

Complete calculus revision

IB Mathematics AI SL

200 questions | Core fluency, applications and exam preparation

Learning goal

Build secure calculus methods and apply them in original IB-style questions.

Syllabus link

Topic 5, SL 5.1–5.8: limits, derivatives, tangents and normals, integration, stationary points, optimisation and the trapezoidal rule.

Structure

60 short + 60 extended + 80 exam-style questions; full answer key.

Technology

 A GDC is permitted throughout, as in both AI SL examination papers.

Instructions. Show your method, not only calculator output. Give exact values where convenient; otherwise use 3 significant figures unless stated, and money to the nearest cent. Use radians for trigonometry. State units and interpret contextual answers. Work on separate paper.

Revision route. Build fluency in Q1–60; develop linked methods in Q61–120; then attempt Q121–200 as exam practice. Use the topic labels to target gaps and check the solution outlines after each set.

1. Short response — Questions 1–60

Limits and rates

1. The table samples $g(x)$ near $x = 1$. Estimate $\lim_{x \rightarrow 1} g(x)$. Does the table determine $g(1)$?

x	0.90	0.99	1.01	1.10
y	1.90	1.99	2.01	2.10

2. The table samples $g(x)$ near $x = 2$. Estimate $\lim_{x \rightarrow 2} g(x)$. Does the table determine $g(2)$?

x	1.90	1.99	2.01	2.10
y	3.90	3.99	4.01	4.10

3. The table samples $g(x)$ near $x = 3$. Estimate $\lim_{x \rightarrow 3} g(x)$. Does the table determine $g(3)$?

x	2.90	2.99	3.01	3.10
y	5.90	5.99	6.01	6.10

4. A table gives $f(1.99) = 6.9601$ and $f(2.01) = 7.0401$. Estimate $f'(2)$ using the secant through these points.

5. A cost function $C(q)$ is measured in euros and q is the number of items. Interpret $C'(50) = 3.2$, including units.

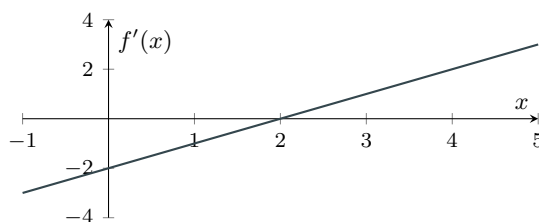
6. The volume of water $V(t)$ is measured in litres and time t in minutes. Interpret $V'(8) = -4.5$.

7. The tangent at $(2, 7)$ passes through $(5, 13)$. Find the gradient of the curve at $(2, 7)$.

8. A function approaches 4 as x approaches 2 from the left, and approaches 7 from the right. Does a two-sided limit exist at 2? Explain.

Increasing and decreasing

9. Given $f'(x) = x + 2$, state where f is increasing and decreasing.
10. Given $f'(x) = x - 1$, state where f is increasing and decreasing.
11. Given $f'(x) = x - 4$, state where f is increasing and decreasing.
12. Given $f'(x) = -(x - 1)(x - 5)$, state where f is increasing.
13. The derivative is positive on both sides of $x = 3$, but $f'(3) = 0$. Must f have a local maximum or minimum there?
14. Can a function have a negative value and still be increasing? Give an example.
15. The graph shows $y = f'(x)$. State where f is decreasing.



16. If $f'(x) < 0$ throughout $2 < x < 6$, compare $f(3)$ and $f(5)$.

Differentiation

17. Differentiate $f(x) = 2x^5$.
18. Find $f'(1)$ for $f(x) = -3x^4 + 5x + 7$.
19. Differentiate $f(x) = 4x^3 - 3x^2 + 2x$.
20. Differentiate $f(x) = 6x^{-1}$.
21. Find $f'(1)$ for $f(x) = 3x + 4x^{-2}$.
22. Differentiate $f(x) = 5x^2 - 2x^{-3}$.
23. Differentiate $f(x) = \frac{1}{3}x^6 - 4$.



24. Find $f'(1)$ for $f(x) = x^3 - 7x + 2$.

25. Differentiate $f(x) = 7x^2 - 3x^{-1}$.

26. Differentiate $f(x) = \frac{1}{2}x^4 - 5x^{-2}$.

27. Find $f'(1)$ for $f(x) = 9$.

28. Differentiate $f(x) = -2x^3 + 8x^2 + 3x^{-2}$.

Tangents and normals

29. Find the tangent to $y = x^2 + 1$ at $x = 2$.

30. Find the tangent to $y = x^3 - 2x$ at $x = 1$.

31. Find the tangent to $y = 8x^{-1}$ at $x = 2$.

32. Find the tangent to $y = 2x^2 - x + 3$ at $x = 1$.

33. A curve passes through $(1, 4)$ with tangent gradient 2. Find the normal there.

34. A curve passes through $(2, 5)$ with tangent gradient -3 . Find the normal there.

35. A curve passes through $(4, -1)$ with tangent gradient $\frac{1}{2}$. Find the normal there.

36. A curve has a horizontal tangent at $(3, 8)$. Write the equations of its tangent and normal.

Integration

37. Find $\int (6x^2) dx$.

38. Find $\int (8x^3 + 3) dx$.

39. Find $\int (4x - 5) dx$.

40. Find $\int (6x^{-2}) dx$.



41. Find $\int (2x - 4x^{-3}) dx$.

42. Find $\int (5x^4 - 3x^2 + 2) dx$.

43. Given $f'(x) = 4x$ and $f(1) = 7$, find $f(x)$.

44. Given $f'(x) = 3x^2 - 2$ and $f(0) = 5$, find $f(x)$.

45. Given $f'(x) = 2x^{-2}$ and $f(1) = 6$, find $f(x)$.

46. Write a definite integral for the area under $y = x^2 + 3$ between $x = 1$ and $x = 4$. Do not evaluate it.

Stationary points

47. Find and classify the stationary point of $f(x) = x^2 - 4x + 9$. Use a graph or the first derivative.

48. Find and classify the stationary point of $f(x) = x^2 + 2x + 4$. Use a graph or the first derivative.

49. Find and classify the stationary point of $f(x) = x^2 - 8x + 14$. Use a graph or the first derivative.

50. For $f(x) = x^3 - 3x$, find both stationary x -coordinates.

51. On $0 \leq x \leq 4$, which x -values must be checked to find the greatest value of a differentiable function?

52. The GDC reports a local maximum at $(1.4, 8.2)$ and a local minimum at $(3.7, -2.1)$. State the stationary values of x .

Optimisation

53. A rectangle has perimeter 36 cm. One side is x cm. Express its area in terms of x and state the physical domain.

54. A firm has revenue $R(q) = 40q - q^2$ and cost $C(q) = 6q + 20$. Form the profit function.

55. An open box has square base side x and volume 90. Express its height and material area in terms of x .

56. A model has a maximum at $q = 18.6$, but q must be a whole number. Which two neighbouring feasible values should be compared?

Trapezoidal rule

57. The interval $1 \leq x \leq 9$ is divided into four equal strips. Find the strip width h .

58. Use the trapezoidal rule to estimate the area represented by the table.

x	0	1	2
y	2	5	8

59. For five equally spaced ordinates y_0, y_1, y_2, y_3, y_4 , write the trapezoidal estimate in terms of h and the ordinates.

60. Explain why the equal-width composite trapezoidal formula cannot be applied directly to values at $x = 0, 1, 3, 4$.

2. Extended response — Questions 61–120

Limits and local rates

61. The function $g(x) = \frac{x^3 - 8}{x - 2}$ is undefined at $x = 2$. Use the table to estimate its limit there. Explain why substituting $x = 2$ directly does not give the limit.

x	1.90	1.99	2.01	2.10
y	11.4100	11.9401	12.0601	12.6100

62. The function $g(x) = \frac{x^3 - 64}{x - 4}$ is undefined at $x = 4$. Use the table to estimate its limit there. Explain why substituting $x = 4$ directly does not give the limit.

x	3.90	3.99	4.01	4.10
y	46.8100	47.8801	48.1201	49.2100

63. A tank volume is modelled by $F(t) = -t^2 + 12t + 50$. The input and output units make F' have units of L/min.

(a) Find the average rate of change from $t = 3$ to $t = 4$.

(b) Find the instantaneous rate at $t = 3$ and interpret its sign.

64. A production cost is modelled by $F(t) = \frac{1}{5}t^2 + 3t + 100$. The input and output units make F' have units of euros/item.

(a) Find the average rate of change from $t = 10$ to $t = 11$.

(b) Find the instantaneous rate at $t = 10$ and interpret its sign.

65. A culture population is modelled by $F(t) = 2t^3 - 3t^2 + 4t + 80$. The input and output units make F' have units of cells/hour.

(a) Find the average rate of change from $t = 2$ to $t = 3$.

(b) Find the instantaneous rate at $t = 2$ and interpret its sign.

66. A cooling model is modelled by $F(t) = \frac{1}{2}t^2 - 8t + 60$. The input and output units make F' have units of degrees C/min.

(a) Find the average rate of change from $t = 4$ to $t = 5$.

(b) Find the instantaneous rate at $t = 4$ and interpret its sign.

67. Given $f(4) = 12$ and $f'(4) = 2.5$, use the tangent line to estimate $f(4.08)$. Explain why the estimate is local.

68. A smooth function is sampled near $x = 3$.

x	2.9	2.99	3	3.01	3.1
y	5.61	5.9601	6	6.0401	6.41

- Estimate $f'(3)$ from the two nearest points on either side.
- State whether the function is increasing near 3, giving evidence.

Tangents and normals

69. For $f(x) = x^3 - 2x^2 + 4$,

- find $f(2)$ and $f'(2)$;
- find the tangent and normal at $x = 2$.

70. For $f(x) = 3x^2 + 2x^{-1}$,

- find $f(1)$ and $f'(1)$;
- find the tangent and normal at $x = 1$.

71. For $f(x) = x + 12x^{-2}$,

- find $f(2)$ and $f'(2)$;
- find the tangent and normal at $x = 2$.

72. For $f(x) = x^4 - 3x^2 + 2$,

- find $f(1)$ and $f'(1)$;
- find the tangent and normal at $x = 1$.

73. The curve $y = ax^2 + bx + 1$ passes through $(1, 6)$ and has tangent gradient 7 there. Find a and b .

74. The curve $y = ax^2 + bx + 1$ passes through $(2, 9)$ and has tangent gradient 10 there. Find a and b .

75. For $f(x) = 3e^{0.4x} + 2$, use your GDC to find the numerical derivative at $x = 1$, then find the tangent and normal equations. Analytical differentiation is not required.

76. For $f(x) = 5\ln(x + 2)$, use your GDC to find the numerical derivative at $x = 2$, then find the tangent and normal equations. Analytical differentiation is not required.

77. The tangent to $y = x^2 + 6$ at $x = 2$ meets the axes at A and B .

- Find its equation.
- Find the area of triangle OAB , where O is the origin.

78. The tangent to $y = x^2 + 11$ at $x = 3$ meets the axes at A and B .

- Find its equation.
- Find the area of triangle OAB , where O is the origin.

Integration and area

79. Given $f'(x) = 6x^2 + 2$ and $f(0) = 3$,

(a) find $f(x)$;

(b) find $f(2)$.

80. Given $f'(x) = 4x - 3$ and $f(2) = 7$,

(a) find $f(x)$;

(b) find $f(4)$.

