

The trapezoidal rule

IB Mathematics AI SL

50 questions | Functions, tables, contexts and error interpretation

Learning goal

Estimate areas and accumulated quantities from functions and equally spaced data.

Syllabus link

SL 5.8: approximating areas using the trapezoidal rule; links to SL 5.5 and SL 1.6.

Structure

15 short + 15 extended + 20 exam-style questions.

Technology



A GDC is permitted throughout, including exam-style questions (as in AI Paper 1).

Instructions. Show the strip width and substituted trapezoidal sum. Give non-exact numerical answers to 3 significant figures, unless stated otherwise; give money to the nearest cent. Use radians for trigonometric functions. Work on separate paper.

1. Short response

- The interval $1 \leq x \leq 9$ is divided into four equal strips. Find the strip width h .
- An interval of length 15 is divided into strips of width 2.5. State the number of trapezoids and the number of ordinates required.
- Use one trapezoid to estimate the area under a positive curve between $x = 0$ and $x = 3$, given $f(0) = 4$ and $f(3) = 10$.

- Use the trapezoidal rule to estimate the area represented by the table.

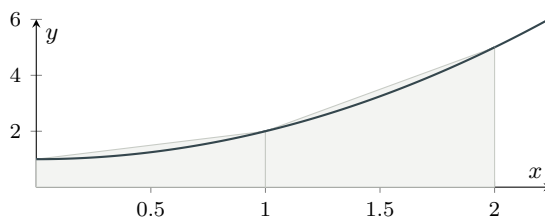
x	0	1	2
y	2	5	8

- Estimate the area under the curve from $x = 1$ to $x = 5$.

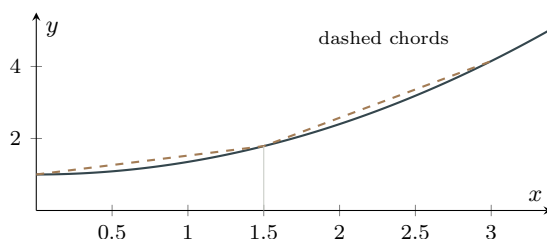
x	1	2	3	4	5
y	3	4	6	5	2

- For five equally spaced ordinates y_0, y_1, y_2, y_3, y_4 , write the trapezoidal estimate in terms of h and the ordinates.
- A student writes $T = \frac{h}{2}[2y_0 + y_1 + y_2 + 2y_3]$ for three strips. Correct the expression.

- Use two equal trapezoids to estimate $\int_0^2 (x^2 + 1) dx$.



9. Use three equal trapezoids to estimate $\int_0^3 (2x + 1) dx$.
10. The horizontal coordinate is measured in metres and $f(x)$ is measured in metres. State the units of the trapezoidal area estimate.
11. A discharge rate is measured in litres per minute. A trapezoidal estimate is made over an interval measured in minutes. State what the estimate represents and give its units.
12. The trapezoidal approximation to an area is 24.8 cm^2 ; the exact area is 24 cm^2 . Find the absolute error and percentage error.
13. Explain why the equal-width composite trapezoidal formula cannot be applied directly to values at $x = 0, 1, 3, 4$.
14. A positive straight-line graph is approximated using trapezoids. Explain why the estimated area is exact.
15. The curve is positive and concave up. Straight chords join consecutive sampled points. State, with a reason, whether the trapezoidal estimate is above or below the true area.

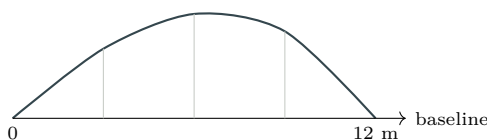


2. Extended response

16. A pond has widths measured at equal distances along a straight baseline.

Distance (m)	0	3	6	9	12
Width (m)	0	4	6	5	0

- (a) Estimate the surface area of the pond.
- (b) State one modelling assumption.



17. For $f(x) = x^2 + 2$, on $0 \leq x \leq 4$,
- (a) complete a table with step size 1;
- (b) find the trapezoidal estimate using four strips;
- (c) use your GDC to find the integral and compare it with the estimate.
18. Use two strips and then four strips to estimate $\int_0^4 \sqrt{x+1} dx$. Compare both with a GDC numerical integral.

