maximum; local comparisons alone do not find the greatest value on the full domain.