81. Given $f'(x) = 3 + 8x^{-2}$ and $f(1) = 4$,

(a) find $f(x)$;

(b) find $f(2)$.

82. Given $f'(x) = 4x^3 - 2x$ and $f(1) = 5$,

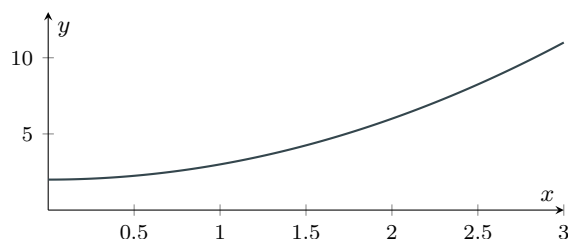
(a) find $f(x)$;

(b) find $f(3)$.

83. The curve $y = x^2 + 2$ is positive for $0 \leq x \leq 3$.

(a) Write the definite integral giving the area under the curve.

(b) Use your GDC to evaluate the area to three significant figures.



84. The curve $y = 2e^{0.3x}$ is positive for $1 \leq x \leq 4$.

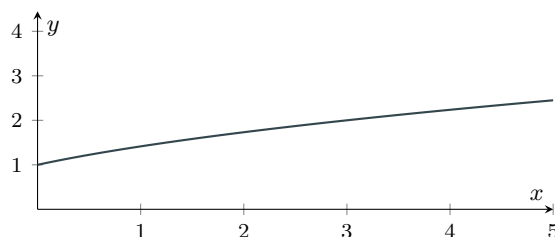
(a) Write the definite integral giving the area under the curve.

(b) Use your GDC to evaluate the area to three significant figures.

85. The curve $y = \sqrt{x+1}$ is positive for $0 \leq x \leq 5$.

(a) Write the definite integral giving the area under the curve.

(b) Use your GDC to evaluate the area to three significant figures.



86. The curve $y = 4 + \sin x$ is positive for $0 \leq x \leq \pi$.

- Write the definite integral giving the area under the curve.
- Use your GDC to evaluate the area to three significant figures.

87. The region under $y = 2x + 1$ from $x = 0$ to $x = k$, where $k > 0$, has area 20.

- Form an equation involving an integral.
- Find k .

88. The area under $y = a(x^2 + 1)$ from 0 to 2 is 14, with $a > 0$. Find a .

Stationary points

89. Let $f(x) = 2x^3 + 3x^2 - 12x + 3$.

- Find the stationary points.
- Classify them using the graph or the sign of the first derivative.
- State where f is decreasing.

90. Let $f(x) = 2x^3 - 6x^2 - 18x + 2$.

- Find the stationary points.
- Classify them using the graph or the sign of the first derivative.
- State where f is decreasing.

91. Let $f(x) = 2x^3 - 15x^2 + 24x + 1$.

- Find the stationary points.
- Classify them using the graph or the sign of the first derivative.
- State where f is decreasing.

92. Let $f(x) = 2x^3 + 3x^2 - 36x + 4$.

- Find the stationary points.
- Classify them using the graph or the sign of the first derivative.
- State where f is decreasing.

93. Use your GDC to find the minimum point of $f(x) = x^2 + \frac{16}{x}$ for $x > 0$. Confirm the derivative is negative just before and positive just after the stationary point.

94. Use your GDC to find the minimum point of $f(x) = x^2 + \frac{54}{x}$ for $x > 0$. Confirm the derivative is negative just before and positive just after the stationary point.

95. Use technology to find the greatest value of $f(x) = xe^{-0.5x}$ on $0 \leq x \leq 8$. State the maximizing x -value and explain how you checked the endpoints.

96. Use technology to find the greatest value of $f(x) = \frac{x}{x^2 + 4}$ on $0 \leq x \leq 8$. State the maximizing x -value and explain how you checked the endpoints.

97. Find the greatest and least values of $f(x) = x^3 - 3x$ on $[-2, 3]$. Include the points where the function is stationary and the endpoints in your comparison.

98. Find the greatest and least values of $f(x) = x^3 - 3x$ on $[-2, 4]$. Include the points where the function is stationary and the endpoints in your comparison.

Optimisation

99. A rectangular garden borders a wall. Only three sides need fencing; 54 m is available. Find the dimensions that maximize area.

100. Two equal adjacent rectangular pens share a dividing fence. A total of 72 m of fencing is used. Let x be the total width and y the depth. Find the largest total area.

101. A closed cuboid has a square base and volume 125 cm^3 . Find the dimensions requiring the least material.

102. An open box is formed by cutting squares of side x from a 20 cm by 12 cm card. Use technology to find the greatest volume and the corresponding dimensions.

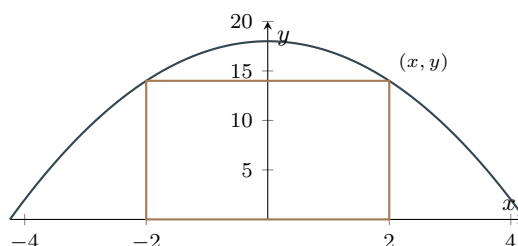
103. A closed cylinder has volume 250 cm^3 . Find its radius and height when its surface area is minimized.

104. A poster must contain 180 cm^2 of printed area, with 2 cm margins at the top and bottom and 1 cm margins at the sides. Find the least possible total area.

105. A shop models the price per item as $p = 60 - 0.2q$ euros, and total cost as $C = 12q + 300$. Find the output, price and maximum profit.

106. The profit from selling n items is $P(n) = -3n^2 + 112n - 80$ euros, where n is a nonnegative integer. Find the integer output giving greatest profit.

107. A rectangle has vertices $(\pm x, 0)$ and $(\pm x, y)$ on $y = 18 - x^2$, with $x, y > 0$. Find its maximum area.



108. A delivery company uses the cost model $C(x) = 2x + 98/x + 15$, for $x > 0$. Find the value of x giving minimum cost and the minimum cost.

109. A campaign has net benefit $B(x) = 120xe^{-0.2x} - 30$, for $0 \leq x \leq 15$. Use your GDC to find the maximum net benefit and the maximizing x .

110. The average cost of producing an item is $C(q) = 0.02q + 800/q + 4$, for $q > 0$. Find the output minimizing average cost and explain why the answer is physically feasible.

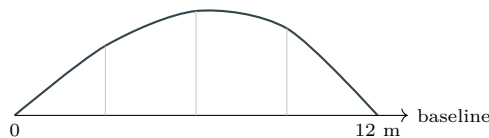
Trapezoidal rule

111. A pond has widths measured at equal distances along a straight baseline.

Distance (m)	0	3	6	9	12
Width (m)	0	4	6	5	0

(a) Estimate the surface area of the pond.

(b) State one modelling assumption.



112. Use two strips and then four strips to estimate $\int_0^4 \sqrt{x+1} dx$. Compare both with a GDC numerical integral.

113. The trapezoidal estimate for the data below is 30 square units. Find k .

x	0	2	4	6
y	2	k	6	4

114. Water enters a tank at the rates shown.

Time (min)	0	2	4	6	8
Rate (L/min)	5	7	8	6	4

(a) Estimate the volume entering during the eight minutes.

(b) The tank initially holds 120 litres. Estimate its final volume.

115. Use four equal strips to estimate $\int_0^2 e^x dx$. Give the estimate and its percentage error relative to a GDC numerical integral.

116. Use four equal strips to estimate the area under $y = \sin x$ for $0 \leq x \leq \pi$. Use radians. Compare your estimate with the exact area, 2.

117. Use four trapezoids and then two trapezoids to estimate the area from the same data. Explain why the table alone cannot prove which is closer to the true area.

x	0	1	2	3	4
y	1	3	4	3	1

118. At equally spaced positions, the widths of a garden are recorded.

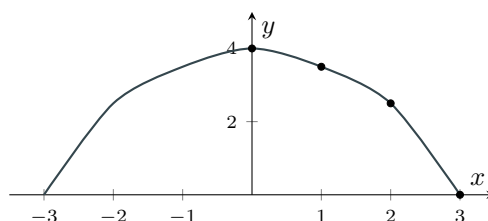
Distance (m)	0	2	4	6	8
Width (m)	2	3	4	3	2

- (a) Estimate the area.
- (b) Every width is known to within 0.1 m. Find a lower and an upper bound for the trapezoidal estimate. Treat the distances as exact.

119. The outline represents a symmetrical cross-section. Heights on its right-hand half are shown.

x	0	1	2	3
y	4	3.5	2.5	0

The full region is reflected in the y -axis. Estimate its total area.



120. A student uses the trapezoidal rule on this table and reports 20.

x	0	1	2	3	4
y	0	4	6	4	0

- (a) Calculate the correct estimate.
- (b) Suggest an endpoint/interior-weighting mistake that would produce 20.

3. IB exam-style practice — Questions 121–200

IB-style: derivatives and tangents

121. Consider $f(x) = 2x^3 - 3x^2 + 4x + 1$.

- (a) Find $f'(x)$. [2 marks]
- (b) Find the equation of the tangent at $x = 1$. [3 marks]
- (c) Find the equation of the normal at the same point. [2 marks]

122. Consider $f(x) = x^3 + 3x - 2$.

- (a) Find $f'(x)$. [2 marks]
- (b) Find the equation of the tangent at $x = 2$. [3 marks]
- (c) Find the equation of the normal at the same point. [2 marks]

123. Consider $f(x) = 2x^2 + 4x^{-1}$.

- (a) Find $f'(x)$. [2 marks]
- (b) Find the equation of the tangent at $x = 2$. [3 marks]
- (c) Find the equation of the normal at the same point. [2 marks]

124. Consider $f(x) = x^4 - 2x^2 + 5$.

- (a) Find $f'(x)$. [2 marks]
- (b) Find the equation of the tangent at $x = 2$. [3 marks]
- (c) Find the equation of the normal at the same point. [2 marks]

125. The curve $y = ax^3 + bx + 4$ passes through $(1, 7)$. Its tangent there has gradient 5.

- (a) Write two equations in a and b . [3 marks]
- (b) Find a and b . [2 marks]
- (c) Write the equation of the normal at the given point. [2 marks]

126. The curve $y = ax^3 + bx + 4$ passes through $(2, 18)$. Its tangent there has gradient 23.

- (a) Write two equations in a and b . [3 marks]
- (b) Find a and b . [2 marks]
- (c) Write the equation of the normal at the given point. [2 marks]

127. The curve $y = ax^3 + bx + 4$ passes through $(1, 8)$. Its tangent there has gradient 10.

- (a) Write two equations in a and b . [3 marks]
- (b) Find a and b . [2 marks]
- (c) Write the equation of the normal at the given point. [2 marks]

128. The curve $y = ax^3 + bx + 4$ passes through $(2, 22)$. Its tangent there has gradient 17.

- (a) Write two equations in a and b . [3 marks]
- (b) Find a and b . [2 marks]
- (c) Write the equation of the normal at the given point. [2 marks]

129. The tangent to $y = 18/x$ at $x = 3$ meets the coordinate axes at A and B .

- (a) Find the tangent equation. [3 marks]
- (b) Find both intercepts. [2 marks]
- (c) Find the area of triangle OAB . [2 marks]