19. The trapezoidal estimate for the data below is 30 square units. Find k .

x	0	2	4	6
y	2	k	6	4

20. Water enters a tank at the rates shown.

Time (min)	0	2	4	6	8
Rate (L/min)	5	7	8	6	4

- (a) Estimate the volume entering during the eight minutes.
 (b) The tank initially holds 120 litres. Estimate its final volume.

21. Use four equal strips to estimate $\int_0^2 e^x dx$. Give the estimate and its percentage error relative to a GDC numerical integral.

22. Use four equal strips to estimate the area under $y = \sin x$ for $0 \leq x \leq \pi$. Use radians. Compare your estimate with the exact area, 2.

23. A trapezoidal estimate for $\int_1^5 f(x) dx$ is 18. Using exactly the same sampling points, find the corresponding estimate for $\int_1^5 [3f(x) + 2] dx$.

24. Use four trapezoids and then two trapezoids to estimate the area from the same data. Explain why the table alone cannot prove which is closer to the true area.

x	0	1	2	3	4
y	1	3	4	3	1

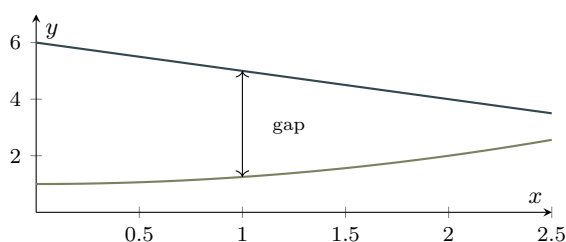
25. At equally spaced positions, the widths of a garden are recorded.

Distance (m)	0	2	4	6	8
Width (m)	2	3	4	3	2

- (a) Estimate the area.
 (b) Every width is known to within 0.1 m. Find a lower and an upper bound for the trapezoidal estimate. Treat the distances as exact.

26. The width of a decorative panel is $w(x) = 4 + 0.2x^2$ metres, for $0 \leq x \leq 6$. Use three equal strips to estimate its area and then the cost of coating it at €12 per square metre.

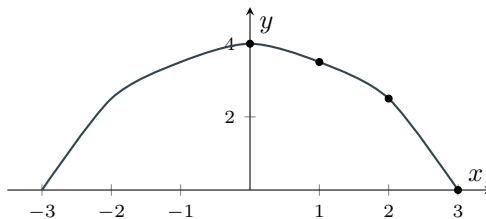
27. The region lies between $y = 6 - x$ and $y = 1 + \frac{1}{4}x^2$, from $x = 0$ to $x = 2$. Use four equal trapezoids applied to the vertical gap to estimate its area.



28. The outline represents a symmetrical cross-section. Heights on its right-hand half are shown.

x	0	1	2	3
y	4	3.5	2.5	0

The full region is reflected in the y -axis. Estimate its total area.



29. A student uses the trapezoidal rule on this table and reports 20.

x	0	1	2	3	4
y	0	4	6	4	0

- (a) Calculate the correct estimate.
(b) Suggest an endpoint/interior-weighting mistake that would produce 20.

30. A smooth positive function has estimated areas $T_2 = 18.6$, $T_4 = 17.4$ and $T_8 = 17.1$.

- (a) State which estimate you would normally choose.
(b) Explain why you cannot claim that its error is exactly 0.3.
(c) State what further information would allow the percentage error to be calculated.

3. IB exam-style practice — calculator permitted

31. A surveyor records the perpendicular width of a small lake at five equally spaced positions along a baseline.

Distance (m)	0	5	10	15	20
Width (m)	0	6	9	7	0

- (a) Write down the width of each strip. [1 marks]
(b) Use the trapezoidal rule to estimate the area of the lake. [3 marks]
(c) A treatment requires 0.08 kg per square metre. Estimate the mass required. [2 marks]

32. Let $f(x) = \frac{1}{x+1}$, for $0 \leq x \leq 2$.

- (a) Construct a table of values at intervals of 0.5. [2 marks]
(b) Use four trapezoids to estimate the area under the curve. [3 marks]
(c) Find a GDC value of the integral and calculate the percentage error of your estimate. [3 marks]

33. The power used by a machine varies over four hours.

Time (hours)	0	1	2	3	4
Power (kW)	2	3	5	4	2

- (a) Estimate the energy used, in kWh. [3 marks]
- (b) Electricity costs €0.24 per kWh. Estimate the cost. [2 marks]
- (c) Explain why joining the points with straight lines is an approximation. [1 marks]