130. The tangent to $y = 20/x$ at $x = 2$ meets the coordinate axes at A and B .

- (a) Find the tangent equation. [3 marks]
- (b) Find both intercepts. [2 marks]
- (c) Find the area of triangle OAB . [2 marks]

131. Let $f(x) = 2 + e^{0.2x}$. Use technology for differentiation.

- (a) Find $f(3)$ and $f'(3)$. [2 marks]
- (b) Find the equation of the tangent at $x = 3$. [2 marks]
- (c) Use the tangent to estimate $f(3.1)$. [2 marks]

132. Let $f(x) = 4 + \sqrt{x+3}$. Use technology for differentiation.

- (a) Find $f(1)$ and $f'(1)$. [2 marks]
- (b) Find the equation of the tangent at $x = 1$. [2 marks]
- (c) Use the tangent to estimate $f(1.1)$. [2 marks]

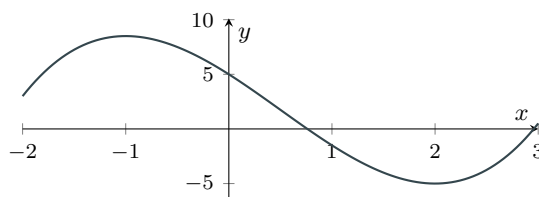
IB-style: rates and stationary points

133. Let $f(x) = x^3 - 12x + 10$.

- (a) Find the two stationary points. [4 marks]
- (b) Classify them using a first-derivative sign change or your graph. [2 marks]
- (c) State the intervals on which the function is increasing. [2 marks]

134. Let $f(x) = x^3 - \frac{3}{2}x^2 - 6x + 5$.

- (a) Find the two stationary points. [4 marks]
- (b) Classify them using a first-derivative sign change or your graph. [2 marks]
- (c) State the intervals on which the function is increasing. [2 marks]



135. Let $f(x) = x^3 - 6x^2 + 9x + 20$.

- (a) Find the two stationary points. [4 marks]
- (b) Classify them using a first-derivative sign change or your graph. [2 marks]
- (c) State the intervals on which the function is increasing. [2 marks]

136. Let $f(x) = x^3 - \frac{21}{2}x^2 + 30x + 40$.

- (a) Find the two stationary points. [4 marks]
- (b) Classify them using a first-derivative sign change or your graph. [2 marks]
- (c) State the intervals on which the function is increasing. [2 marks]

137. The water volume is modelled by $F(t) = -2t^2 + 24t + 80$ litres, where t is measured in minutes and $0 \leq t \leq 12$.

- (a) Find $F'(t)$ and interpret $F'(3)$. [3 marks]
- (b) Find the maximum model value and when it occurs. [3 marks]
- (c) State one limitation of extending the model beyond the stated interval. [1 marks]

138. The profit is modelled by $F(t) = -3t^2 + 42t - 20$ euros, where t is measured in days and $0 \leq t \leq 14$.

- (a) Find $F'(t)$ and interpret $F'(4)$. [3 marks]
- (b) Find the maximum model value and when it occurs. [3 marks]
- (c) State one limitation of extending the model beyond the stated interval. [1 marks]

139. The greenhouse temperature is modelled by $F(t) = -t^2 + 10t + 18$ degrees Celsius, where t is measured in hours and $0 \leq t \leq 10$.

(a) Find $F'(t)$ and interpret $F'(2)$. [3 marks]

(b) Find the maximum model value and when it occurs. [3 marks]

(c) State one limitation of extending the model beyond the stated interval. [1 marks]

140. The nutrient concentration is modelled by $F(t) = -2t^2 + 20t + 6$ mg/L, where t is measured in hours and $0 \leq t \leq 10$.

(a) Find $F'(t)$ and interpret $F'(3)$. [3 marks]

(b) Find the maximum model value and when it occurs. [3 marks]

(c) State one limitation of extending the model beyond the stated interval. [1 marks]

141. A differentiable function has the derivative signs shown.

x	$-\infty$		1		4		$+\infty$
$f'(x)$		-	0	+	0	-	

(a) Classify both stationary points, explaining your reasoning. [4 marks]

(b) State where the function is increasing. [1 marks]

(c) Explain why the sign table does not give the stationary y -coordinates. [1 marks]

142. A differentiable function has the derivative signs shown.

x	$-\infty$		-2		3		$+\infty$
$f'(x)$		+	0	-	0	+	

(a) Classify both stationary points, explaining your reasoning. [4 marks]

(b) State where the function is increasing. [1 marks]

(c) Explain why the sign table does not give the stationary y -coordinates. [1 marks]

143. Let $f(x) = x^3 - 6x^2 + 9x + 1$, for $0 \leq x \leq 5$.

(a) Find all stationary points in the interior. [3 marks]

(b) Find the greatest and least values on the given domain. [3 marks]

(c) Explain why endpoints must be considered. [1 marks]

144. Let $f(x) = x^4 - 8x^2 + 3$, for $-1 \leq x \leq 3$.

(a) Find all stationary points in the interior. [3 marks]

(b) Find the greatest and least values on the given domain. [3 marks]

(c) Explain why endpoints must be considered. [1 marks]

IB-style: integration and area

145. A curve satisfies $f'(x) = 6x$ and $f(0) = 4$.

- (a) Find $f(x)$. [3 marks]
- (b) Write a definite integral for the area under the curve from $x = 0$ to $x = 2$. [1 marks]
- (c) Evaluate the area. [2 marks]

146. A curve satisfies $f'(x) = 3x^2 + 1$ and $f(1) = 5$.

- (a) Find $f(x)$. [3 marks]
- (b) Write a definite integral for the area under the curve from $x = 1$ to $x = 3$. [1 marks]
- (c) Evaluate the area. [2 marks]

147. A curve satisfies $f'(x) = 2x + 4$ and $f(0) = 6$.

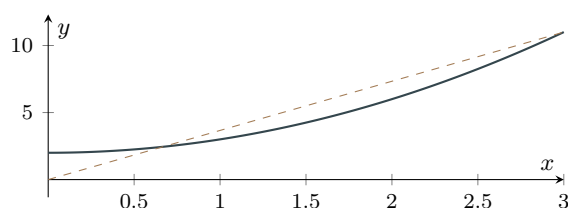
- (a) Find $f(x)$. [3 marks]
- (b) Write a definite integral for the area under the curve from $x = 0$ to $x = 2$. [1 marks]
- (c) Evaluate the area. [2 marks]

148. A curve satisfies $f'(x) = 6x^2 + 2x$ and $f(0) = 2$.

- (a) Find $f(x)$. [3 marks]
- (b) Write a definite integral for the area under the curve from $x = 0$ to $x = 2$. [1 marks]
- (c) Evaluate the area. [2 marks]

149. The region R is under $y = x^2 + 2$, above the x -axis, between $x = 0$ and $x = 3$. The points $O = (0, 0)$, $A = (3, 0)$ and $B = (3, 11)$ form a triangle.

- (a) Find the area of R using a definite integral. [3 marks]
- (b) Find the area of triangle OAB . [2 marks]
- (c) Find the ratio of the area of R to the triangle area. [1 marks]



150. The region R is under $y = x^2 + 3$, above the x -axis, between $x = 0$ and $x = 2$. The points $O = (0, 0)$, $A = (2, 0)$ and $B = (2, 7)$ form a triangle.

- (a) Find the area of R using a definite integral. [3 marks]
- (b) Find the area of triangle OAB . [2 marks]
- (c) Find the ratio of the area of R to the triangle area. [1 marks]



151. The region R is under $y = x^2 + 4$, above the x -axis, between $x = 0$ and $x = 4$. The points $O = (0, 0)$, $A = (4, 0)$ and $B = (4, 20)$ form a triangle.

- (a) Find the area of R using a definite integral. [3 marks]
- (b) Find the area of triangle OAB . [2 marks]
- (c) Find the ratio of the area of R to the triangle area. [1 marks]

152. The region R is under $y = x^2 + 5$, above the x -axis, between $x = 0$ and $x = 2$. The points $O = (0, 0)$, $A = (2, 0)$ and $B = (2, 9)$ form a triangle.

- (a) Find the area of R using a definite integral. [3 marks]
- (b) Find the area of triangle OAB . [2 marks]
- (c) Find the ratio of the area of R to the triangle area. [1 marks]

153. The vertical line $x = k$ divides the region under $y = 2x + 1$ from 0 to 3 into two equal areas.

- (a) Calculate the total area. [2 marks]
- (b) Form an equation involving k . [2 marks]
- (c) Use your GDC to find k . [2 marks]

154. The vertical line $x = k$ divides the region under $y = 2x + 1$ from 0 to 4 into two equal areas.

- (a) Calculate the total area. [2 marks]
- (b) Form an equation involving k . [2 marks]
- (c) Use your GDC to find k . [2 marks]

155. For $f(x) = x^2 + c$, the area under its graph from $x = 0$ to $x = 2$ is 20.

- (a) Write an equation involving a definite integral. [2 marks]
- (b) Find c . [3 marks]
- (c) Check that the function is positive throughout the interval. [1 marks]

156. For $f(x) = x^2 + c$, the area under its graph from $x = 1$ to $x = 3$ is 36.

- (a) Write an equation involving a definite integral. [2 marks]
- (b) Find c . [3 marks]
- (c) Check that the function is positive throughout the interval. [1 marks]

157. The width of a decorative panel is $w(x) = 6e^{-0.25x}$ cm for $0 \leq x \leq 8$, where x is measured in centimetres.

- (a) Write and evaluate a definite integral for its area. [3 marks]
- (b) A coating costs €0.08 per square centimetre. Estimate the cost. [2 marks]
- (c) Explain why the integral represents an area. [1 marks]

- 158.** The width of a decorative panel is $w(x) = 3 + 2\sin(\pi x/6)$ cm for $0 \leq x \leq 6$, where x is measured in centimetres.
- Write and evaluate a definite integral for its area. [3 marks]
 - A coating costs €0.08 per square centimetre. Estimate the cost. [2 marks]
 - Explain why the integral represents an area. [1 marks]

IB-style: optimisation

159. A business models price by $p = 70 - \frac{1}{4}q$ euros and total cost by $C = 10q + 200$ euros. Treat output q as continuous.

- Form an expression for profit and state the feasible domain if price must be nonnegative. [3 marks]
- Find the output and price maximizing profit. [4 marks]
- Find the maximum profit. [1 marks]

160. A business models price by $p = 90 - \frac{1}{2}q$ euros and total cost by $C = 18q + 500$ euros. Treat output q as continuous.

- Form an expression for profit and state the feasible domain if price must be nonnegative. [3 marks]
- Find the output and price maximizing profit. [4 marks]
- Find the maximum profit. [1 marks]

161. An open box is formed from a 18 cm by 12 cm card by removing squares of side x from the corners.

- Show that its volume is $V(x) = x(18 - 2x)(12 - 2x)$ and state the domain. [3 marks]
- Use calculus or a GDC derivative to find the maximizing x . [3 marks]
- Find the maximum volume and justify that it is a maximum. [2 marks]

162. An open box is formed from a 24 cm by 15 cm card by removing squares of side x from the corners.

- Show that its volume is $V(x) = x(24 - 2x)(15 - 2x)$ and state the domain. [3 marks]
- Use calculus or a GDC derivative to find the maximizing x . [3 marks]
- Find the maximum volume and justify that it is a maximum. [2 marks]

163. A lidless cylindrical container has volume 360 cm^3 , radius r and height h .

- Express h in terms of r and form the material-area function. [3 marks]
- Find the dimensions requiring the least material. [4 marks]
- Find the least material area. [1 marks]

164. A closed cylindrical container has volume 500 cm^3 , radius r and height h .

- Express h in terms of r and form the material-area function. [3 marks]
- Find the dimensions requiring the least material. [4 marks]
- Find the least material area. [1 marks]

165. A rectangle has vertices $(\pm x, 0)$ and $(\pm x, y)$ on $y = 15 - x^2$, with $x, y > 0$.

- (a) Show that its area is $A(x) = 2x(15 - x^2)$ and state the domain. [3 marks]
- (b) Find the dimensions and maximum area. [4 marks]
- (c) Explain why the boundary values do not give a positive maximum. [1 marks]

166. A rectangle has vertices $(\pm x, 0)$ and $(\pm x, y)$ on $y = 27 - x^2$, with $x, y > 0$.

- (a) Show that its area is $A(x) = 2x(27 - x^2)$ and state the domain. [3 marks]
- (b) Find the dimensions and maximum area. [4 marks]
- (c) Explain why the boundary values do not give a positive maximum. [1 marks]

167. A page needs 240 cm^2 of printing. The left and right margins are each 1 cm; the top and bottom margins are each 2 cm.

- (a) Let x be the printed width. Express the total page area in terms of x . [3 marks]
- (b) Find the printed dimensions minimizing total page area. [3 marks]
- (c) Find the minimum page area. [2 marks]

168. A page needs 300 cm^2 of printing. The left and right margins are each 2 cm; the top and bottom margins are each 3 cm.

- (a) Let x be the printed width. Express the total page area in terms of x . [3 marks]
- (b) Find the printed dimensions minimizing total page area. [3 marks]
- (c) Find the minimum page area. [2 marks]

169. A rectangular enclosure uses 84 m of fencing, including 1 internal dividing fence parallel to its depth y . Its total width is x .

- (a) Write the fencing constraint and express total area in terms of x . [3 marks]
- (b) Find the maximum total area and the dimensions that achieve it. [4 marks]

170. A rectangular enclosure uses 120 m of fencing, including 2 internal dividing fences parallel to its depth y . Its total width is x .

- (a) Write the fencing constraint and express total area in terms of x . [3 marks]
- (b) Find the maximum total area and the dimensions that achieve it. [4 marks]

171. At a ticket price of €20, a club expects to sell 100 tickets. Each €1 increase reduces sales by 3 tickets. Let n be the whole-number price increase.

- (a) Show that revenue is $R(n) = (20 + n)(100 - 3n)$ and state a feasible integer domain. [3 marks]
- (b) Find the value of n giving the greatest revenue. [4 marks]
- (c) State the optimal price and revenue. [2 marks]

172. An open container has a square base of side x cm, height h cm and total material area 192 cm^2 .

- (a) Express its volume as a function of x and state the domain. [3 marks]
- (b) Find the greatest possible volume and the dimensions. [4 marks]



173. The net return from a fundraising campaign is modelled by $R(x) = 500x/(x+2) - 20x$ euros, where $0 \leq x \leq 30$ is advertising expenditure in units of €10.

- (a) Use your GDC to find the maximizing x . [3 marks]
(b) State the advertising expenditure and maximum net return. [2 marks]
(c) Check that neither endpoint gives a greater return. [2 marks]

174. A rectangle in the first quadrant has one corner at the origin and its opposite corner on $y = 12e^{-0.3x}$.

- (a) Form the area function and state its domain. [2 marks]
(b) Use your GDC to find its maximum area. [3 marks]
(c) Find the corresponding rectangle dimensions. [2 marks]

IB-style: trapezoidal modelling

175. A surveyor records the perpendicular width of a small lake at five equally spaced positions along a baseline.

Distance (m)	0	5	10	15	20
Width (m)	0	6	9	7	0

- (a) Write down the width of each strip. [1 marks]
(b) Use the trapezoidal rule to estimate the area of the lake. [3 marks]
(c) A treatment requires 0.08 kg per square metre. Estimate the mass required. [2 marks]

176. Let $f(x) = \frac{1}{x+1}$, for $0 \leq x \leq 2$.

- (a) Construct a table of values at intervals of 0.5. [2 marks]
(b) Use four trapezoids to estimate the area under the curve. [3 marks]
(c) Find a GDC value of the integral and calculate the percentage error of your estimate. [3 marks]

177. The power used by a machine varies over four hours.

Time (hours)	0	1	2	3	4
Power (kW)	2	3	5	4	2

- (a) Estimate the energy used, in kWh. [3 marks]
(b) Electricity costs €0.24 per kWh. Estimate the cost. [2 marks]
(c) Explain why joining the points with straight lines is an approximation. [1 marks]

178. The area under a positive curve is estimated using four equal trapezoids. The estimate is 46.

x	0	2	4	6	8
y	3	k	8	k	5

- (a) Form an equation in k . [2 marks]
(b) Find k . [2 marks]
(c) The last ordinate is corrected from 5 to 5.6. Find the revised estimate. [2 marks]

179. A laboratory pump delivers a chemical at the following rates.

Time (min)	0	1	2	3	4
Rate (mg/min)	0.8	1.2	1.6	1.0	0.4

- (a) Estimate the mass delivered over four minutes. [3 marks]
- (b) A container already contains 2.5 mg. Estimate the total mass after four minutes. [1 marks]
- (c) The concentration must not exceed 0.05 mg/mL. Using your estimate, find the minimum total solution volume required. [2 marks]

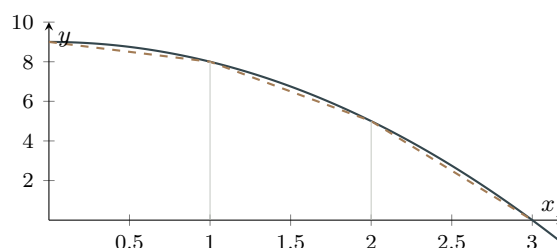
180. A designer measures the height of a logo above a baseline.

Position (cm)	0	2	4	6	8	10
Height (cm)	0	3	5	4	2	0

- (a) Estimate the area of the logo. [3 marks]
- (b) Both dimensions are enlarged by a factor of 1.5. Find the corresponding area estimate for the enlarged logo. [2 marks]
- (c) Explain why multiplying the original area by 1.5 would be incorrect. [1 marks]

181. The curve $y = 9 - x^2$ is shown for $0 \leq x \leq 3$.

- (a) Use three equal trapezoids to estimate the area under the curve. [3 marks]
- (b) Explain geometrically why this is an underestimate. [1 marks]
- (c) The exact area is 18. Find the percentage error. [2 marks]



182. Consider the region under $y = 5e^{-0.4x}$ for $0 \leq x \leq 3$.

- (a) Find the strip width for six equal trapezoids. [1 marks]
- (b) Calculate the trapezoidal area estimate. [4 marks]
- (c) Use a numerical integral to determine whether the estimate is too large or too small. [2 marks]

183. Rainfall intensity is recorded for one hour.

Time (min)	0	15	30	45	60
Intensity (mm/hour)	0	12	24	18	6

- (a) Write down the strip width in hours. [1 marks]
- (b) Estimate the total rainfall depth. [3 marks]
- (c) Estimate the volume collected on a horizontal roof of area 80 m^2 , in litres. Use $1 \text{ mm over } 1 \text{ m}^2 = 1 \text{ litre}$. [2 marks]

184. A storage channel has the cross-section shown by these measurements. Its cross-section is constant along a length of 6 m.

Horizontal position (m)	-4	-2	0	2	4
Depth (m)	0	3	4	3	0

- (a) Estimate the cross-sectional area. [3 marks]
- (b) Hence estimate the capacity of the channel in cubic metres. [2 marks]
- (c) Convert this capacity to litres. [1 marks]

185. The temperature in a chamber is measured every two hours.

Time (hours)	0	2	4	6
Temperature ($^{\circ}\text{C}$)	18	24	27	21

The mean temperature over six hours is defined by $\bar{T} = \frac{1}{6} \int_0^6 T(t) dt$.

- (a) Estimate $\int_0^6 T(t) dt$ using the trapezoidal rule. [3 marks]
- (b) Hence estimate the mean temperature. [2 marks]

186. A trapezoidal estimate uses $h = 0.5$ and seven ordinates. Its value is 12.6.

- (a) Find the total width of the interval. [2 marks]
- (b) One interior ordinate is increased by 0.8. Find the revised estimate. [2 marks]
- (c) Instead, only the final endpoint ordinate is increased by 0.8. Find the revised estimate. [2 marks]

187. The following data are available.

x	0	1	2	4	6
y	1	2	4	5	3

- (a) Select four ordinates that can be used for three equal trapezoids over $0 \leq x \leq 6$. [2 marks]
- (b) Use these ordinates to estimate the area. [3 marks]
- (c) Explain why using all five ordinates with $h = 1.5$ would be invalid. [1 marks]

188. Five equally spaced widths of a plot are 0, 8, 11, 7, 0 metres. The trapezoidal area estimate is 156 m^2 .

- (a) Find the distance h between consecutive measurements. [3 marks]
- (b) Find the total length of the baseline. [1 marks]
- (c) Fertilizer covers 20 m^2 per bag. Find the smallest whole number of bags required according to the estimate. [2 marks]

IB-style: mixed calculus

189. The volume of water in a tank is $V(t) = -t^2 + 12t + 40$ litres for $0 \leq t \leq 10$ minutes.

- (a) Find and interpret $V'(2)$. [3 marks]
- (b) Find the greatest volume and when it occurs. [3 marks]
- (c) Use the tangent at $t = 2$ to estimate $V(2.1)$. [3 marks]

190. A supplier models the selling price by $p = 60 - 0.2q$ euros and cost by $C = 10q + 400$ euros, where q is output. Treat q as continuous.

- Form the profit function and state a feasible domain with nonnegative price. [3 marks]
- Find the profit-maximizing output, selling price and profit. [4 marks]
- Find the interval of output for which the model predicts a positive profit. [3 marks]

191. A function satisfies $f'(x) = 3x^2 - 6x$ and $f(1) = 5$.

- Find $f(x)$. [3 marks]
- Find and classify both stationary points. [4 marks]
- Calculate the area under f from $x = 0$ to $x = 3$. [3 marks]

192. Let $f(x) = x^2 + 4/x$, for $1 \leq x \leq 4$.

- Find its minimum point, using a derivative or your GDC. [3 marks]
- Use three equal trapezoids to estimate the area under the curve. [3 marks]
- Find a numerical integral and the percentage error of the estimate. [3 marks]

193. The graph of $f(x) = kx^2 + 3$ passes through $(2, 11)$.

- Find k and the tangent at $x = 2$. [4 marks]
- Find the area under the graph from 0 to 2. [2 marks]
- Estimate the same area using two trapezoids, and find the percentage error. [4 marks]

194. The perpendicular widths of a flower bed are recorded along a baseline.

Distance (m)	0	2	4	6	8
Width (m)	0	3	5	4	0

- Estimate its area with the trapezoidal rule. [3 marks]
- The width at 4 m was understated by 0.4 m. Find the corrected estimate. [2 marks]
- Using the corrected estimate, find the cost of covering the bed at €9.50 per square metre. [2 marks]
- Explain why the correction does not remove all uncertainty in the true area. [1 marks]

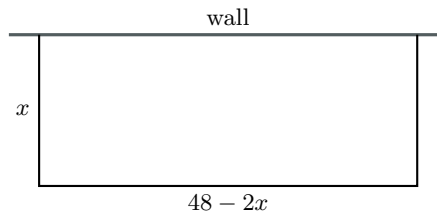
195. A smooth cost model $C(q)$, in euros, has these values near output $q = 100$.

q	99.9	100	100.1
$C(q)$	699.6001	700	700.4001

- Estimate $C'(100)$ using the points on either side. [2 marks]
- State the units and interpretation of your answer. [2 marks]
- Write an approximate tangent equation and use it to estimate $C(101)$. [3 marks]
- Explain why the same tangent may be unsuitable for estimating $C(500)$. [1 marks]

196. A rectangular garden borders a straight wall. The other three sides use 48 m of fencing. The depth perpendicular to the wall is x m.