34. The area under a positive curve is estimated using four equal trapezoids. The estimate is 46.

x	0	2	4	6	8
y	3	k	8	k	5

- (a) Form an equation in k . [2 marks]
- (b) Find k . [2 marks]
- (c) The last ordinate is corrected from 5 to 5.6. Find the revised estimate. [2 marks]

35. A laboratory pump delivers a chemical at the following rates.

Time (min)	0	1	2	3	4
Rate (mg/min)	0.8	1.2	1.6	1.0	0.4

- (a) Estimate the mass delivered over four minutes. [3 marks]
- (b) A container already contains 2.5 mg. Estimate the total mass after four minutes. [1 marks]
- (c) The concentration must not exceed 0.05 mg/mL. Using your estimate, find the minimum total solution volume required. [2 marks]

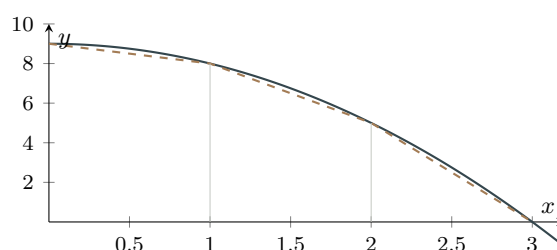
36. A designer measures the height of a logo above a baseline.

Position (cm)	0	2	4	6	8	10
Height (cm)	0	3	5	4	2	0

- (a) Estimate the area of the logo. [3 marks]
- (b) Both dimensions are enlarged by a factor of 1.5. Find the corresponding area estimate for the enlarged logo. [2 marks]
- (c) Explain why multiplying the original area by 1.5 would be incorrect. [1 marks]

37. The curve $y = 9 - x^2$ is shown for $0 \leq x \leq 3$.

- (a) Use three equal trapezoids to estimate the area under the curve. [3 marks]
- (b) Explain geometrically why this is an underestimate. [1 marks]
- (c) The exact area is 18. Find the percentage error. [2 marks]



38. Consider the region under $y = 5e^{-0.4x}$ for $0 \leq x \leq 3$.

- (a) Find the strip width for six equal trapezoids. [1 marks]
- (b) Calculate the trapezoidal area estimate. [4 marks]
- (c) Use a numerical integral to determine whether the estimate is too large or too small. [2 marks]

39. Rainfall intensity is recorded for one hour.

Time (min)	0	15	30	45	60
Intensity (mm/hour)	0	12	24	18	6

- (a) Write down the strip width in hours. [1 marks]
- (b) Estimate the total rainfall depth. [3 marks]
- (c) Estimate the volume collected on a horizontal roof of area 80 m^2 , in litres. Use $1 \text{ mm over } 1 \text{ m}^2 = 1 \text{ litre}$. [2 marks]

40. A storage channel has the cross-section shown by these measurements. Its cross-section is constant along a length of 6 m.

Horizontal position (m)	-4	-2	0	2	4
Depth (m)	0	3	4	3	0

- (a) Estimate the cross-sectional area. [3 marks]
- (b) Hence estimate the capacity of the channel in cubic metres. [2 marks]
- (c) Convert this capacity to litres. [1 marks]

41. The temperature in a chamber is measured every two hours.

Time (hours)	0	2	4	6
Temperature ($^{\circ}\text{C}$)	18	24	27	21

The mean temperature over six hours is defined by $\bar{T} = \frac{1}{6} \int_0^6 T(t) dt$.

- (a) Estimate $\int_0^6 T(t) dt$ using the trapezoidal rule. [3 marks]
- (b) Hence estimate the mean temperature. [2 marks]

42. A trapezoidal estimate uses $h = 0.5$ and seven ordinates. Its value is 12.6.

- (a) Find the total width of the interval. [2 marks]
- (b) One interior ordinate is increased by 0.8. Find the revised estimate. [2 marks]
- (c) Instead, only the final endpoint ordinate is increased by 0.8. Find the revised estimate. [2 marks]

43. The following data are available.

x	0	1	2	4	6
y	1	2	4	5	3

- (a) Select four ordinates that can be used for three equal trapezoids over $0 \leq x \leq 6$. [2 marks]
- (b) Use these ordinates to estimate the area. [3 marks]
- (c) Explain why using all five ordinates with $h = 1.5$ would be invalid. [1 marks]