- (a) Show that $A(x) = 48x - 2x^2$ and state the domain. [3 marks]
- (b) Find the dimensions and maximum area. [4 marks]
- (c) A gate restricts the depth to $x \leq 10$. Find the largest feasible area under this extra restriction. [3 marks]

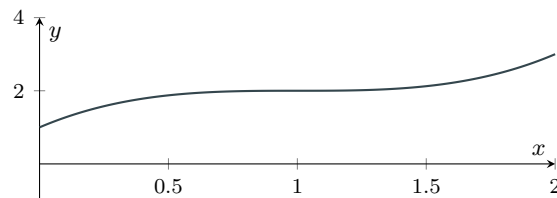


197. A decorative panel has width $w(x) = 3 + 2e^{-0.4x}$ metres for $0 \leq x \leq 4$, where x is in metres. Use technology.

- (a) Find the gradient of the width function at $x = 2$. [2 marks]
- (b) Estimate the panel area using four equal trapezoids. [4 marks]
- (c) Find the numerical integral and comment on the direction and size of the trapezoidal error. [3 marks]

198. Let $f(x) = x^3 - 3x^2 + 3x + 1$.

- (a) Show that $f'(x) = 3(x - 1)^2$. [2 marks]
- (b) Find the stationary point and explain why it is neither a local maximum nor a local minimum. [3 marks]
- (c) Write the tangent and normal there. [2 marks]
- (d) Find the area under the curve on $[0, 2]$. [3 marks]



199. Profit is modelled by $P(q) = -aq^2 + 60q - 100$ euros, with $a > 0$ and $q \geq 0$. The greatest profit occurs at $q = 10$.

- (a) Use the derivative to find a . [3 marks]
- (b) Find the maximum profit and justify it. [3 marks]
- (c) Find the output interval on which profit is positive. [3 marks]

200. The width of a curved walkway is $w(x) = 2 + 0.5x - 0.05x^2$ metres for $0 \leq x \leq 8$, where x is measured along a straight baseline in metres.

- (a) Find the greatest width and where it occurs. [3 marks]
- (b) Find the area of the walkway using a definite integral. [3 marks]
- (c) Use four equal trapezoids to estimate the area. [3 marks]
- (d) Paving costs €12.50 per square metre. Find the amount by which the trapezoidal cost estimate differs from the integral-based cost. [2 marks]

Answer key and solution outlines

Use unrounded intermediate values. Equivalent exact forms and correctly rounded answers are accepted. All exam-style questions are original practice questions, not official IB examination items.

1. Short response — Questions 1–60

1. The values approach 2. The table does not give the value at $x = 1$; a limit and the function value need not coincide.
2. The values approach 4. The table does not give the value at $x = 2$; a limit and the function value need not coincide.
3. The values approach 6. The table does not give the value at $x = 3$; a limit and the function value need not coincide.
4. $f'(2) \approx [7.0401 - 6.9601]/(2.01 - 1.99) = 4$.
5. At an output of 50 items, cost is increasing at approximately €3.20 per additional item.
6. At 8 minutes, the volume is decreasing at 4.5 litres per minute.
7. Gradient $= (13 - 7)/(5 - 2) = 2$.
8. No: the left-hand and right-hand limiting values differ.
9. f decreases for $x < -2$ and increases for $x > -2$.
10. f decreases for $x < 1$ and increases for $x > 1$.
11. f decreases for $x < 4$ and increases for $x > 4$.
12. $f' > 0$ between its roots, so f increases for $1 < x < 5$.
13. No. The function increases on both sides, so there is no local extremum.
14. Yes, for example $f(x) = x - 10$ at $x = 0$: $f(0) = -10$, but $f' = 1 > 0$.
15. The derivative is below the axis for $x < 2$, so f decreases there.
16. $f(3) > f(5)$ because f decreases as x increases.
17. Using $\frac{d}{dx}(ax^n) = anx^{n-1}$, $f'(x) = 10x^4$.
18. $f'(x) = -12x^3 + 5$, hence $f'(1) = -7$.
19. Using $\frac{d}{dx}(ax^n) = anx^{n-1}$, $f'(x) = 12x^2 - 6x + 2$.
20. Using $\frac{d}{dx}(ax^n) = anx^{n-1}$, $f'(x) = -6x^{-2}$.
21. $f'(x) = 3 - 8x^{-3}$, hence $f'(1) = -5$.
22. Using $\frac{d}{dx}(ax^n) = anx^{n-1}$, $f'(x) = 10x + 6x^{-4}$.
23. Using $\frac{d}{dx}(ax^n) = anx^{n-1}$, $f'(x) = 2x^5$.
24. $f'(x) = 3x^2 - 7$, hence $f'(1) = -4$.
25. Using $\frac{d}{dx}(ax^n) = anx^{n-1}$, $f'(x) = 14x + 3x^{-2}$.
26. Using $\frac{d}{dx}(ax^n) = anx^{n-1}$, $f'(x) = 2x^3 + 10x^{-3}$.
27. $f'(x) = 0$, hence $f'(1) = 0$.
28. Using $\frac{d}{dx}(ax^n) = anx^{n-1}$, $f'(x) = -6x^2 + 16x - 6x^{-3}$.
29. $y_0 = 5$ and $m = f'(2) = 4$. Thus $y - 5 = 4(x - 2)$, or $y = 4x - 3$.
30. $y_0 = -1$ and $m = f'(1) = 1$. Thus $y + 1 = 1(x - 1)$, or $y = x - 2$.
31. $y_0 = 4$ and $m = f'(2) = -2$. Thus $y - 4 = -2(x - 2)$, or $y = -2x + 8$.
32. $y_0 = 4$ and $m = f'(1) = 3$. Thus $y - 4 = 3(x - 1)$, or $y = 3x + 1$.
33. $m_n = -1/m_t = -\frac{1}{2}$, so $y = -\frac{1}{2}x + \frac{9}{2}$.

34. $m_n = -1/m_t = \frac{1}{3}$, so $y = \frac{1}{3}x + \frac{13}{3}$.
35. $m_n = -1/m_t = -2$, so $y = -2x + 7$.
36. Tangent $y = 8$; normal $x = 3$.
37. $2x^3 + C$, using $\int ax^n dx = \frac{a}{n+1}x^{n+1} + C$.
38. $2x^4 + 3x + C$, using $\int ax^n dx = \frac{a}{n+1}x^{n+1} + C$.
39. $2x^2 - 5x + C$, using $\int ax^n dx = \frac{a}{n+1}x^{n+1} + C$.
40. $-6x^{-1} + C$, using $\int ax^n dx = \frac{a}{n+1}x^{n+1} + C$.
41. $x^2 + 2x^{-2} + C$, using $\int ax^n dx = \frac{a}{n+1}x^{n+1} + C$.
42. $x^5 - x^3 + 2x + C$, using $\int ax^n dx = \frac{a}{n+1}x^{n+1} + C$.
43. Integrate: $f(x) = 2x^2 + C$. The condition gives $C = 5$, so $f(x) = 2x^2 + 5$.
44. Integrate: $f(x) = x^3 - 2x + C$. The condition gives $C = 5$, so $f(x) = x^3 - 2x + 5$.
45. Integrate: $f(x) = -2x^{-1} + C$. The condition gives $C = 8$, so $f(x) = 8 - 2x^{-1}$.
46. $A = \int_1^4 (x^2 + 3) dx$.
47. $f'(x) = 2x - 4 = 0$ at $x = 2$; signs change $- \rightarrow +$. Minimum $(2, 5)$.
48. $f'(x) = 2x + 2 = 0$ at $x = -1$; signs change $- \rightarrow +$. Minimum $(-1, 3)$.
49. $f'(x) = 2x - 8 = 0$ at $x = 4$; signs change $- \rightarrow +$. Minimum $(4, -2)$.
50. $f' = 3x^2 - 3 = 0$, so $x = -1$ and $x = 1$.
51. Check both endpoints, $x = 0, 4$, and every stationary point in the interior.
52. $x = 1.4$ and $x = 3.7$.
53. $y = 18 - x$; $A = x(18 - x)$, $0 < x < 18$.
54. $P = R - C = -q^2 + 34q - 20$.
55. $h = 90/x^2$ and $S = x^2 + 4xh = x^2 + 360/x$, for $x > 0$.
56. Compare the objective at $q = 18$ and $q = 19$; do not round the optimizer without checking.
57. $h = (9 - 1)/4 = 2$.
58. $T = \frac{1}{2}(2 + 2(5) + 8) = 10$ square units.
59. $T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4)$.
60. The widths are 1, 2, 1, so there is no single common strip width h .

2. Extended response — Questions 61–120

61. The values approach 12. Direct substitution gives $0/0$, which is undefined; the table describes values close to, but not at, $x = 2$. No formal limit manipulation is needed.
62. The values approach 48. Direct substitution gives $0/0$, which is undefined; the table describes values close to, but not at, $x = 4$. No formal limit manipulation is needed.
63. Average rate $= [F(4) - F(3)]/1 = 5$ L/min. $F'(t) = -2t + 12$; $F'(3) = 6$ L/min, so the volume is increasing.
64. Average rate $= [F(11) - F(10)]/1 = \frac{36}{5}$ euros/item. $F'(t) = \frac{2}{5}t + 3$; $F'(10) = 7$ euros/item, so the cost is increasing.
65. Average rate $= [F(3) - F(2)]/1 = 27$ cells/hour. $F'(t) = 6t^2 - 6t + 4$; $F'(2) = 16$ cells/hour, so the population is increasing.
66. Average rate $= [F(5) - F(4)]/1 = -\frac{7}{2}$ degrees C/min. $F'(t) = t - 8$; $F'(4) = -4$ degrees C/min, so the temperature is decreasing.

- 67.** $f(4+h) \approx f(4) + hf'(4)$, so $f(4.08) \approx 12 + 0.08(2.5) = 12.2$. The tangent only closely follows a smooth curve near the point of contact.
- 68.** $f'(3) \approx (6.0401 - 5.9601)/0.02 = 4$. The positive gradient, and increasing table values, indicate that the function is increasing locally.
- 69.** $f'(x) = 3x^2 - 4x$, so $f(2) = 4$, $m_t = 4$. Tangent: $y = 4x - 4$. Normal gradient $m_n = -1/m_t = -\frac{1}{4}$; normal: $y = -\frac{1}{4}x + \frac{9}{2}$.
- 70.** $f'(x) = 6x - 2x^{-2}$, so $f(1) = 5$, $m_t = 4$. Tangent: $y = 4x + 1$. Normal gradient $m_n = -1/m_t = -\frac{1}{4}$; normal: $y = -\frac{1}{4}x + \frac{21}{4}$.
- 71.** $f'(x) = 1 - 24x^{-3}$, so $f(2) = 5$, $m_t = -2$. Tangent: $y = -2x + 9$. Normal gradient $m_n = -1/m_t = \frac{1}{2}$; normal: $y = \frac{1}{2}x + 4$.
- 72.** $f'(x) = 4x^3 - 6x$, so $f(1) = 0$, $m_t = -2$. Tangent: $y = -2x + 2$. Normal gradient $m_n = -1/m_t = \frac{1}{2}$; normal: $y = \frac{1}{2}x - \frac{1}{2}$.
- 73.** Point condition: $1a + 1b = 5$. Gradient condition from $y' = 2ax + b$: $2a + b = 7$. Solving gives $a = 2$, $b = 3$.
- 74.** Point condition: $4a + 2b = 8$. Gradient condition from $y' = 2ax + b$: $4a + b = 10$. Solving gives $a = 3$, $b = -2$.
- 75.** GDC: $f(1) \approx 6.48$, $f'(1) \approx 1.79$. Tangent $y - 6.48 = 1.79(x - 1)$; normal $y - 6.48 = -0.559(x - 1)$. Retain full calculator precision when finding intercept form.
- 76.** GDC: $f(2) \approx 6.93$, $f'(2) \approx 1.25$. Tangent $y - 6.93 = 1.25(x - 2)$; normal $y - 6.93 = -0.8(x - 2)$. Retain full calculator precision when finding intercept form.
- 77.** $m = 2x = 4$ and the point is $(2, 10)$. Tangent $y = 4x + 2$, so intercepts are $(-\frac{1}{2}, 0)$ and $(0, 2)$. Triangle area $= \frac{1}{2} \left| -\frac{1}{2} \right| (2) = \frac{1}{2}$ square units.
- 78.** $m = 2x = 6$ and the point is $(3, 20)$. Tangent $y = 6x + 2$, so intercepts are $(-\frac{1}{3}, 0)$ and $(0, 2)$. Triangle area $= \frac{1}{2} \left| -\frac{1}{3} \right| (2) = \frac{1}{3}$ square units.
- 79.** $f(x) = 2x^3 + 2x + C$. Substituting $x = 0$ gives $C = 3$, hence $f(x) = 2x^3 + 2x + 3$ and $f(2) = 23$.
- 80.** $f(x) = 2x^2 - 3x + C$. Substituting $x = 2$ gives $C = 5$, hence $f(x) = 2x^2 - 3x + 5$ and $f(4) = 25$.
- 81.** $f(x) = 3x - 8x^{-1} + C$. Substituting $x = 1$ gives $C = 9$, hence $f(x) = 3x + 9 - 8x^{-1}$ and $f(2) = 11$.
- 82.** $f(x) = x^4 - x^2 + C$. Substituting $x = 1$ gives $C = 5$, hence $f(x) = x^4 - x^2 + 5$ and $f(3) = 77$.
- 83.** $A = \int_0^3 (x^2 + 2) dx \approx 15$ square units.
- 84.** $A = \int_1^4 (2e^{0.3x}) dx \approx 13.1$ square units.
- 85.** $A = \int_0^5 (\sqrt{x+1}) dx \approx 9.13$ square units.
- 86.** $A = \int_0^\pi (4 + \sin x) dx \approx 14.6$ square units.
- 87.** $\int_0^k (2x + 1) dx = 20$. Equivalently $k^2 + k = 20$, so $(k + 5)(k - 4) = 0$ and $k = 4$.
- 88.** $a \int_0^2 (x^2 + 1) dx = 14$. The integral is $14/3$, so $a = 3$.
- 89.** $f'(x) = 6(x+2)(x-1)$ has signs $+, -, +$. Maximum $(-2, 23)$; minimum $(1, -4)$. Decreasing for $-2 < x < 1$.
- 90.** $f'(x) = 6(x+1)(x-3)$ has signs $+, -, +$. Maximum $(-1, 12)$; minimum $(3, -52)$. Decreasing for $-1 < x < 3$.
- 91.** $f'(x) = 6(x-1)(x-4)$ has signs $+, -, +$. Maximum $(1, 12)$; minimum $(4, -15)$. Decreasing for $1 < x < 4$.
- 92.** $f'(x) = 6(x+3)(x-2)$ has signs $+, -, +$. Maximum $(-3, 85)$; minimum $(2, -40)$. Decreasing for $-3 < x < 2$.
- 93.** $f'(x) = 2x - 16/x^2$. Solving $f' = 0$ gives $x \approx 2$, $y \approx 12$. The derivative changes from negative to positive, confirming a minimum.

- 94.** $f'(x) = 2x - 54/x^2$. Solving $f' = 0$ gives $x \approx 3$, $y \approx 27$. The derivative changes from negative to positive, confirming a minimum.
- 95.** GDC stationary point: $x \approx 2$, $f(x) \approx 0.736$. Endpoint values are $f(0) = 0$, $f(8) \approx 0.147$; both are smaller, so this is the greatest value on the domain.
- 96.** GDC stationary point: $x \approx 2$, $f(x) \approx 0.25$. Endpoint values are $f(0) = 0$, $f(8) \approx 0.118$; both are smaller, so this is the greatest value on the domain.
- 97.** Stationary values occur at $x = \pm 1$. Candidate values: $f(-2) = -2$, $f(-1) = 2$, $f(1) = -2$, $f(3) = 18$. Greatest 18; least -2 .
- 98.** Stationary values occur at $x = \pm 1$. Candidate values: $f(-2) = -2$, $f(-1) = 2$, $f(1) = -2$, $f(4) = 52$. Greatest 52; least -2 .
- 99.** Let the two perpendicular sides be x ; the third is $54 - 2x$. $A = 54x - 2x^2$, $0 < x < 27$. $A' = 54 - 4x = 0$ at $x = 13.5$; derivative changes $+$ $\rightarrow$ $-$. Dimensions 13.5 m by 27 m; maximum 364.5 m^2 .
- 100.** Constraint $2x + 3y = 72$, so $A = x(72 - 2x)/3 = 24x - \frac{2}{3}x^2$. Maximum at $A' = 24 - 4x/3 = 0$, giving $x = 18$, $y = 12$; total area 216 m^2 .
- 101.** $h = 125/x^2$ and $S = 2x^2 + 500/x$. Solve $S' = 4x - 500/x^2 = 0$: $x = 5$, $h = 5$. The derivative changes $- \rightarrow +$; minimum area 150 cm^2 .
- 102.** $V = x(20 - 2x)(12 - 2x)$, $0 < x < 6$. Solve $V' = 12x^2 - 128x + 240 = 0$; valid root $x = (16 - 2\sqrt{19})/3 \approx 2.43$. Dimensions $15.145 \times 7.145 \times 2.427 \text{ cm}$; maximum $\approx 263 \text{ cm}^3$.
- 103.** $h = 250/(\pi r^2)$, $S = 2\pi r^2 + 500/r$. Solve $S' = 4\pi r - 500/r^2 = 0$: $r = (125/\pi)^{1/3} \approx 3.41 \text{ cm}$, $h = 2r \approx 6.83 \text{ cm}$. The derivative changes $- \rightarrow +$.
- 104.** Printed width x , height $180/x$. $A = (x + 2)(180/x + 4) = 188 + 4x + 360/x$. $A' = 4 - 360/x^2 = 0$ at $x = \sqrt{90}$. Minimum area $188 + 16\sqrt{22.5} \approx 264 \text{ cm}^2$; outer dimensions $\approx 11.5 \text{ cm}$ by 23.0 cm .
- 105.** $P = q(60 - 0.2q) - (12q + 300) = -0.2q^2 + 48q - 300$. $P' = 48 - 0.4q = 0$ gives $q = 120$; price €36; maximum profit €2580.
- 106.** The continuous optimizer solves $-6n + 112 = 0$, so $n = 56/3 \approx 18.7$. Compare $P(18) = 964$ and $P(19) = 965$. Greatest integer profit is €965 at $n = 19$.
- 107.** $A = 2x(18 - x^2)$, $0 < x < \sqrt{18}$. $A' = 36 - 6x^2 = 0$ gives $x = \sqrt{6}$ and $y = 12$. Maximum $24\sqrt{6} \approx 58.8$ square units.
- 108.** $C' = 2 - 98/x^2 = 0 \Rightarrow x = 7$. The derivative changes $- \rightarrow +$; minimum cost $C(7) = 43$.
- 109.** GDC maximum at $x = 5.00$, where $B \approx 191$. Check endpoints: $B(0) = -30$, $B(15) \approx 59.6$; both are smaller. No analytical exponential differentiation is required.
- 110.** $C' = 0.02 - 800/q^2 = 0 \Rightarrow q = 200$. The derivative changes $- \rightarrow +$; minimum average cost $C(200) = 12$. The output is positive and an integer.
- 111.** $T = \frac{3}{2}[0 + 2(4 + 6 + 5) + 0] = 45 \text{ m}^2$. The boundary is approximated by straight segments between measurements.
- 112.** $T_2 = \frac{2}{2}[1 + 2\sqrt{3} + \sqrt{5}] \approx 6.70$. $T_4 = \frac{1}{2}[1 + 2(\sqrt{2} + \sqrt{3} + 2) + \sqrt{5}] \approx 6.76$. Integral ≈ 6.787 ; four strips give the closer estimate.
- 113.** $30 = \frac{2}{2}[2 + 2(k + 6) + 4] = 18 + 2k$, so $k = 6$.
- 114.** $T = \frac{2}{2}[5 + 2(7 + 8 + 6) + 4] = 51 \text{ litres}$. Final volume = $120 + 51 = 171 \text{ litres}$.
- 115.** $h = 0.5$, $T = \frac{0.5}{2}[1 + 2(e^{0.5} + e^1 + e^{1.5}) + e^2] \approx 6.52$. Integral $e^2 - 1 \approx 6.38906$; percentage error $\approx 2.07\%$.
- 116.** $h = \pi/4$; $T = \frac{\pi}{8}[0 + 2(\frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2}) + 0] = \frac{\pi}{4}(1 + \sqrt{2}) \approx 1.90$. Underestimate by ≈ 0.104 .
- 117.** $T_4 = \frac{1}{2}[1 + 2(3 + 4 + 3) + 1] = 11$. $T_2 = \frac{2}{2}[1 + 2(4) + 1] = 10$. Values between the sample points are unknown, so the true area is not determined by the table.
- 118.** $T = \frac{2}{2}[2 + 2(3 + 4 + 3) + 2] = 24 \text{ m}^2$. Maximum change = $\frac{2}{2}(0.1 + 6(0.1) + 0.1) = 0.8$. Bounds: 23.2 and 24.8 m^2 ; these bound the estimate, not necessarily the true area.
- 119.** Half-area = $\frac{1}{2}[4 + 2(3.5 + 2.5) + 0] = 8$. Total area ≈ 16 square units.
- 120.** Correct $T = \frac{1}{2}[0 + 2(4 + 6 + 4) + 0] = 14$. For example, counting the middle ordinate twice inside the interior sum gives $\frac{1}{2}[2(4 + 6 + 6 + 4)] = 20$.