44. Five equally spaced widths of a plot are 0, 8, 11, 7, 0 metres. The trapezoidal area estimate is 156 m^2 .

- (a) Find the distance h between consecutive measurements. [3 marks]
- (b) Find the total length of the baseline. [1 marks]
- (c) Fertilizer covers 20 m^2 per bag. Find the smallest whole number of bags required according to the estimate. [2 marks]

45. A curved boundary is sampled as follows.

x	0	1	2	3	4
y	2	3	5	6	8

- (a) Calculate the area estimate using two equal trapezoids. [2 marks]
- (b) Calculate the estimate using four equal trapezoids. [3 marks]
- (c) Explain why the difference between these estimates is not necessarily the error in the four-strip estimate. [1 marks]

46. Let $f(x) = a + x^2$, where $a > 0$. Two equal trapezoids over $0 \leq x \leq 2$ give an area estimate of 14.

- (a) Express the three ordinates in terms of a . [2 marks]
- (b) Find a . [3 marks]
- (c) For this value of a , calculate the estimate using four equal trapezoids. [3 marks]

47. A strip of land lies between two boundaries. Their perpendicular distances from the same baseline are shown.

Position (m)	0	2	4	6
Outer boundary (m)	7	9	10	8

Position (m)	0	2	4	6
Inner boundary (m)	2	3	4	3

- (a) Write down the four widths of the strip of land. [2 marks]
- (b) Estimate the area of the strip. [3 marks]
- (c) Estimate its value at €45 per square metre. [1 marks]

48. A model for a pond width is $w(x) = 2 + \sin(\pi x/6)$ metres, for $0 \leq x \leq 6$.

- (a) Use six equal trapezoids to estimate the surface area. [4 marks]
- (b) Use your GDC to obtain a numerical integral. [1 marks]
- (c) Calculate the percentage error and decide whether it is below 1%. [2 marks]

49. The widths of a fabric panel are measured to the nearest centimetre; its length positions are exact.

Position (cm)	0	10	20	30	40
Width (cm)	12	18	22	18	12

- (a) Estimate the panel area. [3 marks]
- (b) Find lower and upper bounds for the trapezoidal estimate due only to rounding the widths. [3 marks]
- (c) Explain why these are not guaranteed bounds for the true area of the panel. [1 marks]



50. The rate at which a reservoir receives water is modelled by $r(t) = 20 + 8 \sin(\pi t/12)$ cubic metres per hour, for $0 \leq t \leq 12$.

- (a) Use six equal trapezoids to estimate the volume entering in twelve hours. [4 marks]
- (b) The reservoir initially contains 500 m^3 . During the same period, 50 m^3 is removed. Estimate its final volume. [2 marks]
- (c) Use a numerical integral to assess whether the estimated inflow is too large or too small. [2 marks]

Answer key and solution outlines

All trapezoidal estimates use $T = \frac{h}{2} [y_0 + 2(y_1 + \dots + y_{n-1}) + y_n]$, where $h = (b - a)/n$. Unrounded values are used unless stated otherwise.

1. Short response

1. $h = (9 - 1)/4 = 2$.
2. $n = 15/2.5 = 6$ trapezoids; $n + 1 = 7$ ordinates.
3. $T = \frac{3}{2}(4 + 10) = 21$ square units.
4. $T = \frac{1}{2}(2 + 2(5) + 8) = 10$ square units.
5. $T = \frac{1}{2}[3 + 2(4 + 6 + 5) + 2] = 17.5$ square units.
6. $T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4)$.
7. $T = \frac{h}{2}[y_0 + 2y_1 + 2y_2 + y_3]$: interior ordinates occur twice; endpoints occur once.
8. $h = 1$, ordinates 1, 2, 5; $T = \frac{1}{2}(1 + 4 + 5) = 5$.
9. Ordinates 1, 3, 5, 7; $T = \frac{1}{2}[1 + 2(3 + 5) + 7] = 12$.
10. Square metres, m^2 .
11. The total volume discharged, in litres.
12. Absolute error = 0.8 cm^2 ; percentage error = $\frac{0.8}{24} \times 100\% = 3.33\%$.
13. The widths are 1, 2, 1, so there is no single common strip width h .
14. Each straight upper edge coincides with the graph, so no area is added or omitted.
15. Above: each chord lies above the curve, so the trapezoids include extra area.