3. IB exam-style practice — Questions 121–200

- 121.** $f'(x) = 6x^2 - 6x + 4$. At $x = 1$, $y = 4$, $m_t = 4$. Tangent $y = 4x$. Since $m_n = -1/m_t = -\frac{1}{4}$, normal $y = -\frac{1}{4}x + \frac{17}{4}$.
- 122.** $f'(x) = 3x^2 + 3$. At $x = 2$, $y = 12$, $m_t = 15$. Tangent $y = 15x - 18$. Since $m_n = -1/m_t = -\frac{1}{15}$, normal $y = -\frac{1}{15}x + \frac{182}{15}$.
- 123.** $f'(x) = 4x - 4x^{-2}$. At $x = 2$, $y = 10$, $m_t = 7$. Tangent $y = 7x - 4$. Since $m_n = -1/m_t = -\frac{1}{7}$, normal $y = -\frac{1}{7}x + \frac{72}{7}$.
- 124.** $f'(x) = 4x^3 - 4x$. At $x = 2$, $y = 13$, $m_t = 24$. Tangent $y = 24x - 35$. Since $m_n = -1/m_t = -\frac{1}{24}$, normal $y = -\frac{1}{24}x + \frac{157}{12}$.
- 125.** $a(1)^3 + b(1) + 4 = 7$ and $3a(1)^2 + b = 5$. Solving gives $a = 1$, $b = 2$. The normal has gradient $-\frac{1}{5}$ and equation $y = -\frac{1}{5}x + \frac{36}{5}$.
- 126.** $a(2)^3 + b(2) + 4 = 18$ and $3a(2)^2 + b = 23$. Solving gives $a = 2$, $b = -1$. The normal has gradient $-\frac{1}{23}$ and equation $y = -\frac{1}{23}x + \frac{416}{23}$.
- 127.** $a(1)^3 + b(1) + 4 = 8$ and $3a(1)^2 + b = 10$. Solving gives $a = 3$, $b = 1$. The normal has gradient $-\frac{1}{10}$ and equation $y = -\frac{1}{10}x + \frac{81}{10}$.
- 128.** $a(2)^3 + b(2) + 4 = 22$ and $3a(2)^2 + b = 17$. Solving gives $a = 1$, $b = 5$. The normal has gradient $-\frac{1}{17}$ and equation $y = -\frac{1}{17}x + \frac{376}{17}$.
- 129.** $f'(x) = -18/x^2$, giving $m = -2$ at $(3, 6)$. Tangent $y = -2x + 12$. Intercepts $(6, 0)$ and $(0, 12)$. Area $= \frac{1}{2}(6)(12) = 36$ square units.
- 130.** $f'(x) = -20/x^2$, giving $m = -5$ at $(2, 10)$. Tangent $y = -5x + 20$. Intercepts $(4, 0)$ and $(0, 20)$. Area $= \frac{1}{2}(4)(20) = 40$ square units.
- 131.** $f(3) \approx 3.82$, $f'(3) \approx 0.364$. Tangent $y - 3.82 = 0.364(x - 3)$. Local estimate $f(3.1) \approx f(3) + 0.1f'(3) \approx 3.86$.
- 132.** $f(1) \approx 6$, $f'(1) \approx 0.25$. Tangent $y - 6 = 0.25(x - 1)$. Local estimate $f(1.1) \approx f(1) + 0.1f'(1) \approx 6.02$.
- 133.** $f'(x) = 3(x + 2)(x - 2)$. Signs are $+$, $-$, $+$: maximum $(-2, 26)$, minimum $(2, -6)$. Increasing for $x < -2$ and $x > 2$.
- 134.** $f'(x) = 3(x + 1)(x - 2)$. Signs are $+$, $-$, $+$: maximum $(-1, \frac{17}{2})$, minimum $(2, -5)$. Increasing for $x < -1$ and $x > 2$.
- 135.** $f'(x) = 3(x - 1)(x - 3)$. Signs are $+$, $-$, $+$: maximum $(1, 24)$, minimum $(3, 20)$. Increasing for $x < 1$ and $x > 3$.
- 136.** $f'(x) = 3(x - 2)(x - 5)$. Signs are $+$, $-$, $+$: maximum $(2, 66)$, minimum $(5, \frac{105}{2})$. Increasing for $x < 2$ and $x > 5$.
- 137.** $F'(t) = -4t + 24$; $F'(3) = 12$ L/min, so the water volume is increasing at that rate. $F' = 0$ at $t = 6$ minutes, with signs $+$ $\rightarrow$ $-$, giving maximum 152 litres. Beyond the domain the model may fail to fit observations or predict unrealistic values.
- 138.** $F'(t) = -6t + 42$; $F'(4) = 18$ euros/day, so the profit is increasing at that rate. $F' = 0$ at $t = 7$ days, with signs $+$ $\rightarrow$ $-$, giving maximum 127 euros. Beyond the domain the model may fail to fit observations or predict unrealistic values.
- 139.** $F'(t) = -2t + 10$; $F'(2) = 6$ degrees Celsius/hour, so the greenhouse temperature is increasing at that rate. $F' = 0$ at $t = 5$ hours, with signs $+$ $\rightarrow$ $-$, giving maximum 43 degrees Celsius. Beyond the domain the model may fail to fit observations or predict unrealistic values.
- 140.** $F'(t) = -4t + 20$; $F'(3) = 8$ mg/L per hour, so the nutrient concentration is increasing at that rate. $F' = 0$ at $t = 5$ hours, with signs $+$ $\rightarrow$ $-$, giving maximum 56 mg/L. Beyond the domain the model may fail to fit observations or predict unrealistic values.
- 141.** At $x = 1$: minimum; at $x = 4$: maximum. Each follows from the derivative sign change. Increasing for $1 < x < 4$. The signs contain no function-height information.
- 142.** At $x = -2$: maximum; at $x = 3$: minimum. Each follows from the derivative sign change. Increasing for $x < -2$ and $x > 3$. The signs contain no function-height information.
- 143.** $f'(x) = 3x^2 - 12x + 9$. Candidate pairs $(0, 1)$, $(5, 21)$, $(3, 1)$, $(1, 5)$. Greatest value 21; least 1. Endpoints can exceed or fall below interior local extrema.

- 144.** $f'(x) = 4x^3 - 16x$. Candidate pairs $(-1, -4)$, $(3, 12)$, $(2, -13)$, $(0, 3)$. Greatest value 12; least -13 . Endpoints can exceed or fall below interior local extrema.
- 145.** $f(x) = 3x^2 + C$. The boundary condition gives $C = 4$, so $f(x) = 3x^2 + 4$. The function is positive on the interval; $A = \int_0^2 (3x^2 + 4) dx = 16$ square units.
- 146.** $f(x) = x^3 + x + C$. The boundary condition gives $C = 3$, so $f(x) = x^3 + x + 3$. The function is positive on the interval; $A = \int_1^3 (x^3 + x + 3) dx = 30$ square units.
- 147.** $f(x) = x^2 + 4x + C$. The boundary condition gives $C = 6$, so $f(x) = x^2 + 4x + 6$. The function is positive on the interval; $A = \int_0^2 (x^2 + 4x + 6) dx = \frac{68}{3}$ square units.
- 148.** $f(x) = 2x^3 + x^2 + C$. The boundary condition gives $C = 2$, so $f(x) = 2x^3 + x^2 + 2$. The function is positive on the interval; $A = \int_0^2 (2x^3 + x^2 + 2) dx = \frac{44}{3}$ square units.
- 149.** $A_R = \int_0^3 (x^2 + 2) dx = 15$. Triangle area $= \frac{1}{2}(3)(11) = \frac{33}{2}$. Ratio $= \frac{10}{11}$.
- 150.** $A_R = \int_0^2 (x^2 + 3) dx = \frac{26}{3}$. Triangle area $= \frac{1}{2}(2)(7) = 7$. Ratio $= \frac{26}{21}$.
- 151.** $A_R = \int_0^4 (x^2 + 4) dx = \frac{112}{3}$. Triangle area $= \frac{1}{2}(4)(20) = 40$. Ratio $= \frac{14}{15}$.
- 152.** $A_R = \int_0^2 (x^2 + 5) dx = \frac{38}{3}$. Triangle area $= \frac{1}{2}(2)(9) = 9$. Ratio $= \frac{38}{27}$.
- 153.** Total area $= \int_0^3 (2x + 1) dx = 12$. Require $\int_0^k (2x + 1) dx = 6$, so $k^2 + k = 6$. Positive solution $k \approx 2$.
- 154.** Total area $= \int_0^4 (2x + 1) dx = 20$. Require $\int_0^k (2x + 1) dx = 10$, so $k^2 + k = 10$. Positive solution $k \approx 2.7$.
- 155.** $\int_0^2 (x^2 + c) dx = 20$ gives $\frac{8}{3} + 2c = 20$, so $c = \frac{26}{3}$. Since $c > 0$ and $x^2 \geq 0$, $f(x) > 0$.
- 156.** $\int_1^3 (x^2 + c) dx = 36$ gives $\frac{26}{3} + 2c = 36$, so $c = \frac{41}{3}$. Since $c > 0$ and $x^2 \geq 0$, $f(x) > 0$.
- 157.** $A = \int_0^8 (6e^{-0.25x}) dx \approx 20.8 \text{ cm}^2$. Cost $\approx \text{€}1.66$. Each thin strip has approximate area $w(x) dx$, with dimensions cm by cm.
- 158.** $A = \int_0^6 (3 + 2 \sin(\pi x/6)) dx \approx 25.6 \text{ cm}^2$. Cost $\approx \text{€}2.05$. Each thin strip has approximate area $w(x) dx$, with dimensions cm by cm.
- 159.** $P = q(70 - \frac{1}{4}q) - (10q + 200) = -\frac{1}{4}q^2 + 60q - 200$; $0 \leq q \leq 280$. $P' = 60 - \frac{1}{2}q = 0$ at $q = 120$. Derivative changes $+$ $\rightarrow$ $-$; price $\text{€}40$, profit $\text{€}3400$.
- 160.** $P = q(90 - \frac{1}{2}q) - (18q + 500) = -\frac{1}{2}q^2 + 72q - 500$; $0 \leq q \leq 180$. $P' = 72 - q = 0$ at $q = 72$. Derivative changes $+$ $\rightarrow$ $-$; price $\text{€}54$, profit $\text{€}2092$.
- 161.** $V = 4x^3 - 60x^2 + 216x$, with $0 < x < 6$. $V' = 12x^2 - 120x + 216$. The feasible zero is $x \approx 2.35$ cm. V' changes $+$ $\rightarrow$ $-$ there; maximum volume $\approx 228 \text{ cm}^3$.
- 162.** $V = 4x^3 - 78x^2 + 360x$, with $0 < x < 7.5$. $V' = 12x^2 - 156x + 360$. The feasible zero is $x \approx 3$ cm. V' changes $+$ $\rightarrow$ $-$ there; maximum volume $\approx 486 \text{ cm}^3$.
- 163.** $h = 360/(\pi r^2)$; $S = 1\pi r^2 + 720/r$. $S' = 2\pi r - 720/r^2 = 0$ gives $r \approx 4.86$ cm, $h \approx 4.86$ cm. The derivative changes $-$ $\rightarrow$ $+$; minimum $S \approx 222 \text{ cm}^2$.
- 164.** $h = 500/(\pi r^2)$; $S = 2\pi r^2 + 1000/r$. $S' = 4\pi r - 1000/r^2 = 0$ gives $r \approx 4.3$ cm, $h \approx 8.6$ cm. The derivative changes $-$ $\rightarrow$ $+$; minimum $S \approx 349 \text{ cm}^2$.
- 165.** $A = 30x - 2x^3$, $0 < x < \sqrt{15}$. $A' = 30 - 6x^2 = 0$ gives $x = \sqrt{5} \approx 2.24$. Width ≈ 4.47 , height 10; maximum area ≈ 44.7 . Area tends to zero at both boundaries.
- 166.** $A = 54x - 2x^3$, $0 < x < \sqrt{27}$. $A' = 54 - 6x^2 = 0$ gives $x = \sqrt{9} \approx 3$. Width ≈ 6 , height 18; maximum area ≈ 108 . Area tends to zero at both boundaries.
- 167.** $A = (x + 2)(240/x + 4) = 248 + 4x + 480/x$. $A' = 4 - 480/x^2 = 0$, so printed dimensions $x \approx 11$ cm, $y \approx 21.9$ cm. The sign changes $-$ $\rightarrow$ $+$; total area $\approx 336 \text{ cm}^2$.
- 168.** $A = (x + 4)(300/x + 6) = 324 + 6x + 1200/x$. $A' = 6 - 1200/x^2 = 0$, so printed dimensions $x \approx 14.1$ cm, $y \approx 21.2$ cm. The sign changes $-$ $\rightarrow$ $+$; total area $\approx 494 \text{ cm}^2$.
- 169.** $2x + 3y = 84$, so $A = x(84 - 2x)/3$, $0 < x < 42$. The maximum occurs at $A' = (84 - 4x)/3 = 0$, giving $x = 21$ m and $y = 14$ m. The derivative changes $+$ $\rightarrow$ $-$; maximum 294 m^2 .