2. Extended response

16. $T = \frac{3}{2}[0 + 2(4 + 6 + 5) + 0] = 45 \text{ m}^2$. The boundary is approximated by straight segments between measurements.
17. Ordinates 2, 3, 6, 11, 18. $T = \frac{1}{2}[2 + 2(3 + 6 + 11) + 18] = 30$. GDC integral = $88/3 \approx 29.3$; the estimate is $2/3$ too large.
18. $T_2 = \frac{2}{2}[1 + 2\sqrt{3} + \sqrt{5}] \approx 6.70$. $T_4 = \frac{1}{2}[1 + 2(\sqrt{2} + \sqrt{3} + 2) + \sqrt{5}] \approx 6.76$. Integral ≈ 6.787 ; four strips give the closer estimate.
19. $30 = \frac{2}{2}[2 + 2(k + 6) + 4] = 18 + 2k$, so $k = 6$.
20. $T = \frac{2}{2}[5 + 2(7 + 8 + 6) + 4] = 51$ litres. Final volume = $120 + 51 = 171$ litres.
21. $h = 0.5$, $T = \frac{0.5}{2}[1 + 2(e^{0.5} + e^1 + e^{1.5}) + e^2] \approx 6.52$. Integral $e^2 - 1 \approx 6.38906$; percentage error $\approx 2.07\%$.
22. $h = \pi/4$; $T = \frac{\pi}{8}[0 + 2(\frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2}) + 0] = \frac{\pi}{4}(1 + \sqrt{2}) \approx 1.90$. Underestimate by ≈ 0.104 .
23. $3T + 2(5 - 1) = 3(18) + 8 = 62$. The constant contributes its exact rectangular area.
24. $T_4 = \frac{1}{2}[1 + 2(3 + 4 + 3) + 1] = 11$. $T_2 = \frac{2}{2}[1 + 2(4) + 1] = 10$. Values between the sample points are unknown, so the true area is not determined by the table.
25. $T = \frac{2}{2}[2 + 2(3 + 4 + 3) + 2] = 24 \text{ m}^2$. Maximum change = $\frac{2}{2}(0.1 + 6(0.1) + 0.1) = 0.8$. Bounds: 23.2 and 24.8 m^2 ; these bound the estimate, not necessarily the true area.
26. $h = 2$; widths 4, 4.8, 7.2, 11.2. $T = \frac{2}{2}[4 + 2(4.8 + 7.2) + 11.2] = 39.2 \text{ m}^2$; cost = €470.40.
27. Gap $g(x) = 5 - x - \frac{1}{4}x^2$ gives 5, 4.4375, 3.75, 2.9375, 2. Thus $T = \frac{0.5}{2}[5 + 2(4.4375 + 3.75 + 2.9375) + 2] = 7.3125 \approx 7.31$ square units.
28. Half-area = $\frac{1}{2}[4 + 2(3.5 + 2.5) + 0] = 8$. Total area ≈ 16 square units.
29. Correct $T = \frac{1}{2}[0 + 2(4 + 6 + 4) + 0] = 14$. For example, counting the middle ordinate twice inside the interior sum gives $\frac{1}{2}[2(4 + 6 + 6 + 4)] = 20$.

30. Normally choose T_8 , using narrower strips. 0.3 is the change between estimates, not the distance from the true area. The true area, or an independently sufficiently accurate value, is needed.

3. IB exam-style practice

- 31.** (a) $h = 5$ m. (b) $T = \frac{5}{2}[0 + 2(6 + 9 + 7) + 0] = 110 \text{ m}^2$. (c) $0.08(110) = 8.80$ kg.
- 32.** (a) $1, \frac{2}{3}, \frac{1}{2}, \frac{2}{5}, \frac{1}{3}$. (b) $T = \frac{1}{4}[1 + 2(\frac{2}{3} + \frac{1}{2} + \frac{2}{5}) + \frac{1}{3}] = \frac{67}{60} \approx 1.12$. (c) Integral ≈ 1.09861 ; error $\approx 1.65\%$.
- 33.** (a) $T = \frac{1}{2}[2 + 2(3 + 5 + 4) + 2] = 14$ kWh. (b) Cost = €3.36. (c) Power may vary non-linearly between measurements.
- 34.** (a) $46 = \frac{2}{2}[3 + 2(k + 8 + k) + 5] = 24 + 4k$. (b) $k = 5.5$. (c) Increase = $\frac{2}{2}(0.6) = 0.6$; revised estimate 46.6.
- 35.** (a) $T = \frac{1}{2}[0.8 + 2(1.2 + 1.6 + 1.0) + 0.4] = 4.4$ mg. (b) 6.9 mg. (c) $6.9/V \leq 0.05$, so $V \geq 138$ mL.
- 36.** (a) $T = \frac{2}{2}[0 + 2(3 + 5 + 4 + 2) + 0] = 28 \text{ cm}^2$. (b) $1.5^2(28) = 63 \text{ cm}^2$. (c) Area scales with the square of the length scale factor.
- 37.** (a) Ordinates 9, 8, 5, 0; $T = \frac{1}{2}[9 + 2(8 + 5) + 0] = 17.5$. (b) Chords lie below this concave-down curve. (c) $\frac{0.5}{18} \times 100\% = 2.78\%$.
- 38.** (a) $h = 0.5$. (b) $T = \frac{0.5}{2}[5 + 2\sum_{i=1}^5 5e^{-0.2i} + 5e^{-1.2}] \approx 8.76$. (c) Integral ≈ 8.73507 ; $T \approx 8.76417$ is too large.
- 39.** (a) $h = 0.25$ hour. (b) $T = \frac{0.25}{2}[0 + 2(12 + 24 + 18) + 6] = 14.25$ mm. (c) $80(14.25) = 1140$ litres.
- 40.** (a) $T = \frac{2}{2}[0 + 2(3 + 4 + 3) + 0] = 20 \text{ m}^2$. (b) $V \approx 6(20) = 120 \text{ m}^3$. (c) 120 000 litres.
- 41.** (a) Estimate = $\frac{2}{2}[18 + 2(24 + 27) + 21] = 141^\circ\text{C} \cdot \text{h}$. (b) Mean = $141/6 = 23.5^\circ\text{C}$.
- 42.** (a) Six strips give width $6(0.5) = 3$. (b) Increase = $h(0.8) = 0.4$; 13.0. (c) Increase = $\frac{h}{2}(0.8) = 0.2$; 12.8.
- 43.** (a) Use $x = 0, 2, 4, 6$, with ordinates 1, 4, 5, 3. (b) $T = \frac{2}{2}[1 + 2(4 + 5) + 3] = 22$. (c) The five available points are not spaced 1.5 units apart.
- 44.** (a) $156 = \frac{h}{2}[0 + 2(8 + 11 + 7) + 0] = 26h$, so $h = 6$ m. (b) Length $4h = 24$ m. (c) $\lceil 156/20 \rceil = 8$ bags.
- 45.** (a) $T_2 = 20$. (b) $T_4 = 19$, using $\frac{1}{2}[2 + 2(3 + 5 + 6) + 8]$. (c) Both are approximations; the true area is unknown. Their difference is 1, which need not equal either error.
- 46.** (a) $a, a + 1, a + 4$. (b) $14 = \frac{1}{2}[a + 2(a + 1) + a + 4] = 2a + 3$, so $a = \frac{11}{2}$. (c) Ordinates 5.5, 5.75, 6.5, 7.75, 9.5; $T_4 = 13.75$.
- 47.** (a) Widths 5, 6, 6, 5 m. (b) $T = \frac{2}{2}[5 + 2(6 + 6) + 5] = 34 \text{ m}^2$. (c) Value $\approx$ €1530.
- 48.** (a) Widths $2, 2.5, 2 + \frac{\sqrt{3}}{2}, 3, 2 + \frac{\sqrt{3}}{2}, 2.5, 2$. $T = 14 + \sqrt{3} \approx 15.7 \text{ m}^2$. (b) Integral = $12 + 12/\pi \approx 15.8197$. (c) Error $\approx 0.554\%$, below 1%.
- 49.** (a) $T = \frac{10}{2}[12 + 2(18 + 22 + 18) + 12] = 700 \text{ cm}^2$. (b) Each width has uncertainty 0.5 cm, giving area uncertainty $40(0.5) = 20 \text{ cm}^2$. Bounds 680, 720 cm^2 . (c) Curvature between measurements creates an additional approximation error.
- 50.** (a) $h = 2$; rates 20, 24, $20 + 4\sqrt{3}$, 28, $20 + 4\sqrt{3}$, 24, 20. $T = 272 + 16\sqrt{3} \approx 299.713 \text{ m}^3$ (300 to 3 s.f.). (b) $500 + T - 50 \approx 749.713 \text{ m}^3$ (750 to 3 s.f.). (c) Integral = $240 + 192/\pi \approx 301.115$; the trapezoidal estimate is too small.