- 170.** $2x + 4y = 120$, so $A = x(120 - 2x)/4$, $0 < x < 60$. The maximum occurs at $A' = (120 - 4x)/4 = 0$, giving $x = 30$ m and $y = 15$ m. The derivative changes $+$ $\rightarrow$ $-$; maximum 450 m^2 .
- 171.** $R = 2000 + 40n - 3n^2$, with $n = 0, 1, \dots, 33$. Continuous maximum at $n = 20/3$. Compare $R(6) = 2132$ and $R(7) = 2133$. Thus $n = 7$, price €27, revenue €2133.
- 172.** $x^2 + 4xh = 192$, so $h = (192 - x^2)/(4x)$ and $V = 48x - x^3/4$, $0 < x < \sqrt{192}$. $V' = 48 - 3x^2/4 = 0 \Rightarrow x = 8$, $h = 4$. Derivative changes $+$ $\rightarrow$ $-$; maximum 256 cm^3 .
- 173.** A GDC gives $x \approx 5.07$, with maximum $R \approx \text{€}257$. Expenditure is €50.71 (using the unrounded optimizer). Endpoints: $R(0) = 0$, $R(30) = -131.25$, both smaller.
- 174.** $A(x) = 12xe^{-0.3x}$, $x > 0$. GDC maximum at $x = 10/3 \approx 3.33$; height $12/e \approx 4.41$. Maximum area $40/e \approx 14.7$ square units. The area rises then falls around the maximum.
- 175.** (a) $h = 5$ m. (b) $T = \frac{5}{2}[0 + 2(6 + 9 + 7) + 0] = 110 \text{ m}^2$. (c) $0.08(110) = 8.80$ kg.
- 176.** (a) $1, \frac{2}{3}, \frac{1}{2}, \frac{2}{5}, \frac{1}{3}$. (b) $T = \frac{1}{4}[1 + 2(\frac{2}{3} + \frac{1}{2} + \frac{2}{5}) + \frac{1}{3}] = \frac{67}{60} \approx 1.12$. (c) Integral ≈ 1.09861 ; error $\approx 1.65\%$.
- 177.** (a) $T = \frac{1}{2}[2 + 2(3 + 5 + 4) + 2] = 14$ kWh. (b) Cost = €3.36. (c) Power may vary non-linearly between measurements.
- 178.** (a) $46 = \frac{2}{2}[3 + 2(k + 8 + k) + 5] = 24 + 4k$. (b) $k = 5.5$. (c) Increase $= \frac{2}{2}(0.6) = 0.6$; revised estimate 46.6.
- 179.** (a) $T = \frac{1}{2}[0.8 + 2(1.2 + 1.6 + 1.0) + 0.4] = 4.4$ mg. (b) 6.9 mg. (c) $6.9/V \leq 0.05$, so $V \geq 138$ mL.
- 180.** (a) $T = \frac{2}{2}[0 + 2(3 + 5 + 4 + 2) + 0] = 28 \text{ cm}^2$. (b) $1.5^2(28) = 63 \text{ cm}^2$. (c) Area scales with the square of the length scale factor.
- 181.** (a) Ordinates 9, 8, 5, 0; $T = \frac{1}{2}[9 + 2(8 + 5) + 0] = 17.5$. (b) Chords lie below this concave-down curve. (c) $\frac{0.5}{18} \times 100\% = 2.78\%$.
- 182.** (a) $h = 0.5$. (b) $T = \frac{0.5}{2} \left[5 + 2 \sum_{i=1}^5 5e^{-0.2i} + 5e^{-1.2} \right] \approx 8.76$. (c) Integral ≈ 8.73507 ; $T \approx 8.76417$ is too large.
- 183.** (a) $h = 0.25$ hour. (b) $T = \frac{0.25}{2}[0 + 2(12 + 24 + 18) + 6] = 14.25$ mm. (c) $80(14.25) = 1140$ litres.
- 184.** (a) $T = \frac{2}{2}[0 + 2(3 + 4 + 3) + 0] = 20 \text{ m}^2$. (b) $V \approx 6(20) = 120 \text{ m}^3$. (c) 120 000 litres.
- 185.** (a) Estimate $= \frac{2}{2}[18 + 2(24 + 27) + 21] = 141$ °C · h. (b) Mean $= 141/6 = 23.5$ °C.
- 186.** (a) Six strips give width $6(0.5) = 3$. (b) Increase $= h(0.8) = 0.4$; 13.0. (c) Increase $= \frac{h}{2}(0.8) = 0.2$; 12.8.
- 187.** (a) Use $x = 0, 2, 4, 6$, with ordinates 1, 4, 5, 3. (b) $T = \frac{2}{2}[1 + 2(4 + 5) + 3] = 22$. (c) The five available points are not spaced 1.5 units apart.
- 188.** (a) $156 = \frac{h}{2}[0 + 2(8 + 11 + 7) + 0] = 26h$, so $h = 6$ m. (b) Length $4h = 24$ m. (c) $\lceil 156/20 \rceil = 8$ bags.
- 189.** $V' = 12 - 2t$; $V'(2) = 8$ L/min, so the volume is increasing. The derivative changes $+$ $\rightarrow$ $-$ at $t = 6$, giving $V(6) = 76$ L (endpoints: 40, 60 L). Since $V(2) = 60$, the tangent estimate is $60 + 8(0.1) = 60.8$ L.
- 190.** $P = -0.2q^2 + 50q - 400$, $0 \leq q \leq 300$. $P' = 50 - 0.4q$ changes $+$ $\rightarrow$ $-$ at $q = 125$. Price €35; profit €2725. GDC roots are $q \approx 8.27, 241.73$; positive profit for approximately $8.27 < q < 241.73$ (integer outputs 9 to 241).
- 191.** $f = x^3 - 3x^2 + C$; $5 = 1 - 3 + C$, so $C = 7$. $f' = 3x(x - 2)$ has signs $+$, $-$, $+$: maximum $(0, 7)$, minimum $(2, 3)$. The curve stays positive on $[0, 3]$, so area $\int_0^3 (x^3 - 3x^2 + 7) dx = 57/4 = 14.25$.

x	$-\infty$	0	2	$+\infty$		
$f'(x)$		+	0	-	0	+
$f(x)$			Max	Min		

- 192.** $f' = 2x - 4/x^2 = 0$ gives $x = \sqrt[3]{2} \approx 1.26$, $f(x) \approx 4.76$; signs $- \rightarrow +$. Ordinates 5, 6, $31/3$, 17 give $T = 82/3 \approx 27.3$. GDC integral ≈ 26.54518 ; percentage error $100|T - I|/I \approx 2.97\%$.
- 193.** $11 = 4k + 3$, so $k = 2$. $f' = 4x$; tangent $y - 11 = 8(x - 2)$, or $y = 8x - 5$. Area $= \int_0^2 (2x^2 + 3) dx = 34/3$. Ordinates 3, 5, 11 give $T = (3 + 10 + 11)/2 = 12$; percentage error $100(12 - 34/3)/(34/3) \approx 5.88\%$.

- 194.** $T = \frac{2}{2}[0 + 2(3 + 5 + 4) + 0] = 24 \text{ m}^2$. The interior correction adds $h(0.4) = 0.8 \text{ m}^2$, giving 24.8 m^2 . Cost €235.60. The boundary between measured positions is still approximated by straight segments.
- 195.** $C'(100) \approx (700.4001 - 699.6001)/0.2 = 4$ euros per item: locally, one more item adds about €4 to cost. Tangent $C \approx 700 + 4(q - 100)$; $C(101) \approx 704$ euros. A tangent is a local approximation, and the model can curve substantially over a large interval.
- 196.** The width is $48 - 2x$, so $A = x(48 - 2x)$, $0 < x < 24$. $A' = 48 - 4x$ changes $+$ $\rightarrow$ $-$ at $x = 12$: width 24 m and area 288 m^2 . On $0 < x \leq 10$, $A' > 0$, so the constrained maximum is at $x = 10$: width 28 m and area 280 m^2 .
- 197.** GDC derivative $w'(2) \approx -0.359$. Ordinates approximately 5.00000, 4.34064, 3.89866, 3.60239, 3.40379. Using unrounded ordinates gives $T \approx 16 \text{ m}^2$. Integral $I \approx 16 \text{ m}^2$; overestimate by 0.0531 m^2 (about 0.332%).
- 198.** $f' = 3x^2 - 6x + 3 = 3(x - 1)^2$. At $x = 1$, $f(1) = 2$. The derivative is positive on both sides: no local extremum. Tangent $y = 2$; normal $x = 1$. Since f is positive on $[0, 2]$, area $\int_0^2 f(x) dx = 4$.
- 199.** $P' = -2aq + 60$; $P'(10) = 0$ gives $a = 3$. The derivative changes $+$ $\rightarrow$ $-$ at 10, giving maximum €200. Roots of $-3q^2 + 60q - 100 = 0$ are $10 \pm \frac{10\sqrt{6}}{3}$, approximately 1.84, 18.2. Profit is positive between these roots.
- 200.** $w' = 0.5 - 0.1x = 0$ at $x = 5$, giving maximum width 3.25 m. $I = \int_0^8 w(x) dx = \frac{352}{15} \approx 23.5 \text{ m}^2$. Ordinates 2, 2.8, 3.2, 3.2, 2.8 give $T = \frac{116}{5} = 23.2 \text{ m}^2$. Costs: €290.00 and €293.33; the trapezoidal estimate is €3.33 lower.